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Enhancement of the filamentation instability due to collisions

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Since the seminal paper of Tabak *et al.* [1], the fast ignition (FI) approach to obtain the thermonuclear fusion by inertial confinement has received an immediate and increasing interest, also due to promising experimental results [2]. The advance in understanding and controlling the complex physics involved in the FI allowed by numerical codes has been accompanied by an intense theoretical activity. One of the most extensively explored subject of the last years has been the Weibel-like filamentation instability (WI) [3].

Also invoked in astrophysics as mechanism to generate magnetic fields in GRBs [4,5], the WI shows up in the coronal region of the FI fuel target, when the MA-current carried by the MeV-beam electrons is compensated by a return current as natural response of the background plasma. Filaments are formed during the linear stage of the instability; after that they start to merge. This non-linear dissipative process can be either deleterious, if it occurs in the coronal region by preventing the beam propagation, or useful by depositing the beam energy into the pellet core. Moving from the corona towards the centre of the pellet, the huge particle densities in the inner regions of the FI target make the collision frequency significant in comparison with the electron plasma frequency. The transition from the collisionless to the collisional regime and, in particular, how this affects the WI represents a crucial but not yet well understood topic in FI.

A non-relativistic analytical study [6] showed that the collisional WI always occurs, regardless of the transverse temperature: for temperatures at which the collisionless instability is suppressed, collisions allow a small but not negligible growth rate. However, implicit PIC simulations [7] demonstrated instability suppression by collisions. These opposite conclusions show the need to clarify the effects of collisions on the WI, both theoretically and numerically.

In this paper, we use relativistic kinetic theory to analyze the linear phase of the WI triggered by an electron beam and a counter-streaming background electron return current, by comparing the collisionless and collisional growth rates. We consider warm species, different densities and space charge effects (i.e., $\boldsymbol{k}\cdot\boldsymbol{E}\neq 0$). We show that by varying parameters such as the density and the transverse temperature of the beam, when collisions are taken into account it is possible to find scenarios for which the growth rate Γ is larger than the one of the corresponding collisionless cases for a wide range of wavelengths. Our theory shows the enhancement of the WI due to collisions in relativistic scenarios, and fully relativistic PIC simulations including binary collisions confirm this effect. The preferential formation of larger filaments in collisional scenarios than in collisionless ones predicted by the model is also numerically confirmed.

In order to obtain the dispersion relation for waves propagating in the z direction with wave vector $\boldsymbol{k} = k\boldsymbol{e}_z$, we use the relativistic Maxwell-Boltzmann with the BGK (or Krook) particle-number conserving model as the collisional term and Maxwell's equations. Considering that only the return current plasma electrons (e) make collisions with the ions, the corresponding perturbed Maxwell-Boltzmann equation reads

$$\frac{\partial f_{1e}}{\partial t}+\mathbf{v}_{0e}\cdot\nabla f_{1e}+\frac{e}{m_e}\left(\mathbf{E}_1+\frac{\mathbf{v}_{0e}}{c}\times\mathbf{B}_1\right)\cdot\nabla_{\mathbf{p}} f_{0e} = -\nu_e\left(f_{1e}-n_{1e}F_{0e}\right) \quad (1)$$

where f_{1e} (f_{0e}) is the perturbed (unperturbed) distribution function, n_{1e} is the perturbed density, $F_{0e} = f_{0e}/n_{0e}$ represents the unperturbed normalized distribution function and ν_e is the plasma electron collision frequency. For non-colliding species (beam electrons and ions), the corresponding equation is a relativistic Vlasov equation (i.e., the term on the right-hand side of (1) is zero). As well as in [5], we choose a waterbag distribution function such that $F_{j0} = \left[1/\left(2p_{zj0}\right)\right]\delta\left(p_x - p_{xj0}\right)\delta\left(p_y\right)\left\{\Theta\left(p_z + p_{zj0}\right) - \Theta\left(p_z - p_{zj0}\right)\right\}$ where p_{xj0} is the momentum in the x direction and p_{zj0} is the momentum thermal spread in the z direction of the j species, while $\theta(x)$ is the Heaviside step function. Thus, we consider cold species along the y direction, propagating along the x direction, with z defining the perpendicular direction. The interested reader can find all the calculations yielding the dispersion relation in reference [5]; the only difference is represented by the coefficients for the colliding plasma electrons appearing in the dispersion relation, which are

