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Analysis of RWM feedback systems with internal coils

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1. Introduction

One of the β limiting phenomena in stationary tokamaks can be the external kink modes which stabilization by the vacuum vessel wall is incomplete because of the wall resistivity, so-called Resistive Wall Modes, RWMs. Such modes and methods for their stabilization have been systematically studied in the DIII-D tokamak in last years. From the beginning, the experiments on RWM stabilization in the DIII-D have been done with external stabilizing coils (the so-called C-coils) [1-5]. Recently a new set of the coils for stability control has been installed inside the DIII-D vacuum vessel [4-6]. In the experiments, the feedback using these new Internal-Coils was found to be more effective when compared to previously operated system with the External-Coils located outside the vacuum vessel [4-6].

Here we analyse the RWM feedback system with internal coils using analytical approach similar to that described in [7-9]. The approach is based on cylindrical approximation. The model, which was tested against the numerical results and was found reliable [10, 11], is generalized so that it allows now to consider the RWM feedback control with either external (EC) or internal (IC) correction coils, or with both. The IC and EC feedback schemes are compared via the gain factor K necessary for suppressing the mode with a given growth rate. For quantitative estimates, a proportional control is assumed.

2. Theoretical model

We use a model described in detail in [7-9]. Here its main features only are outlined with emphasis on new elements that appear when internal coils are included into the analysis.

In the cylindrical approximation the amplitude of the (m,n) harmonic of the radial perturbed magnetic field on the wall, B_m, is described by the equation

$$\tau_w \frac{\partial B_m}{\partial t} = -2\mu B_m^{wall}, \tag{1}$$

where $\tau_w = \mu_0 \sigma r_w d$ is the 'wall time', σ, r_w, and d are the conductivity, minor radius and thickness of the wall, respectively, $\mu = |m|$, and B_m^{wall} is the part of B_m created by the currents induced in the wall. Equation (1) is a direct consequence of the Maxwell equations and Ohm's law for a conducting wall and long-wavelength perturbations. In a general case, we can distinguish four sources of the magnetic perturbation $\mathbf{b}$: plasma, internal correction coils (IC), the vessel wall, and the currents in the external region (behind the wall). These sources make additive inputs to the perturbation, so that

$$B_m = B_m^{pl} + B_m^{IC} + B_m^{wall} + B_m^{ext}. \tag{2}$$

Equation (1) gives B_m^{wall} if the time dependence of B_m is known. To convert equation (1) into equation for B_m, we need an additional relationship between B_m^{wall} and B_m. This can be obtained from boundary conditions for the perturbed magnetic field at the plasma surface, as discussed in [8, 9]. Finally, following the logic described in [8, 9], we come to the linear dependence

$$B_m^{wall} + B_m^{ext} = -(\Gamma_m B_m + \alpha B_m^{IC}) / 2\mu, \tag{3}$$

where

$$\alpha = \Gamma_m \left(x_c^{-2\mu} - 1 \right) + 2\mu x_c^{-2\mu} , \tag{4}$$

and Γ_m is a parameter depending on the properties of the equilibrium configuration. Using (3), we can transform equation (1) into

$$\tau_w \frac{\partial B_m}{\partial t} = \Gamma_m B_m + \alpha B_m^{IC} + 2\mu B_m^{ext} . \tag{5}$$

This is the main equation for the further analysis. In this equation, the internal and external correction coils are presented separately by terms with B_m^{IC} and B_m^{ext}, which provides a direct way for comparison of different stabilizing systems. Such equation with $B_m^{IC} = 0$ was used for analysis of the RWM feedback stabilization with the external correction coils [7, 8] and in studies of the error field amplification [9, 12]. Similarity of (5) and the equation used earlier allows the same technique be applied here. It worth noting, however, that the terms with B_m^{IC} and B_m^{ext} in (5) may produce essentially different effects since α depends on Γ_m, while the other coefficient 2μ is constant.

In equation (5), B_m^{IC} is a known quantity determined by the geometry of the internal coils and the IC currents. It can be shown that Γ_m is determined by the perturbed magnetic field in the plasma [8, 9]. At the same time, it follows from (5) that

$$\Gamma_m = \tau_w \left(\gamma_0 + in\Omega_0 \right), \tag{6}$$

where γ_0 is the instantaneous growth/decay rate of the mode, and Ω_0 is the angular frequency of the mode toroidal rotation, when $B_m^{ext} = B_m^{IC} = 0$. They can be found from magnetic measurements outside the plasma, as discussed in [9, 12]. This allows one to close the problem by using experimental data without finding $b_m(r)$ in the plasma. Below Γ_m is assumed constant.

3. Feedback algorithms

Equation (5) allows one to consider the RWM feedback control with either external or internal correction coils, or with both. For analysis of a feedback system we must prescribe a dependence of the feedback-produced magnetic field B_f (either B_m^{IC} or B_m^{ext}) on the input signal. We consider here a simple proportional control

$$B_f = -K \times (\text{input signal}) . \tag{7}$$

In [7, 8] equation (5) was used for EC systems with input signals that can be measured by radial or poloidal sensors. Here we perform similar analysis for the feedback with $B_f = B_m^{IC}$.

