*32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-5.080 (2005)*

Linearization of ITER Plasma Equilibrium Model on DINA Code

Y.V. Mitrishkin[1], V.N. Dokuka[2], R.R. Khayrutdinov[2]

[1]*Inst. of Control Sciences, 65 Profsoyuznaya St., Moscow, Russia*
[2]*SRC RF TRINITI, Moscow Region, Russia*

Abstract. Plasma in ITER has the vertical position instability inherent in vertically elongated plasmas; therefore it is a potentially dangerous plant under control and has a high risk of crashes [1]. This circumstance requires of all-round and detailed development of the ITER plasma magnetic control system. In that connection the purpose of the paper presented is to carry out numerical linearization of ITER plasma nonlinear equilibrium model by means of the DINA code [2], comparison of linear and nonlinear models behavior in a feedback with the same controllers and disturbances, comparison of open linear models in frequency domain to find out linearization accuracy features. The linear models obtained can be used for analysis and optimization of plant functional controllability [3], for an estimation of admissible disturbances, which do not take out the unstable plant from the limited controllability state space area [4], and also for synthesis and analysis of both linear and nonlinear robust controllers of plasma position, current, and shape taking into account the constraints on inputs actions and output signals [5].

The mathematical model of plasma equilibrium in tokamak is represented by the set of interconnected differential equations (1)-(3) [2]. The nonlinearity in (3) is caused by the presence of plasma contribution Ψ^i_{pl} to the poloidal flux obtained from the solution of the nonlinear equations (1)-(2).

- Grad-Shafranov equation

$$\Delta^*\Psi = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right)+\frac{\partial^2\Psi}{\partial z^2} = \begin{cases} -\dfrac{8\pi^2}{c}j_t, & (r,z)\in S_{pl} \\ -\dfrac{8\pi^2}{c}\sum_{k=1}^{n} rI_k\delta(r-r_k)\delta(z-z_k), & (r,z)\notin S_{pl} \end{cases} \quad (1)$$

$$j_t = r2\pi c\frac{dp(\Psi)}{d\Psi}+\frac{1}{cr}\frac{d\left[F^2(\Psi)\right]}{d\Psi}.$$

- Magnetic field diffusion equation

$$\frac{d\Phi}{d\rho}\dot{\Psi}-\frac{d\Psi}{d\rho}\dot{\Phi} = \frac{4\pi}{\sigma''}\left[J\frac{dF}{d\rho}-F\frac{dJ}{d\rho}\right],\ J(\rho)=\iint_{S_p} j_t drdz \quad (2)$$

- Plasma configuration with a free boundary at given plasma current and its profile is defined by currents in poloidal active coils and passive contours. The dynamics of these currents is described by the Kirchoff equations:

$$\frac{d}{dt}\left(L_iI_i+\sum_{j\neq i}M_{ij}I_j+\Psi^i_{pl}\right)+R_iI_i=U_i \quad (3)$$

The linearization of ITER plasma equilibrium model (1)-(3) was carried out with the use of DINA code in the assumption of ideal conductivity of plasma. In the linear model obtained (4)

$$\frac{d}{dt}(\delta I) = A\delta I + B\delta U + E\frac{d}{dt}(\delta\xi),\ y = C\delta I + D\delta U + F\delta\xi \quad (4)$$

the currents increments $\delta I(t) = I(t) - I_0$ about their setpoint values in poloidal coils and passive contours appropriate to the examined equilibrium are used as states where $I(t)$ is the currents vector at the moment t, I_0 is the currents vector of setpoint plasma configuration at the moment $t=t_0$. The input vector $U(t)$ consists of voltages on PF coils. The dimension of the state vector δI is equal to $n_{pf}+n_{vv}$, where n_{pf} is a number of PF coils and n_{vv} is a number of passive contours. The presence of β_p and l_i perturbations is taken into account by introducing a vector $\delta\xi$ with $\dim(\delta\xi) = 2$. Hence the sizes of matrices in equations (4) are as follows A: $(n_{pf}+n_{vv})\times(n_{pf}+n_{vv})$, B: $(n_{pf}+n_{vv})\times n_{pf}$, E: $(n_{pf}+n_{vv})\times\dim(\delta\xi)$. Output vector $y=[\delta g, \delta R_{mag}, \delta Z_{mag}, \delta I_{pl}]$ is formed from variations of observed values about their reference outputs namely gaps $g_{1\div 6}$ between plasma boundary and the first wall, coordinates of magnetic axis (R_{mag}, Z_{mag}) and plasma current I_{pl}. Therefore the dimension of output vector y is equal to 9 and the dimensions of matrixes C, D and F are equal to $\dim(y)\times(n_{pf}+n_{vv})$, $\dim(y)\times(n_{pf}+n_{vv})$ and $\dim(y)\times\dim(\delta\xi)$ accordingly. The linearization procedure is reduced to the calculation of matrices A, B, C, D, E, F in (4) [6]. In doing so, the deviations of currents and disturbances from their setpoints are prescribed; new plasma equilibrium configuration is calculated by the DINA code, and then the deviations obtained of poloidal fluxes in contours and of components in output vector y are used to determine the matrices in (4). Linearization is carried out for various R-Z calculation grids of the area of the Grad-Shafranov equation solution and different numbers of passive contours, that are, (33×65, 131); (65×65, 131); (33×65, 248); (65×65, 248); (65×65, 248) (the last one does not contain mutual inductances of contours). The aim of the study of various divisions of passive structures and the area of the equilibrium equation solution is to clear up their influence on linearization accuracy of the working linear model.

