Parametric decay instability accompanying electron Bernstein wave heating in MAST

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Introduction

Electron cyclotron heating was shown to be effective in stellarator and tokamak plasmas. Unfortunately its application to spherical tokamaks (STs) is substantially limited by high plasma density and relatively low magnetic field typical for these devices. This ST feature has a strong effect on the electromagnetic wave propagation. In the microwave frequency region, characteristic surfaces, like the upper hybrid resonance (UHR) and cut-offs are very close to the plasma edge. As a result electromagnetic (EM) waves are unable to penetrate into the plasma interior. The only way to overcome this difficulty is to use a linear conversion of the incident EM wave into the electron Bernstein wave (EBW) at the UHR. EBW has no density limit and can, in principle, carry the radio frequency power deep into the plasma. The feasibility of this plasma heating scheme is under investigation now on the MAST tokamak at Culham, UK.

The wave propagation in the UHR, where the wave electric field increases, is usually distorted by nonlinear effects at a power density level exceeding $\sim 1 \text{ kW/cm}^2$ typical for EBW heating experiments [1–3]. In particular, parametric decay instabilities can cause an anomalous reflection and absorption of the incident power. In experiments [1, 2] generation of low-frequency (LF) waves has been observed. These waves were identified as lower-hybrid (LH) waves, which are the only waves able to reach the small scales of the pump UH wave and satisfy the wavelength matching condition for the decay instability. In the present paper the first observations of LF waves generated in the EBW heating experiments on MAST are reported. The radiation is postulated to arise due to the parametric decay of the pump UH wave into another UH wave and LH wave. The theoretical study of this instability is performed and the threshold power is calculated.

Experimental results

The EBW heating system in the MAST tokamak allows up to 1 MW microwave power at 60 GHz to be launched into the plasma in 7 beams. The coupling to EBW is optimized by a system of steerable focusing mirrors. At a certain orientation of the microwave antenna the steep growth of 100 MHz RF radiation (fig. 1c) during the heating pulse (fig. 1a) was registered by the specially designed RF antenna situated in the edge plasma at 5 cm outside the separatrix.
Fig. 1: a) RF power injected into the plasma, b) line integrated density, c) lower hybrid emission (LHE) signal measured at 101 MHz, d) LHE spectrum measured at different time slices.

The maximum of the registered RF radiation corresponds to the over-dense plasma (fig. 1b), which provides the necessary conditions for the heating wave mode conversion to EBW. Due to the fact that the observed RF signals have frequencies close to the LH resonance frequency in the UHR region we postulate that the radiation is generated by the parametric decay instability occurring at the UHR.

**Theoretical analysis**

We consider a slab plasma model and 1D problem of parametric decay, assuming that the pump UH wave propagates along the inhomogeneity direction \( x \) as well as the reflected UH wave. We analyze the “high-frequency” case, specific for MAST conditions, when the pump frequency is more than twice the ECR frequency in the UHR: \( \omega_0 > 2\omega_{ce} \). In this case the UH waves dispersion curves (fig. 2a) do not possess a turning point and transformation to Bernstein wave occurs without a change of group velocity sign. The opposite direction of the daughter waves propagation between the decay points (fig. 2a) corresponds to the situation where a feedback loop and, therefore, an absolute parametric decay instability [4] is possible. We are interested here in the fundamental mode of this instability, which usually has a minimal threshold and corresponds to the decay points \( x_{d1} \) and \( x_{d2} \) (fig. 2b) being close to each other \( (x_{d1} \simeq x_{d2} \equiv x_d) \) and to the LHW turning point. In this case the situation under consideration is similar to the decay problem, when two daughter waves have the cut-off. Namely our problem is equivalent

Fig. 2: a), b) dispersion curves for pump UH wave \( k_0 \), reflected UH wave \( k_1 \) and LH wave \( k_2 \). Arrows denote group velocity direction. c) Three-wave interaction, when two waves possess turning points.
to that studied in [5], where one of the cut-offs is far from the feed-back loop, where the decay wave energy is circulating (fig. 2c).

Keeping in mind this analogy, we consider the equations, describing the complex amplitudes \( \phi_1, \phi_2 \) of the daughter waves potentials \( \phi_{uh} = \frac{1}{2} [ \phi_1(x) e^{iklx} + c.c.], \phi_{lh} = \frac{1}{2} [ \phi_2(x) e^{iklx} + c.c.] \).

We use the following notation: \( k_0, k_1 \) are the constant values denoting the wavenumbers of the UH waves in the vicinity of the LHW turning point. The transformation wavenumbers \( \nu_e, k_e \) of LH and UH waves are given by \( \nu_e^2 = \omega_{pe} N_{lh} / [ e \ell_T(\omega_2)], k_e^2 = \omega_{le} \sqrt{1 + N_{lh}^2 / [ e \ell_T(\omega_3)]}, \) where \( \ell_T^2(\omega) = \frac{3}{2} \left( \frac{\omega_{pe}^2}{\omega^2 - \omega_e^2} - \frac{v_F^2}{\omega^2} + \frac{\omega_{pe}^2 v_F^2}{\omega^4} \right) \) and \( N_{lh}, N_{uh} \) are the toroidal refraction coefficients of the corresponding waves.

