

Effect of plasma absorption on the total force acting on a dust grain in highly collisional drifting plasma

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“Complex” or “dusty” plasma consists of electron, ion, charged micron sized dust particles and neutral gas. Very often complex plasma are subjected to a large scale electric field which has a direct consequence to generate electric force on the charged grain. However presence of electric field also produce ion drag and electron drag forces which are nothing but the momentum transfer from flowing ions and electrons to the charged grain. The competition between these different types of forces are responsible to determine different types of static and dynamic properties of grains, affect wave phenomena etc. [1] In general the ion drag dominates over the electron drag force because of large ion-to-electron mass ratio. However in presence of large scale electric field the situation may change where the electrons drift much faster than ions because of its much higher mobility. It was shown in an earlier work by Khrapak and Morfill [2] that in the collisionless regime the electron drag force can indeed dominate over the electric and ion drag forces provided the electron-to-ion temperature is not too high.

In this work we have analysed the electric force, ion and electron drag forces in highly collisional regime subject to a weak electric field taking into account plasma absorption on the grain surface. The importance of plasma absorption on the grain surface has been discussed recently. [3, 4] The total force which is the sum of electric, ion drag and electron drag forces is proportional to the electric field and always act in the direction of the electric field. The proportionality constant represents the effective charge such as $\mathbf{F} = Q_{\text{eff}}\mathbf{E}$.

In our model we consider a small spherical negatively charged grain which is placed in highly collisional, weakly ionized plasma with a constant electric field \mathbf{E}_0 . The grain is stationary and absorbs plasma on its surface. The ions drift in the direction of the electric field, whereas electrons drift in the opposite direction. The electric field is sufficiently weak so that both electron and ion drifts are subthermal $M_{T_\alpha} = |u_\alpha|/v_{T_\alpha} < 1$ where $v_{T_\alpha} = \sqrt{T_\alpha/m_\alpha}$ is the thermal velocity and M_{T_α} is the thermal Mach number for the corresponding species. Here $\alpha = i(e)$ for ions (electrons). There are no plasma sources or sinks except at the grain surface, which is fully absorbing. In the highly collisional regime both the ion and electron components are suitably described by the hydrodynamic equations. The corresponding continuity and momentum equations are:

$$\nabla(n_\alpha \mathbf{v}_\alpha) = -J_\alpha \delta(\mathbf{r}), \quad (1)$$

$$(\mathbf{v}_\alpha \nabla) \mathbf{v}_\alpha = q_\alpha (e/m_\alpha) \mathbf{E} - (\nabla n_\alpha / n_\alpha) v_{T\alpha}^2 - \mathbf{v}_\alpha \nabla \mathbf{v}_\alpha, \quad (2)$$

where n_α , \mathbf{v}_α are the density and velocity of the corresponding species, $J_{i(e)}$ denotes the ion (electron) fluxes that the grain collects from the plasma and \mathbf{E} is the electric field. In Equation (2) $q_i = +1$ and $q_e = -1$. The above set of equations is closed with the Poisson equation:

$$\Delta \phi = -4\pi e(n_i - n_e) - 4\pi Q \delta(\mathbf{r}). \quad (3)$$

The self consistent electrostatic potential around the absorbing point grain has been calculated using linear dielectric response formalism. Assuming the plasma perturbation to be proportional to $\propto \exp(i\mathbf{k}\mathbf{r})$ we get [3]

$$\phi_P(\mathbf{r}) = \frac{4\pi Q}{(2\pi)^3} \int \frac{\exp(i\mathbf{k}\mathbf{r}) d\mathbf{k}}{\chi_1} + \frac{4\pi e}{(2\pi)^3} \sum_{\alpha=i,e} \int \frac{\exp(i\mathbf{k}\mathbf{r}) d\mathbf{k}}{\chi_\alpha}, \quad (4)$$

where

$$\chi_1 = k^2 \left[1 + \sum_{\alpha=i,e} \left(\frac{\omega_{p\alpha}}{\Omega_\alpha} \right)^2 \right], \quad \chi_\alpha = iq_\alpha \left[\frac{\chi_1 \Omega_\alpha^2}{J_\alpha(\mathbf{k}\mathbf{u}_\alpha - i\nu_\alpha)} \right].$$

Here $\omega_{p\alpha} = \sqrt{4\pi n_\alpha e^2 / m_\alpha}$ is the plasma frequency of the corresponding species and $\Omega_\alpha^2 = k^2 v_{T\alpha}^2 - \mathbf{k}\mathbf{u}_\alpha(\mathbf{k}\mathbf{u}_\alpha - i\nu_\alpha)$. The first term in Eq.(4) represents the potential of a non-absorbing point-like grain, [5] while the second term represents the sum of the contribution to the potential due to ion and electron absorptions respectively. [3]

