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Recurrence quantification analysis of electrostatic fluctuations in tokamaks

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A major challenge in the study of advanced tokamak scenarios is to understand the causes and associated rates of anomalously large cross-field transport, which is thought to be caused by plasma turbulence. One experimental signature of plasma turbulence in the plasma edge of a tokamak is the fluctuating behavior of the electrostatic floating potential. Such signals often display a broad fluctuation spectra, as observed in the Brazilian tokamak TCABR [1]. A number of probes have been built in TCABR to measure the particle density and temperature fluctuations in the edge region. The experimental results suggest that the turbulent transport is mainly electrostatic in nature [2, 3]. These fluctuations are known to depend critically on the radial position such that the electrostatic turbulent fluctuations should also exhibit some radial dependence. However, the radial variation of the deterministic content of the plasma turbulence may not be immediately apparent from the experimental data obtained in tokamaks. This problem calls for alternative quantitative descriptions of chaoticity and stochasticity in turbulent time series.

One promising technique to analyze turbulent fluctuations data in tokamaks is the use of recurrence plots. We start from a univariate time series sampled at equally spaced time intervals $x_i = x(t = ih)$, where h is the inverse of the sampling frequency. The second step consists of defining vectors with time-delayed points of the original series $\mathbf{x}_i = \{x_i, x_{i+\tau}, \cdots, x_{i+(d-1)\tau}\}$, where d is the embedding dimension and τ is the time delay. The basic idea of a recurrence plot is to start from such a phase space embedding and compare the embedding vectors with each other, drawing pixels when the Euclidean distance between vectors is below some threshold ε, defined as a small fraction of the standard deviation of the time series being considered. More formally, recurrence plots are graphical representations of the matrix [4] $\mathbf{R}_{i,j} = \Theta(\varepsilon - ||\mathbf{x}_i - \mathbf{x}_j||)$, $i, j = 1, 2, \cdots N$, where $\mathbf{x}_i$ is a time-delayed vector, ε is the threshold, $\Theta(.)$ is the unit step function, $||\cdots||$ stands for the Euclidean norm, and N is the total number of points. The RP is thus obtained by assigning a black (white) dot to the points for which $\mathbf{R}_{i,j} = 1$ (0).

Recurrence plots can be used to study non-stationarity of a time series as well as to indicate its degree of aperiodicity. In particular, this method does not demand that the time series be stationary, very long, or with low noise, what makes recurrence plots a highly desirable way to investigate experimental data in a many fields [4]. In this work we used recurrence plots as a tool to quantify the recurrence properties of electrostatic fluctuations in the tokamak plasma edge, in order to provide a quantification of the amount of deterministic chaos present in a turbulent signal, as well as its dependence on the radial position at the plasma edge.

The many kinds of structures present in a recurrence plot are useful to characterize the dynamical properties of the underlying time series, which can be investigated through the so-called recurrence quantification analysis. There are many measures available in the recurrence quantification analysis, such as [4] the *recurrence rate* (RR), which is the fraction of recurrence points (for which $\mathbf{R}_{i,j} = 1$) in the entire plot, excluding points from the main diagonal line (which are always recurrent by definition). Another measure is the *determinism* (DET): is the fraction of recurrence points belonging to diagonal lines, which are structures parallel to the main diagonal line. Along a diagonal line, two pieces of a trajectory undergo for a certain time (the length of the diagonal) a similar evolution and visit the same region of phase space at different times. Hence the existence of many diagonal lines is a signature of determinism. The quantity DET is related with the predictability of the dynamical system, because a random process would have a recurrence plot with almost only single dots and very few diagonal lines, whereas a deterministic process has a recurrence plot with very few single dots but many long diagonal lines.

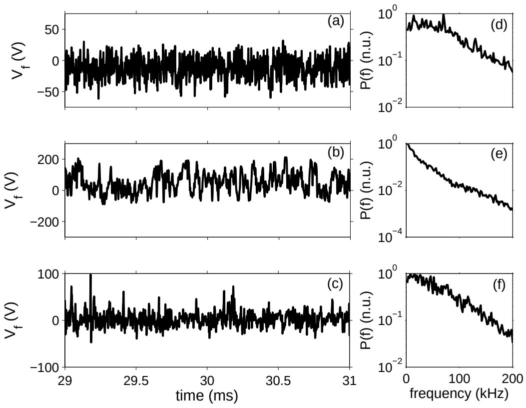

Figure 1: Time evolution of the floating potential measured at radii (a) $r = 16.5$ cm; (b) $r = 18.0$ cm, and (c) $r = 21.0$ cm, with the corresponding power spectra.

