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Reformulation of Hamiltonian dynamics for dust particle interactions in complex plasma

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It is well known that the dynamics of two dust particles suspended in the complex plasma environment of the sheath cannot be completely described using the Hamiltonian formalism. The non-Hamiltonian dynamics of the particles includes non-reciprocal (positional) forces due to ion focusing, dissipative forces such as friction of the particles against stationary neutrals and spatial variations of the particle charges [1]. In particular, the non-reciprocal wake-particle interaction breaks the symmetry of the otherwise mutually repulsive Debye interaction. Symmetry can be restored if the particles are made to satisfy the symmetries broken by the non-potential part of the dust interaction [2]. The interaction is then completely described by the Hamiltonian. As the system departs from the symmetrical configuration, its dynamics becomes increasingly non-Hamiltonian. We are attempting to classify all systems which display such asymptotic Hamiltonian behaviour. A generalised reformulation of Hamiltonian dynamics is underway which exploits the remaining symmetry of the system to define a potential for the non-Hamiltonian dynamics.

The study of plasma systems containing ensembles of microparticles (dust) is a rapidly developing field of complex systems research with applications extending from solid-state physics to astrophysics [3, 4, 5]. Complex systems typically elicit several signature characteristics including scaling, criticality and universality. The multi-component mixture of plasma and dust particles is a characteristically complex system with many interwoven, strongly non-linear particle interactions [7].

One of the general features of complex plasma systems is the presence of non-conservative interaction forces between the dust particles due the dynamic interaction between the dust particles and the plasma in which they are immersed [1, 8, 9, 10, 11]. Generally, systems of charged dust particles in an anisotropic plasma with ion flow are characterized by non-reciprocal interactions when momentum is brought and taken away by the ions. Strictly speaking, such a system cannot be described by a Hamiltonian, since the energy is not conserved because of the openness of the system due to plasma-particle interaction. Under some conditions, however, (e.g., when the energy flow in and out of the system is balanced) the Hamiltonian treatment provides useful insights, as in the case of the direct analogy between the asymmetric particle interactions and Cooper pairing [13]. Furthermore, although the non-potential nature of the interaction forces makes the the system inherently non-Hamiltonian, when the system is made to satisfy the symmetries which are broken by the underlying anisotropic background field, a potential can still be defined so that for a certain set of parameters, the system is described by the Hamiltonian. When the system loses stability, the traditional Hamiltonian analysis fails and we must explicitly to analyse the dynamical equations of motion. Non-hamiltonian dynamics is of fundamental interest due to its ubiquity in complex systems, and fractional calculus has proven to be a very useful tool in the field. Since most systems in nature are non-Hamiltonian, it can be useful to redefine the Hamiltonian to include non-potential forces. The reformulation of Hamiltonian dynamics for non-Hamiltonian systems poses several theoretical challenges. For example, due to the complexity of the particle interactions, it is not clear that a relationship should always exist amongst the components, thereby allowing a potential to be defined through an appropriate differential operator. The potentialisation of a dynamical system can thus be thought of as a transfer of information from the dynamics to differential operators. When there is symmetry present in the system, as in the case of two particles, judicious choices of operators can made to help simplify the problem. To our knowledge, the inclusion of non-reciprocal interaction forces of the kind found in dusty plasma experiments has not been achieved. A reformulation of Hamiltonian dynamics using fractional derivatives [15] has been shown to encompass a wider class of systems than the conventional Hamiltonian, which is limited to non-dissipative systems. In the theory of fractional calculus, a generalized differential operator is defined by

$$\mathbf{D}_x^\alpha = \frac{1}{\Gamma(m-\alpha)} \frac{\partial^m}{\partial x^m} \int_0^x \frac{f(y)dy}{(x-y)^{\alpha-m+1}}$$

where the order of differentiation α can take on any non-negative real value, and m is defined to be the ceiling of α. By replacing the derivatives in the theory by those of non-integer order, it has been possible to incorporate the dissipative frictional force [16]. Although necessary and sufficient conditions have been developed for the identification of fractionally Hamiltonian systems, it is not known whether all dynamical systems can be described in this way.

In this presentation, we will focus on using the Hamiltonian to model non-Hamiltonian systems, using the two particle complex plasma as an example. The nature of particle arrangements in systems containing large numbers of particles will be explored by considering simplified systems of just two particles. In the complex plasma community circles, it is well known that even the interaction between two particles is highly complicated. Using the Hamiltonian of the system, we find there exist deep parallels between the dynamical instability which occurs in two particles in the presence of anisotropic ion flow, and thermodynamic phase transitions. The amazing property of critical phenomena is their universality when similar scaling appears in different systems: e.g., magnets and gases follow simple power laws for the order parameter, specific heat capacity, susceptibility, compressibility, etc. [17]. In thermodynamic systems, phase transitions take place at a critical temperature T_c when the coefficients that characterize the linear response of the system to external perturbations diverge [18]. The corresponding theory of critical phenomena has mostly been explored from the perspective of statistical thermodynamics [19]. In so-called extensive systems, the number of interacting particles is of the order of Avogadro's number, so the assumption of an infinite uniform system is justified. In non-extensive systems where the number of particles may be fewer than 10^3, the thermodynamic limit cannot be applied, since the extent of the particle interaction is comparable with the size of the system. The question of whether universal scaling also takes place in non-extensive class systems is still open. The presented analysis of the two particle system indicates that the system may belong to the Ising Universality class characterized by the set of critical exponents $\{\alpha = 0, \beta = 1/2, \gamma = 1, \delta = 3\}$. In addition, a new universality class is suggested for a critical point associated with spatial symmetry breaking induced by the anisotropic ion flow.

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