

Collective processes in collisional dusty plasmas of planetary rings

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The presence of dust in many astrophysical objects has been known for a long time and dust particles immersed in ambient plasmas and radiative environments become electrically charged by variety of processes. The heavy plasma component modifies the plasma modes and also leads to the existence of the extremely low-frequency wave perturbations associated with dust dynamics. A variety of laboratory experiments confirmed the existence of such dust-density wave modes and suggested useful approaches to plasma diagnostics. Typical peculiarities of the wave experiments are the following: (i) laboratory complex plasmas are usually weakly ionized and thus strongly collisional, and ion-neutral, ion-dust and dust-neutral collisions become essential factors modifying the dust modes; (ii) the plasma electrons and ions stream through the more inertial microparticles due to the discharge electric fields, increasing the role of the ion drag force and leading to various kind of instabilities. Space dusty plasmas often exhibit many features in common with laboratory complex plasmas. For example, the dusty plasma of planetary rings consists of dust particles imbedded in an electron-ion plasma. Measurements made by the Cassini Plasma Spectrometer Instrument over the main (A and B) rings confirmed the existence of an oxygen ionosphere/plasma in the direct vicinity of Saturn's rings. This plasma is predicted to be weakly ionized and one can then expect that the collisions of charged particles with neutrals cannot be ignored. Furthermore, the dust particles exhibit an azimuthal drift relative to the plasma. Fast relative motion and existence of the ring ionosphere create the necessary prerequisites for the plasma drag force to become a significant factor affecting low-frequency collective modes in the dusty plasma of the planetary rings. Therefore it is necessary to investigate the role of dissipative processes in dust plasma waves developing on timescales comparable with collisional time scales.

We deal with a four-component collisional dusty plasma, consisting of plasma electrons (subscript e), ions (i), neutrals (n) and charged dust particles (d). For simplicity, we consider waves propagating along the ring and assume that that only electrons and ions are subjected to the planetary magnetic field and co-rotate with the planet, with equilibrium velocities $v_{e0,i0} = \Omega_p r$, with Ω_p being the planetary rotation frequency and r is the radial distance from the planet. The dust particles move around the planet with Keplerian velocity $v_{d0} = \sqrt{GM_p/r}$ (G is the gravitational constant, M_p denotes the mass of the planet). The two velocities become equal at the

synchronous orbit r^* , at distances $r < r^*$, the dust grains move faster than the electrons and ions, while for $r > r^*$, the plasma overtakes the dust particles. The relative velocity $\Delta v = |v_{i0} - v_{d0}|$ varies in a wide range from zero at $r = r^*$ to significant values of the order of the ion thermal velocity $\Delta v \sim 5 \times 10^5 - 10^6$ cm/s at the boundary of the main rings, thus leading to the ion relative velocities $U = |v_{i0} - v_{d0}|/v_{Ti} \leq 1$.

If there is a relative motion between the dust and plasma components, there inevitably arises a force related to the exchange of momentum transfer from a flowing plasma to charged dust grains. Because of their much larger masses, ions mainly contribute to this drag force. Recently, there has been a considerable theoretical development in modeling of the ion drag force (see [1]). For typical plasma parameters in the vicinity of the rings, the ion drag force is mainly due to the collection of ions by the charged grain, $F_i \simeq v_{di}v_i$, where the dust-ion momentum-transfer frequency is determined by $v_{di} \simeq (8\sqrt{2\pi}/3)a^2n_im_iv_{Ti}/m_d$ and the ion-dust momentum-transfer frequency v_{id} can be approximated as $v_{id} \simeq (8\sqrt{2\pi}/3)a^2n_dv_{Ti}$.

The dust density modes of low-frequencies are derived from the standard fluid approach and model presentation of the drag forces relevant to the parameter regime of the dusty plasma of planetary rings. Note that the most estimation of the plasma parameters is based on the observations of the Cassini spacecraft in the vicinity of the main rings (see Table 1 in [2]). Typical relations between the plasma parameters and transition to the frame of reference moving with the dust grains allow us to obtain the dispersion law describing small low-frequency wave perturbations as

$$1 - \frac{k^2 u_{ia}^2}{L_i} - \frac{k^2 u_{da}^2}{(\omega - kv_{d0})(\omega - kv_{d0} + iv_{dn})} + ik^2 u_{da}^2 v_i p^{-1} \times \frac{(\omega - kv_{i0})}{(\omega - kv_{d0})(\omega - kv_{d0} + iv_{dn}) L_i} = 0, \quad (1)$$

where $L_i = \omega^2 + i\omega(v_i + v_{in}) - k^2 v_{Ti}^2$, an ion-sound velocity is given by $u_{ia} = \lambda_{De} \omega_{pi} \simeq v_{Ti}$ (as a result of the assumption $T_e \simeq T_i, n_{e0} \simeq n_{i0}$) and a dust-sound velocity is $u_{da} = \lambda_{De} \omega_{pd} \simeq \lambda_{Di} \omega_{pd}$. The appearance of the last term in the dispersion relation (1) is new, resulting from the coupling between perturbations of the ion and dust particle populations (densities) due to the ion drag force. It should be noted that this contribution (due to the momentum exchange between the dust component and plasma ions) can be positive or negative, depending on the distance from the planet and the relation between Δv and v_{Ti} . As a result one can expect the existence of spatially restricted domains where the dust-wave be generated.