In the vicinity of the LHW turning point the equations in question [6] can be represented as

\[
\begin{align*}
\phi_1'' + \alpha_1(x - x_1) \phi_1 &= V_{12} e^{-iK_x} \phi_2 \\
\phi_2'' + \alpha_2(x - x_2) \phi_2 &= V_{21} e^{iK_x} \phi_1
\end{align*}
\]

where \( \mu = (k_e/k_1)^4, \phi_1 = \phi_1^0 e^{-i\Delta k x}, \Delta k = k_1 (1 + \mu) / (5 + \mu), K = k_0 - k_1 - \nu_e + \Delta k, \alpha_1 = [(5 + \mu) \ell_T^2(\omega_1) L(x_d)]^{-1}, \alpha_2 = [4 \ell_T^2(\omega_2) L(x_d)]^{-1} \) and \( L(x_d) \) is the inhomogeneity scale: \( L^{-1}(x) = d \ln n / dx + 2 \omega_e^2 / \omega_{pe}^2 \cdot d \ln B / dx \). The quantity \( x_1 \) denotes a virtual turning point for the UH wave \( x_1 = x_d - \ell_T^2(\omega_1) L(x_d) k_e^2 (1 + \mu)^2 / (5 + \mu) \), while \( x_2 \) gives the real turning point of the LH wave \( x_2 = x_d + 2i \omega_e^2 \omega_{pe}^2 L(x_d) / (\omega_2 \omega_e^2) \), which is slightly shifted to the complex plane due to \( \omega''_e \), which is the imaginary part of the LH wave frequency \( \omega_2 = \omega'_e + i \omega''_e \), describing the growth rate of the instability. The interaction coefficients take the form

\[
V_{12} = \frac{-ie}{2 \omega_e^2 (5 + \mu) \ell_T^2(\omega_1)} \frac{\nu_e^2 E_0^*}{k_1}, \quad V_{21} = \frac{ie}{2 \omega_e^2 \ell_T^2(\omega_2)} \frac{k_1 E_0}{\nu_e^2}
\]

where \( E_0 \) is an amplitude of the pump wave electric field. According to [5] the absolute instability threshold for the fundamental mode of the system (1) is determined by the equation \( V_{12} V_{21} \left( \alpha_2^2 - \alpha_1^2 \right)^{-1/3} \approx 0.2 \). This gives the following expression for the parametric decay, producing the LH wave with \( N_{lh}^2 \approx c (5V_{12} \ell_T), \) which can be shown to have the minimal threshold

\[
\frac{P_f}{\pi \rho^2} \left[ \frac{W}{\text{cm}^2} \right] = 2 \cdot 10^{-3} \left[ \frac{W}{\text{cm}^2 / \text{T}^{1/3} \text{GHz}^{1/3} \text{eV}^{13/6}} \right] \left[ \frac{f_0^{1/3} T_1^{11/12} T_e^{5/4} B_1^{1/3}}{L^{4/3}} \right]
\]

where \( T = T_e + 4T_i \) and \( \rho \) is the electric field e-fold radius of the heating beam. All the plasma parameters here should be taken at the UHR position.

Discussion

For MAST experiment parameters \( f_0 = \omega_0 / (2\pi) = 60 \text{ GHz}, T_e \sim T_i = 140 \text{ eV}, B = 0.38 \text{ T}, L = 3 \text{ cm} \) the threshold (2) is equal to \( P_f / (\pi \rho^2) \approx 260 \text{ W/cm}^2 \), which gives \( P_f \approx 80 \text{ kW} \) for the beam of \( \rho = 10 \text{ cm} \) radius. This power was well exceeded in every heating beam in the recent MAST experiments.
It should be noted that the actual power $P_i^*$ is less than the heating beam power due to the conversion efficiency in the UHR, which is less than 100%. In case of the MAST experiments the UHR accessibility window position is rather hard to control and its angular width is small ($3^\circ \div 5^\circ$). Under these conditions the decay instability excitation can be considered as a proof of the successful injection of the power exceeding the threshold, given by (2), into the plasma. It can be concluded that in these experiments the coupling efficiency of at least one of the heating beams was not less than 50%.

Due to the inhomogeneity scale in the denominator of (2), which is a rapidly growing function of the heating frequency (e.g., for linear density profile $L \propto f_0^2$), staying in the denominator of (2), the instability threshold increases with decreasing frequency. Thus, to optimize the EBWH experiment, the decrease of the heating frequency can be recommended. It leads additionally to the broadening of the accessibility window, hence simplifying the design of the experiment. Unfortunately ion and electron temperatures in the UHR are decreased at lower frequencies due to the UHR shift to the plasma periphery. That gives the reverse tendency to the threshold behaviour, which however can be expected to be overcome by minor heating power deposition in the UHR vicinity.

**Conclusion**

Generation of lower hybrid waves has been observed in EBW heating experiments on MAST for the first time. Theoretical study of corresponding decay instability of the pump wave induced backscattering was performed. The threshold power was estimated for typical parameters in MAST. It was shown that the backscattering parametric decay instability can arise only if the pump power exceeds 80 kW in any beam, so there is an indication of substantial coupling of the microwave power to the UHR in the MAST experiment.

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**References**

[5] Piliya A D and Fedorov V I 1975 *Zhurnal Eksperimental’noi i Teoreticheskoi Fiziki (JETP)* **68** 987