$$\begin{pmatrix} C_{lmn,e} \\ D_{l,e} \end{pmatrix} = \omega_{p0,e}^2 m_e \int d\mathbf{p}\left\{\frac{\omega v_l}{\omega^+ - kv_z}\begin{pmatrix}\alpha_{mn} \\ \partial_{p_z}\end{pmatrix}F_{0e} - i\nu_e\frac{\omega v_l}{\omega^+ - kv_z}\frac{F_{0e}}{1-\left(i\nu_e/2kc\beta_{th0,e}\right)L}\int d\mathbf{p}\frac{1}{\omega^+ - kv_z}\begin{pmatrix}\alpha_{mn} \\ \partial_{p_z}\end{pmatrix}F_{0e}\right\} \quad (2)$$

where $\omega^+ = \omega + i\nu_e$, $\alpha_{mn} = \left[\left(1 - kv_z/\omega\right)\partial_{p_m} + \left(kv_m/\omega\right)\partial_{p_n}\right]$ and $L = \ln\left[\left(\omega^+ + kc\beta_{th0,e}\right)/\left(\omega^+ - kc\beta_{th0,e}\right)\right]$, being $\beta_{th0,e}$ the transverse thermal velocity. Due to its complexity, the dispersion relation will be solved numerically.

By considering an electron beam with $\gamma_b = \left(1 - \beta_b{}^2 - \beta_{th,b}{}^2\right)^{-1/2} = 5$, a transverse temperature T_b = 2 keV, a density ratio $\alpha = n_b/n_i = 0.3$ (thus $n_e = (1\text{-}\alpha)n_i$ for charge neutrality) a plasma electron transverse temperature T_e = 10 eV and cold ions, collisional effects are evident when the growth rate of the WI is plotted against the wavenumbers (Fig. 1a). As ν_e (normalized to ω_{pe}) increases, the instability for high modes is weakened [8] and the wavenumber k_{max} corresponding to the maximum growth rate Γ_{max} is shifted toward larger wavelengths (i.e. smaller k's), indicating a preferential formation of larger filaments than those in collisionless scenarios.

Scenarios exist where collisions enhance the growth rate, allowing the occurrence of the WI even at transverse temperatures that completely suppress collisionless unstable modes. Fig. 1b shows the theoretical predictions regarding this effect for $\alpha = 0.1$, $\nu_e = 0.5$ and increasing T_b. Physically, collisions slow down the beam electrons which feed the electric field coming from a finite resistivity $\eta \propto \nu_e$. Since $\Gamma \propto \beta_b(\alpha/\gamma_b)^{1/2}$ from fluid calculations, the collisional Γ will be larger than the collisionless one, provided that γ_b be large enough (i.e. $\gamma_b > 1.8$). Therefore the collisional WI can show larger growth rates and can even occur at transverse temperatures for which the collisionless WI is suppressed. It is worth to point out that the small Γ corresponding to the largest temperature of the collisionless scenarios in Fig. 1b is solely due to the ions, which are allowed to move ($\Gamma = 0$ otherwise).

Large-scale PIC simulations have been carried out with osiris 2.0 [9], including binary collisions, to test the effects predicted by the theoretical model. The strong numerical noise in collisional simulations is reduced by using a large number of particles-per-cell (ppc) and a very high spatial resolution (i.e., small grid size Δx); moreover a large simulation box

