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Stimulated emission without inversion in ensembles of classical electrons

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Introduction. The stimulated emission (lasing) without inversion (LWI) as one of the basic effects in the physics of radiation processes has been intensively studied for the last several years. This effect was predicted initially for quantum systems [1]. But like the usual stimulated emission, the LWI has been found as a general physical effect. The classical LWI models include both monochromatic systems with spatial division of electron beam into two fractions [2] and systems where spatially homogeneous ensemble of electrons interacts with the bichromatic radiation [3]. In the systems of the second type, discussed here, the parametric coupling of radiation components by means of medium oscillations underlies the LWI effect.

Here we present stationary regimes of three-waves "inversionless" cyclotron amplification. The basic idea is borrowed from quantum electronics as well as based on results of previous investigations of "inversionless" amplification of high-frequency (HF) bichromatic radiation in classical plasma systems with given low-frequency (LF) modulation [3]. This effect of "inversionless" instability is revealed in the systems of three-waves interaction in plasma with quadratic nonlinearity under cyclotron (Doppler) synchronism conditions between monochromatic field components and electrons. In presented regimes two HF probe waves are amplified simultaneously due to coupling with permanent LF pumping wave. In the absence of LF pumping the linear regime of interaction between HF waves with the medium is characterized either by the field absorption or by no energy exchange between the medium and the field . As result of these investigations further analogies between classical and quantum electronics, where large number of AWI schemes with permanent LF pumping is known, are revealed. Besides, a thorough research of use environment of the Manley-Rowe relations, which define multi-photon process behaviour, is presented.

1. The first regime. Consider three coherent waves propagating transversely to the constant magnetic field $\vec{B}_0 = B_0\vec{z}_0$:

$$\vec{E} = \vec{y}_0 Re \sum_{i=1}^{2} E_j exp(ik_j x - i\omega_j t + i\pi/2) + \vec{y}_0 Re E_p exp(i\kappa x - i\Omega t + i\pi/2). \tag{1}$$

Probe HF waves are resonant to the relativistic electrons with gamma-factor γ_R at two high harmonics of gyro-frequency. The pumping wave is resonant to the same electrons at the differented harmonic. The following resonant conditions are fulfilled:

$$\omega_1 = N_1\omega_R, \quad \omega_2 = N_2\omega_R, \quad \Omega = (N_1 - N_2)\,\omega_R = L\omega_R, \quad \omega_R = \frac{eB_0}{mc\gamma_R}, \quad k_1 - k_2 = \kappa. \quad (2)$$

Assign the amplitude of the pumping wave to be fixed and find the nonlinear susceptibility at the frequencies ω_1 and ω_2, solving the kinetic equation for the electrons distribution function within the limits of quadratic approximation with respect to the field amplitudes. Assume, that 1) the electrons density and amplitude of pumping wave are small, so that the vacuum variance relation $\omega_j = ck_j$ is slightly perturbed; 2) the energy of electrons in ensemble is slightly disturbed near the resonant value $mc^2\gamma_R$ and the field amplitudes are small, so that the motion of relativistic electrons can be described by the equations of nonlinear pendulum. Under these conditions the following system of equations can be considered:

$$\frac{\partial f}{\partial t} + \omega_H \frac{\partial f}{\partial \theta} + \sum_{j=1}^{2}\left(F_j \frac{\partial f}{\partial w}\right) + F_p \frac{\partial f}{\partial w} = -\nu\,(f - f_0)\,. \quad (3)$$

$$\frac{d}{dt}\alpha_j = -\frac{2\pi e}{mc\omega_j} I_j \quad (4)$$

$$F_{1,2} = -\omega_j G_j Re\left(\alpha_j exp\left(ik_j X + iN_j\theta - i\omega_j t\right)\right),$$

$$F_p = -\Omega G_p Re\left(\alpha_p exp\left(i\kappa X + iL\theta - i\Omega t\right)\right). \quad (5)$$

$$I_j = ecG_j \langle \int dw d\theta\, f exp\left(-iN_j\theta - ik_j X + i\omega_j t\right)\rangle_{t,X}. \quad (6)$$

where $f(w,\theta,t,X)$– the electrons distribution function, $w = \gamma - \gamma_R$, θ– the gyro-phase, $f_0(w)$– the unperturbed distribution function; $\omega_H = eB_0/(mc\gamma)$; $\alpha_j = eE_j/(mc\omega_j)$, $\alpha_p = eE_p/(mc\Omega)$, $G_j = \beta_{\perp R} J'_{N_j}(k_j r_H)$, $J'_{N_j}(k_j r_H)$–the derivative of the Bessel function, r_H– the gyro-radius of the resonant particles (the definition of G_p is analogous). The parameter ν is the rate of some dissipative process in the medium.

Firstly, we found, that in the absence of the dissipation in electron ensemble $(\nu = 0)$[1] the connection of two HF waves by means of pumping LF wave can not lead to the simultaneous amplification of two HF waves in the "inversionless" medium (where $\partial f_0/\partial w(w = 0) < 0$). On the contrary, if the dissipation rate ν is sufficiently large and the pumping wave is strong enough the three-waves coupling can result in the simultaneous amplification of HF waves in "inversionless" electron medium. The following condition must be fulfilled:

$$\nu \gg \frac{\Omega\,\langle\Delta w\rangle}{\gamma_R} \quad (7)$$

[1]Note that the effect of inversionless HF waves amplification in transitional regime with interaction switching on at the initial moment, investigated in [3], is obtained on the basis of equations (3)-(6) with $\nu = 0$.