In the simplest case of the synchronous orbit $r \sim r^*$, where $v_{d0} \simeq v_{i0}$, one finds in the low

frequency regime

$$\Re\omega \simeq ku_{da}/\sqrt{2}, \quad \Im\omega \simeq \frac{u_{da}^2(2v_i - v_{in}p)}{8v_{Ti}^2p} - \frac{v_{dn}}{2}. \quad (2)$$

To derive these expressions we have used the plausible assumptions $v_{Ti}^2 \gg u_{da}^2$ and $p \ll 1$. The obtained solution describes the usual dust-acoustic mode, which can be unstable if

$$v_i > v_{cr} = p(2v_{Ti}^2v_{dn}/u_{da}^2 + v_{in}/2) \simeq 2v_{dn}pv_{Ti}^2/u_{da}^2, \quad (3)$$

which can be simplified to $v_i > v_{cr} \simeq 2v_{dn}m_d/(m_iZ_d)$. This criterion is limited by rather specific conditions: it requires high values of $s = \sqrt{T_i/T_n} > 1$ and $p_n = Z_dn_{d0}/n_n \leq 1$. However, typically $p_n \ll 1$ and $s \geq 1$, so that the dust-acoustic instability due to the ion drag force can hardly develop directly on the synchronous orbit.

To explore the influence of a finite difference in equilibrium velocities on the low-frequency solutions (1), we consider the case when $\Delta v = v_{i0} - v_{d0} < v_{Ti}$ and use the ordering $v_{dn} \ll \omega \ll k\Delta v$. One then easily finds that the dust-acoustic mode drifting along with the dust flow obeys

$$\Re\omega \simeq ku_{da}/\sqrt{2}, \quad \Im\omega \simeq -\frac{\sqrt{2}v_iu_{da}\Delta v}{4v_{Ti}^2p} - \frac{v_{dn}}{2}. \quad (4)$$

For these perturbations, the instability due to the ion drag force can only occur in that part of the planetary rings where the ions move slower than the dust grains, i.e. inside the corotation distance, at orbits $r < r^*$, giving $\Delta v < 0$. The instability threshold is now "reduced" to $v_{cr} = \sqrt{2}v_{Ti}^2pv_{dn}/(u_{da}\Delta v)$, which is $(u_{da}/\Delta v \gg 1)$ times lower than (2), and thus can be easily achieved.

A further growth of the relative dust-plasma velocity, $|\Delta v| = |v_{i0} - v_{d0}| \sim v_{Ti}$ will result in unstable (damped) modifications of the dust-acoustic perturbations, $\Re\omega \sim |\Im\omega|$, developing with

$$\Im\omega \simeq \pm ku_{da} \frac{\sqrt{v_i}}{\sqrt{2pkv_{Ti}}}. \quad (5)$$

These expressions were derived using the reasonable assumptions that $\Re\omega, \Im\omega > v_{dn}$ and $k|\Delta v| > v_{in}, v_i$. The instability (+) due to the ion drag force can only develop for dust orbiting at $r \sim r_T < r^*$, where $-\Delta v(r_T) \sim v_{Ti}$, the reversed position outside the corotation orbit $r \sim 2r^* - r_T > r^*$ (here the ions move faster than the dust grains and thus provide $\Delta v(r) \sim v_{Ti}$) corresponding to the damped mode ("-" sign in Eq. (5)).

Finally, for larger relative dust-plasma velocities, when $\Delta v > v_{Ti}$ or even $\Delta v \gg v_{Ti}$, and $k|\Delta v| > v_{in}, v_i$, one obtains the approximate dust-acoustic solution of (1) in the form

$$\Re\omega \simeq \frac{ku_{da}\Delta v}{(\Delta v^2 - 2v_{Ti}^2)^{1/2}}, \quad \Im\omega \simeq -\frac{v_{dn}}{2} + \frac{1}{2} \frac{u_{da}v_i}{p\Delta v}. \quad (6)$$

As can be seen, the ion-dust collisions can trigger the dust-acoustic instability in the region $r \gg r^*$ (where $\Delta v > 0$) when, on the one hand, the velocity difference is large enough so that $\Delta v(r) > v_{Ti}$, but on the other hand, the unstable threshold requires $\Delta v(r) < v_i u_{da} / (v_{dn} p)$. For typical dusty plasma parameters in the planetary rings, $v_i u_{da} / (v_{dn} p) \geq 10^8$ cm/s, and the instability condition from above is always satisfied. Therefore, the domain of unstable solutions in application to the planetary rings is only restricted from below by the condition $\Delta v(r) > v_{Ti}$ and thus could be of importance for remote Saturn's rings (like E and G).

We have demonstrated that the ion drag force related to the momentum transfer from the moving ions to the charged dust grains can affect stability of the low-frequency dust-density perturbations even in the direct vicinity of the synchronous orbit: inside the co-rotation distance (where the plasma ions move slower than the dust grains) the plasma drag force can be responsible for the excitation of dust-acoustic perturbations due to the dissipative instability.

The growth rate of the dissipative instability is strongly dependent on the radial distance from the planet. The neutral gas damping (associated with dust-neutral momentum transfer) can quench the instability and move the boundary for the onset of the instability inside the co-rotation distance for the negatively charged dust particles. Outside the co-rotation distance (where the plasma ions overtake the dust grains) the collisional processes lead to the damped dust density perturbations, and thus the dust-density waves can hardly be excited at this part of Saturn's rings. Our theory also predicts that the smaller the charged particles are, the larger the growth rate of the dissipative instability and the smaller the distance from the synchronous orbit for the onset of this instability. Estimations of the plasma parameters made on the basis of Cassini data allow us to suggest that the boundary for the onset of the dissipative instability is located in the direct vicinity of the synchronize orbit. It is therefore possible that the discovered instability could be of importance for the formation and evolution of Saturn's "spokes" observed in the vicinity of the synchronous orbit and formed by micron and submicron charged dust grains.

References

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