Comparison of linear models in time and frequency domains. The comparison of the specified variants of DINA linear models and PET linear model were performed in a feedback loop at stabilization of plasma vertical speed about zero. On the control system block diagram in **Fig. 1** the switch S is in the upper position creating a closed loop with a speed proportional controller. The results of comparison are shown in **Fig. 2**. Obviously the proportional controller maintains plasma magnetic axis speed near to zero for all linear models investigated. However there are some distinctions in plasma position behaviour for various models due to different accuracy of linear models and stabilisation of the plasma speed but not its position. Plasma position tends to a slow change because it is impossible to stabilize speed near zero precisely by a proportional controller.

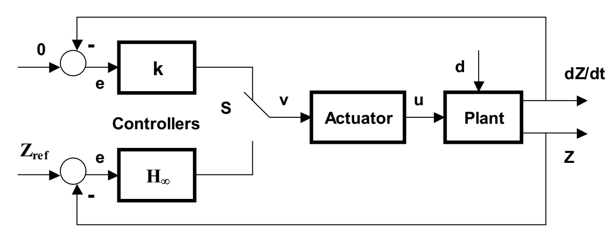

Fig. 1

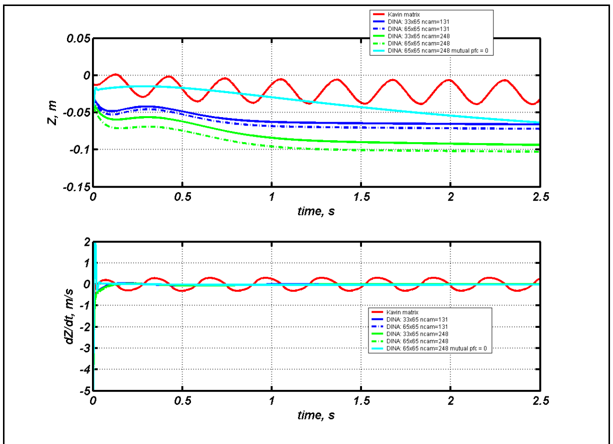

Fig. 2

For comparison of linear models in frequency domain the frequency singular values characteristics of plant scaled matrix transfer function G were used where G was obtained from (4) namely $G(s) = C(sI - A)^{-1} B + D$ at E = 0, F = 0. The singular values σ_i of G are gains of multivariable linear plant in various directions and are

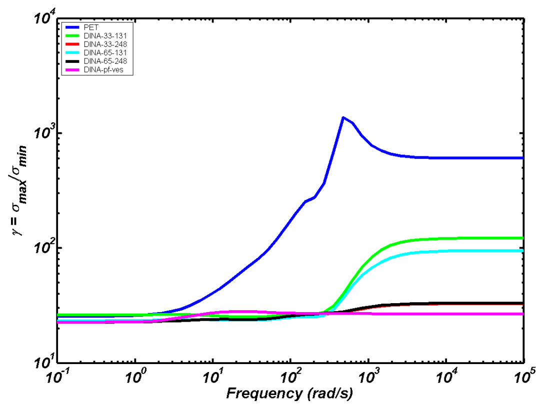

Fig. 3

given by the expression $\sigma_i = +\sqrt{\lambda_i\left(G^H G\right)}, \quad G^H(s) = G^T(\bar{s})$. The condition number $\gamma(\omega) = \sigma_{\max}[G(j\omega)]/\sigma_{\min}[G(j\omega)]$ was chosen as a criterion of frequency comparison of linear models, which reflects the range of plant gains change. The frequency characteristics of γ are shown in **Fig. 3**. There is a good coincidence of γ for all models at lower frequencies up to ~2 rad/sec with about 13% dispersal at 0.1 rad/sec. This is acceptable for engineering estimations because the controller design is done at relatively lower frequencies to achieve good performance of closed loop system.

Comparison of linear and nonlinear models in time domain. The comparison of evolution of Z for linear model (4) and nonlinear model (1)-(3) in a feedback by using a speed controller for the case (65×65, 131) is shown on in **Fig. 4** with a deduction of results of "undisturbed" solution from "disturbed" one for the case of DINA simulation. In the nonlinear case the results were obtained on DINA code for 10% β_p drop, which occurred in the middle of simulation time: δZ is a displacement of a plasma vertical position from the given value, $d(\delta Z)/dt$ is its derivative, U_{VSC} is a voltage on Vertical Stability Converter. Here, as well as in the previous case, there is a rather inexact coincidence of transient response for δZ.