From Equation (4) we get the polarization part of the total force experienced by the test grain using the relation $F_P = -Q \nabla \phi_P |_{r=0}$. This yields

$$F_P = \pi^{-1} \int_0^\infty k^3 dk \int_{-1}^1 \mu d\mu \left[Q^2 \text{Im} \{ \chi_1^{-1}(\mu, k) \} + Qe \sum_{\alpha=i,e} \text{Im} \{ \chi_\alpha^{-1}(\mu, k) \} \right], \quad (5)$$

where $\mu = \cos \theta$ and θ is the angle between \mathbf{k} and \mathbf{E}_0 . In our model we have considered hydrodynamic approach both for ions and electrons, the applicability of which requires the condition $k\ell_\alpha \ll 1$. Using the flux balance condition $J_i = J_e$ for a floating grain and small ion and electron drift velocities, $k\ell_\alpha \gg M_{T\alpha}$ we get after integration

$$F_P = (1/6) Q^2 k_{D,i}^2 (\ell_i k_D)^{-1} M_{T,i} - (1/6) Q^2 k_{D,e}^2 (\ell_e k_D)^{-1} M_{T,e} \\ + (1/6) (Qe/k_D) \sum_{\alpha=i,e} (J_\alpha M_{T\alpha} / \ell_\alpha^2 \nu_{T\alpha}) \left[(2 - k_{D\alpha}^2 / k_D^2) + (\ell_i \nu_{T,i} / \ell_e \nu_{T,e})^{q_\alpha} (k_{D\alpha} / k_D)^2 \right]. \quad (6)$$

The first and second part of the above equation represents the ion and electron drag forces for a non-absorbing grain. The expression for the ion drag force acting on a non-absorbing grain coincides with the earlier expression derived by Ivlev *et al.* [5] using more generalized kinetic approach in the considered limit of highly collisional ions ($\ell_i \ll \lambda_D$). The ion and electron drag forces experienced by a non-absorbing grain are directed along the drift velocity of the corresponding species, i.e. they act in the opposite directions and the ratio of their absolute magnitudes is $(T_e/T_i)^2$. This implies that in one temperature plasmas ($T_e = T_i$) they exactly cancel each other. In highly non-thermal plasma ($T_e \gg T_i$), the ion drag force (directed opposite to the electric force) dominates. The ratio of the ion drag force to the electric one is $(1/6)\beta$ where $\beta = z\tau(a/\lambda_D)$ is the so-called scattering parameter. [6, 7] Here $z = |Q|e/aT_e$ is the dimensionless charge, $\tau = T_e/T_i$ is the electron-to-ion temperature ratio and a is the grain radius.

The second sum in Eq.(6) is the contribution to the drag forces due to ion and electron absorptions. Under most typical plasma conditions $\ell_e/\ell_i \sim 10 - 100$, $v_{T_e}/v_{T_i} \sim 10^2 - 10^3$ and $T_e/T_i \sim 1 - 100$ we can approximate the absorption parts of the ion and electron drag forces: $(F_i)_{abs} \simeq (1/6)(Qe/k_D)(J_i M_{T_i}/\ell_i^2 v_{T_i})(1 + k_{De}^2/k_D^2)$ and $(F_e)_{abs} = (1/6)(Qe/k_D)(J_e M_{T_e}/\ell_i \ell_e v_{T_i})(k_{De}/k_D)^2$ the ratio of which is $|F_i/F_e|_{abs} \approx (T_e/T_i)^2$. It is clear that the effect of ion absorption reduces the absolute magnitude of the total ion drag force for negatively charged grains whereas the effect of electron absorption increases the total electron drag force. Note that neglecting electron absorption ($J_e = 0$) and electron drift ($u_e = 0$) we get back the expression for the ion drag force identical to that in earlier work by Khrapak *et al.* [3].

In the continuum regime ($\ell_\alpha \ll a$) for infinitesimally small grain ($a \ll \lambda_D$) we use simple analytical asymptotic expressions for the charging fluxes i.e $J_e = J_i \simeq 4\pi a z \tau n_0 \ell_i v_{T_i}$ where n_0 is the unperturbed plasma density so that the total drag force can be written as:

$$F_P = -(1/6)Q^2 k_{De}^2 [M_{T_i}(\ell_i k_D)^{-1} (k_{Di}/k_D)^2 + M_{T_e}(\ell_e k_D)^{-1} (1 + k_{Di}^2/k_D^2)] = (1/6)(QE)(\beta/\tau)$$

i.e. both ion and electron drag forces are directed in the same direction (opposite to the electric field). This is associated with the fact that the ion absorption changes the direction of the ion drag force in highly collisional plasmas, [3] whilst electron absorption increases the magnitude of the electron drag force. The absolute ratio of total ion-to-electron drag force is, $|F_i/F_e| \approx (T_e/T_i)$.

In our model it is not possible to estimate the forces associated with the ‘‘drift momentum’’ transfer from ions and electrons absorbed by the grain self consistently which is of minor importance as shown later. This force is roughly $\sim J_0 m_{i(e)} u_{i(e)}$ for ions (electrons). These contributions are smaller than the corresponding drag forces by a factor of $\sim (\tau/\beta)(\ell_i/\lambda_D)^2$ for

ions and $\sim (\tau^2/\beta)(v_{T_i}/v_{T_e})(\ell_i/\lambda_D)(\ell_e/\lambda_D)$ for electrons. Thus, in highly collisional plasma it is reasonable to neglect the above mentioned effect. The grain effective charge in the considered parameter regime can be written as:

$$Q_{\text{eff}}/Q = 1 + (1/6)(\beta/\tau). \quad (7)$$

The application of the linear theory requires $\beta \ll 1$. [5] Thus, we have $Q_{\text{eff}} \approx Q$. This implies that both the ion and electron drag forces acting on an absorbing grain are small compared to the electric force in the considered case.

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