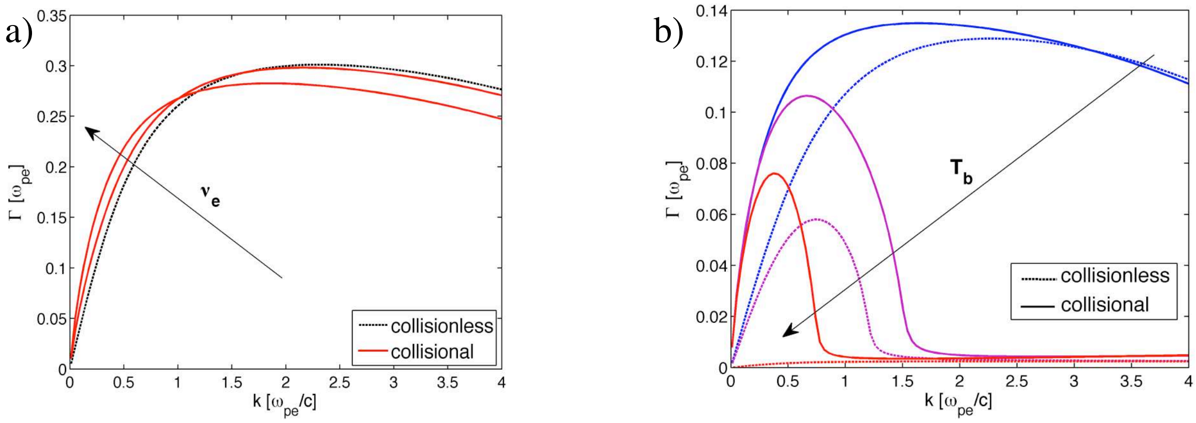

FIG. 1: Collisional effects on the filamentation instability: a) k_{max} shift toward larger wavelengths as ν_e increases (α=0.3, T_b=2 keV, ν_e=0.1,0.5); b) enhancement due to collisions (α=0.1, T_b=0.5,9,34 keV, ν_e=0.5).

is needed to resolve large wavelengths modes, which are important at high collisionality. A quadratic interpolation scheme, which will be soon implemented in the collision module, is expected to reduce even more the numerical noise to better evaluate a sensitive quantity such as the growth rate of the WI. In order to meet all these requirements, we have performed 1D simulations with a simulation box of 5120 c/ω_{pe}, Δx = 0.025 c/ω_{pe} and 5000 ppc, with periodic boundary conditions. The growth rate of the WI for α = 0.1, T_b = 0.5 keV, T_e = 10 eV and ν_{ei} = 0.57 is shown in Fig. 2a. Although the points from collisional simulations still show some noise, the comparison between theory and simulations shows a very good agreement, confirming the theoretical predictions about both the enhancement of the WI and the k_{max} shift toward smaller k's due to collisions. 2D simulations, for the same physical parameters of Fig. 1a, confirm the preferential formation of larger filaments in collisional scenarios as shown in Fig. 2b, where the temporal evolution of the beam electron (absolute) charge density over the linear phase of the WI is presented.

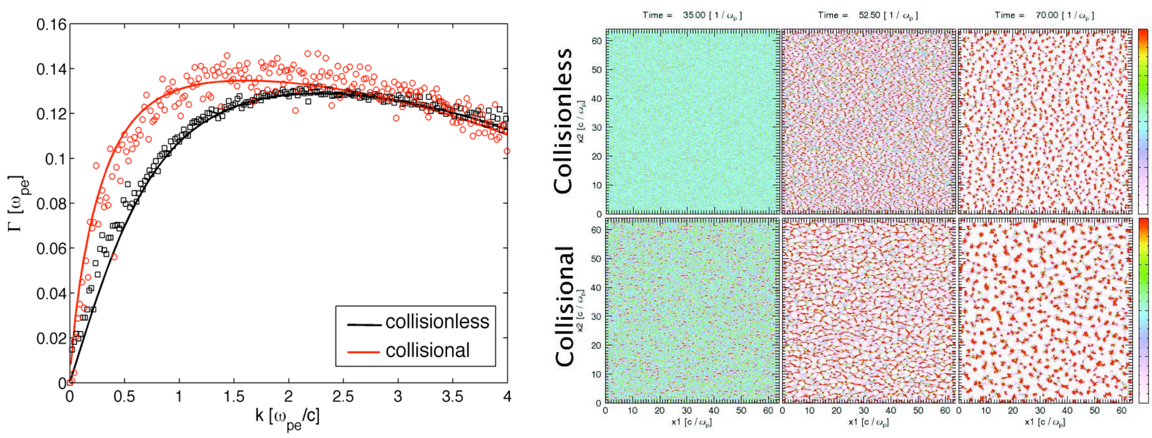

FIG. 2: a) Enhancement of the WI and k_{max} shift due to collisions: comparison between theory (solid lines) and simulations (markers); b) beam electron density: larger filament formation in collisional scenarios.

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