It means that the resonant region with the pumping wave exceeds the particles dispersion in the phase space. Besides let the dissipation rate ν to be sufficiently small so that HF waves interact with electron ensemble in kinetic regime susceptible to the properties of the distribution function:

$$\nu \ll \frac{\omega_j \langle \Delta w \rangle}{\gamma_R} \tag{8}$$

So, we put $\Omega \ll \omega_j$. Then the coupling equations for HF waves take the form:

$$\begin{aligned} \frac{d\alpha_1}{dt} + \frac{G_1^2}{N_1}(\gamma_L + i\delta_L) &= (\Gamma_{NL} + iD_{NL})\frac{N_2}{N_1^2}\alpha_2\alpha_p \\ \frac{d\alpha_2}{dt} + \frac{G_2^2}{N_2}(\gamma_L + i\delta_L) &= (\Gamma_{NL} + iD_{NL})\frac{N_1}{N_2^2}\alpha_1\alpha_p^*, \end{aligned} \tag{9}$$

where

$$\gamma_L + i\delta_L = -\frac{2\pi e^2}{m}\frac{\gamma_R}{\omega_R}\left(i\int \frac{\partial f_0}{\partial w}\frac{\wp}{w}dw + \pi \left.\frac{\partial f_0}{\partial w}\right|_{w=0} \right) \tag{10}$$

$$\Gamma_{NL} + iD_{NL} = \frac{\pi e^2}{m}G_1G_2G_p\frac{\gamma_R}{\omega_R}\frac{\Omega}{\nu}\left(i\int \frac{\partial^2 f_0}{w^2}\frac{\wp}{w}dw + \pi \left.\frac{\partial^2 f_0}{w^2}\right|_{w=0} \right). \tag{11}$$

The condition of "inversionless" amplification of two HF waves in the simplest form (when $G_1 \approx G_2$) is the following:

$$\frac{\Omega}{2\nu}\left|\alpha_p\right| G_p \left|\frac{\partial^2 f_0}{\partial w^2}\right|_{(w=0)} > -\left.\frac{\partial f_0}{\partial w}\right|_{(w=0)}. \tag{12}$$

In this system the mechanism of "inversionless" amplification consists in the parametric interaction of HF waves by means of sufficiently strong modulation of conductivity of the medium under the action of the pumping wave. Note, that the active response of the medium is formed by the particle, *resonant with the HF waves*.

2. The second regime. Consider the system of the second type. Two HF electromagnetic probe waves propagate along the constant magnetic field in opposite direction; the longitudinal wave plays the role of the LF pump:

$$\vec{E}_\perp = \sum_{j=1}^{2} Re\left\{ \vec{e_+}E_j e^{ik_jz - i\omega_jt} \right\}, \quad k_1 > 0, \quad k_2 < 0, \tag{13}$$

$$\vec{E}_p = \vec{z}_0 Re\left\{ E_p e^{i\kappa z - i\Omega t} \right\}, \quad \kappa = k_1 - k_2, \quad \Omega = \omega_1 - \omega_2. \tag{14}$$

We have shown that in this system the simultaneous amplification of two electromagnetic waves is possible *in the absence of electrons resonant with these waves*. As the necessary condition we should again to assume the dissipative process with sufficiently large rate to be included in the system:

$$\nu \gg \Delta_M, \quad \Delta_M = \Omega - \kappa V_{||}. \tag{15}$$

At this condition all electrons are in synchronism condition with the longitudinal pumping wave. Simultaneously the dissipative rate is small, so that the interaction of electromagnetic waves with electrons is considered as *nonresonant*:

$$\nu \ll \left|\Delta_j\right|, \quad \Delta_j = \omega_j - \omega_H - k_j V_{||}. \tag{16}$$

Here $V_{||}$ is the longitudinal velocity of electrons in ensemble. Considering (as an example) the interaction of waves with nonrelativistic plasma, we obtain the following condition of simultaneous "nonresonant" amplification of two electromagnetic waves (in the simplest form, when $\Omega \ll \omega_j$, $k_1 \approx -k_2 = k$, $\omega_j \ll \omega_H$):

$$\frac{1}{2}k\frac{e\left|E_p\right|}{m} > \nu^2\frac{\omega}{\omega_H}. \tag{17}$$

In this system the mechanism of "nonresonant" amplification consists in the parametric interaction of HF waves by means of antiphase modulation of reactive response of the medium, formed by *nonresonant* particles.

Conclusion. We revealed and analyzed the systems of three-waves interaction, where the energy exchange process contradicts, on the one hand, with the customary Manley-Rowe relations and, on the other hand, with "inversion" of the resonant medium as condition of amplification. The principal aspect is the combination of three-waves process with the resonant interaction of the field with electrons-oscillators either at every frequency or at least at the differented (low) frequency. The last case is especially surprising. For this regimes to be realized in classical systems the sufficiently wide frequency band of resonant interaction with LF wave must overlap the spectrum of the corresponding natural vibrations in the medium.

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