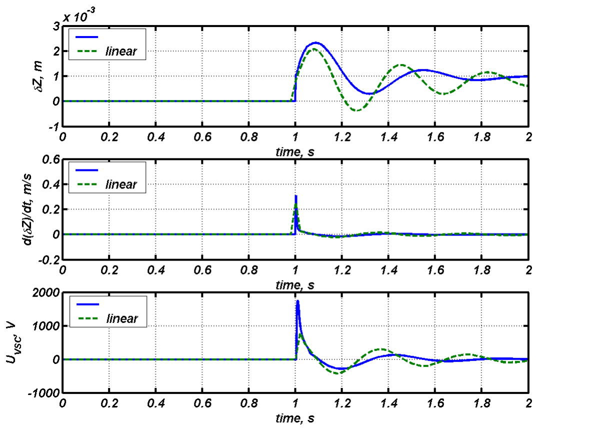

Fig. 4

For this reason instead of speed controller, a new H_∞ controller [5] for stabilization of a proper vertical position of magnetic axis of plasma was designed. The block diagram of the system for this case with a lower position of the switch S is presented in **Fig. 1**. The comparison of processes of plasma vertical position control for linearized model and DINA code were carried out. The processes of tracking stepped command δZ_{ref} by the control system for linear and nonlinear models are shown in **Fig. 5**. The command signal δZ_{ref} has the amplitude of 5 cm and is marked by green colour in the upper part of **Fig. 5**. The red and blue lines correspond to the outputs δZ and δZ_{non} for linearized model and DINA code accordingly.

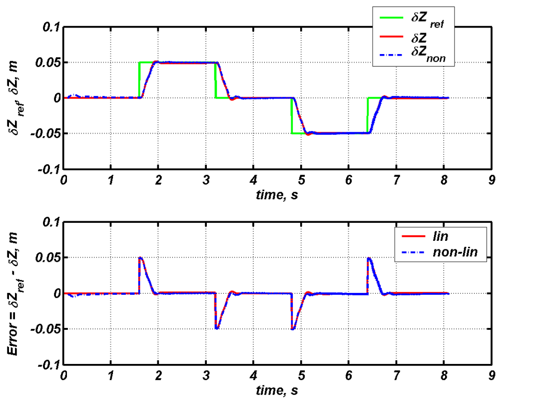

Fig. 5

In the lower part of **Fig. 5** the dynamic errors $e = \delta Z_{ref} - \delta Z$ for the control system with linear model (red colour) and DINA code (blue colour) are shown. According **Fig. 5** the transient responses at steps in δZ_{ref} terminate within 0.5 s. Thus the transient responses of plasma vertical control for linear model and DINA code coincide with high accuracy (about 1%) that confirms high accuracy of calculation of matrixes *A*, *B*, *C*, *D* of linear model (4). Moreover, the fact of good agreement of simulation results on non-linear and linear models with using the same controller of *Z*-position synthesized on the base of *A*, *B*, *C*, *D* matrices, also gives the evidence about high accuracy of their calculation. The matrixes *E*, *F* did not participate in the synthesis of the H_∞ controller.

Conclusions

- The set of linear models with different grid sizes of area of solution of the Grad - Shafranov equation (1) and different numbers of contours for passive structures in the Kirchoff equation (3) was obtained by means of the created linearization procedure of the non-linear model (1)-(3) of plasma equilibrium in ITER based on the DINA code.
- The comparison of linear models in time and frequency domains was carried out, that allowed to choose well-grounded working linear model for synthesis of controllers.
- The comparison of linearized model and DINA code in a feedback for plasma vertical position stabilization by the usage of the H_∞ controller synthesized and for proportional stabilization of *dZ/dt* around zero in a closed loop was performed.
- The comparison showed high accuracy of agreement of transient responses at stabilization *Z*, that confirmed high accuracy of getting *A*, *B*, *C*, *D* matrices of linear model (4) with the help of the created linearization procedure on DINA code, and also weak coincidence of transient responses at stabilization of *dZ/dt* because in this case there was not complete control of coordinate *Z*.

References

1. Stein G., IEEE Control Systems, Vol. 23, No. 4, pp. 12-25, 2003.
2. Khayrutdinov R.R., Lukash V.E., Journal of Computational Physics, 109(2), pp. 193-201, 1993.
3. Mitrishkin Y.V., Proc. of VIII International Workshop on Stability and Oscillations of Non-linear Control Systems, Inst. of Control Sciences, Moscow, Russia, pp. 125-126, 2004.
4. Mitrishkin Y.V., Plasma Devices and Operations, Vol. 4, pp. 111-140, 1995.
5. Mitrishkin Y.V., Proc. of The 8th World Multi-Conference on Systemics, Cybernetics and Informatics, Orlando, FL, USA, Vol. XV, pp. 247-252, 2004.
6. Belyakov V., Kavin A. Proc. of International Conf. on Physics and Control, St.-Petersburg, pp. 1019-1024, 2003.