The experiments were performed in a hydrogen circular plasma in the Brazilian tokamak TCABR [1] (major radius $R = 61$ cm and minor radius $a = 18$ cm). The floating potential fluctuations were measured by Langmuir probes are mounted on a movable shaft that can be displaced radially from $r = 15$ cm to 23 cm, with respect to the center of the plasma column. In this work we shall focus on the range from 16 to 21cm so as to cover both the plasma edge and the so-called scrape-off layer, the latter comprising part of the vacuum layer existent between the plasma column and the vessel wall. The nature of the electrostatic potential fluctuation is dependent on the radial location where the probe is placed, as suggested by Fig. 1, where the first 2 ms of the time window in the plasma current plateau has been selected, as well as the corresponding power spectra. Within the plasma column [Fig. 1(a)] the potential fluctuations present a -50 V - $+50$ V range. As we move outside the plasma column [Fig. 1(b)], just after its radius (imposed by a material limiter which plays no role in this discussion) such fluctuations increase by a factor of 4, indicating that the turbulence level augments as we approach the plasma radius. This increase is not monotonic, though, as revealed by Fig. 1(c), where the floating potential range decreases to an in-between level. Hence the turbulent fluctuations become weaker as we move outside the plasma radius toward the vessel wall. The radial dependence of the electrostatic turbulence level at the vicinity of the plasma radius is a signature of the role played by radial density gradients in the generation of drift waves [5]. However, characterizing turbulence in order to quantify its level and radial dependence is a difficult task, what can be illustrated by Figures 1(d) to (f), where we show the power spectra of the potential fluctuations in the three radial positions just analyzed. All of them are broadband, which is already expected from the chaotic behavior related to turbulence but, apart from some unessential rippling, those spectra do not show a distinguish feature which could be used to quantify the turbulence level and specific different dynamical regimes.

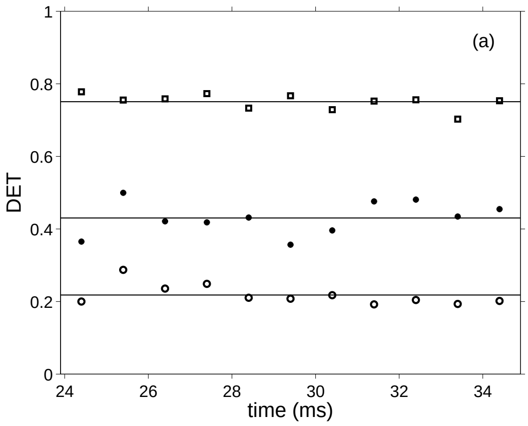

Figure 2: Variation of determinism with the time window (each of them containing $1,000$ points) selected from the original signal for $r = 16.5$ cm (circles), $r = 18$ cm (squares) and $r = 21$ cm (stars).

The success of using these recurrence methods in characterizing the turbulence level observed in tokamak experiments suggests that RQA can be also of interest, especially because it does not impose stationarity nor long series length as necessary conditions, and can also work satisfactorily with moderate noise levels. We have performed a test of stationarity of the data using the consistency of recurrence-based diagnostics, like the determinism, the results being shown in Figure 2, where the determinism (DET) of a sequence of recurrence plots have been determined for consecutive pieces of an original series, each of them with 1 ms of length (equivalent to 1000 points, as before). For data acquired at $r = 16.5$ cm [depicted as empty circles in Fig. 2] the values of DET for each series segment are distributed around a mean value of ≈ 0.17 and with a small dispersion, indicating a consistency of such diagnostic against the total extension of the series and thus suggesting that the original series is stationary enough for our purposes. In

Figure 3 we consider potential fluctuation data from out of 57 discharges involving 9 radial positions of the probe (in each discharge the probe position is held fixed at a given position). The mean values of DET obtained in different discharges are plotted *versus* the radial positions. The line joining those points is a polynomial fit just to guide the eye, i.e. it is not intended to give a radial profile $DET(r)$ but rather a trend: the degree of determinism increases significantly as we approach the plasma radius, and decreases afterwards. This suggests a diminishing contribution of stochastic effects at the plasma edge.

In conclusion, the deterministic content of these fluctuations is not spatially uniform, but it is more pronounced just before the plasma border. Our results suggest that theoretical models for describing such fluctuations, using nonlinear mode coupling of drift waves, can be used in the vicinity of the plasma border, but may not give reliable results when applied in the internal edge plasma or the far scrape-off layer separating the plasma column from the tokamak wall. The radial location of the maximum level of determinism is within the region of steep density gradients observed in tokamaks, the latter being considered as the main cause of drift wave instabilities leading to electrostatic plasma turbulence.

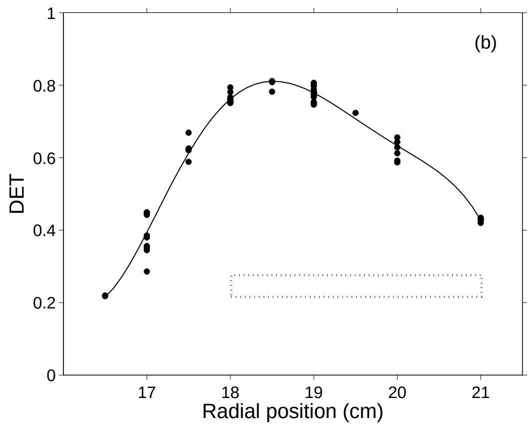

Figure 3: Radial profile for the mean value of determinism for various discharges (see text for details).

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