From the equation $\text{div}\,\mathbf{b} = 0$ we obtain

$$i \frac{m}{\mu} B_\theta^{w-} = B_m \left(\Gamma_m / \mu + 1 \right) + B_m^{IC} \alpha / \mu , \tag{8}$$

where B_θ^{w-} is the amplitude of $b_{\theta m}$ at the inner side of the wall. We have used here the definition (3) and approximate expression

$$b_m = B_m^{in} x^{-\mu-1} + B_m^{out} x^{\mu-1} \tag{9}$$

for the amplitude of the radial component of $\mathbf{b}$ in vacuum. Also we have

$$i \frac{m}{\mu} B_\theta^{w+} = B_m - 2B_m^{ext} , \tag{10}$$

where B_θ^{w+} is the amplitude of $b_{\theta m}$ at the outer side of the wall. When $B_m^{ext} = 0$, this gives B_m, the same signal as measured by the radial probes.

4. IC feedback systems with radial and poloidal sensors

Two cases are considered here: with input signals B_m and B_θ^{w-}, assuming that the control field is created by the internal coils only ($B_m^{ext} = 0$). Note that according to (10) the feedback system with poloidal probes outside the wall must be (within the model) equivalent to that with radial probes when $B_m^{ext} = 0$.

(1). *Radial sensors.* We assume here

$$\text{input signal} = B_m . \tag{11}$$

With

$$B_m^{IC} = -K_r B_m \tag{12}$$

the main equation (5) becomes

$$\tau_w \frac{\partial B_m}{\partial t} = (\Gamma_m - \alpha K_r) B_m . \tag{13}$$

Then for RWM suppression we need

$$K_r > \frac{\Gamma_m}{\alpha} = K_{\max}\left(1 - \frac{1}{1 + x_c^{2\mu} K_0 / K_{\max}}\right), \tag{14}$$

where $K_0 \equiv \Gamma_m / (2\mu)$ and

$$K_{\max} \equiv 1/(x_c^{-2\mu} - 1) . \tag{15}$$

It follows from (14) that, for unstable modes ($K_0 > 0$), a fixed value $K_r = K_{\max}$ would guarantee fulfilment of the stability condition (14). This value is quite reasonable even for the coils placed very close to the wall: for the $m = 2$ mode, we obtain from (15) $K_{\max} < 6$ for $x_c < 0.96$. Smaller x_c results in smaller $K_{\max}$ as shown in Fig. 1.

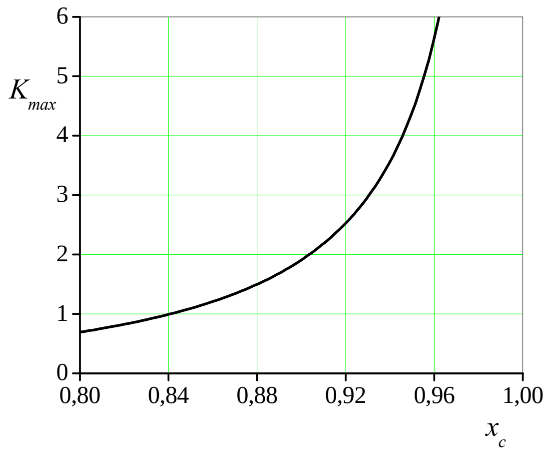

Fig. 1. Dependence of $K_{\max}$ on the in-vessel coil position ($m = 2$).

The result (14) expressed in terms of α allows easy comparison of systems with internal or external coils: in the latter case, α must be replaced by 2μ in (14). The mode is unstable when $\Gamma_m > 0$, while $\alpha > 2\mu$ for $\Gamma_m > -2\mu$ and $x_c < 1$, see (4). Therefore, we can conclude that smaller gains are needed for RWM stabilization with IC system, as compared to similar feedback using the external coils.

With EC system ($\alpha = 2\mu$), larger gains would be needed for the modes with larger growth rates, and with fixed K (for example, $K = K_{\max}$) the RWM stabilization with external coils may be possible only for the modes with $\Gamma_m < 2\mu K$.

(2). *Poloidal sensors inside the vessel*. Consider the feedback system with

$$\text{input signal} = i(m/\mu) B_\theta^{w-} . \tag{16}$$

Then for the proportional control (7) with $K = K_\theta$ for perturbations $\propto \exp(\gamma t)$ we obtain from the main equation (5)

$$\gamma\tau_w = \frac{\Gamma_m - \alpha K_\theta}{1 + \alpha K_\theta / \mu}. \tag{17}$$

The mode becomes stable ($\gamma < 0$) with either $K_\theta > \Gamma_m / \alpha$, which is the same condition as (14), or $K_\theta < -\mu / \alpha$. These two possibilities can be combined into

$$|K_\theta| > \max\left\{\frac{\Gamma_m}{\alpha}, \frac{\mu}{\alpha}\right\}, \tag{18}$$

which can be satisfied by

$$|K_\theta| > K_{\max} \tag{19}$$

for any positive Γ_m. Criterion (19) determines the absolute value of K_θ leaving its sign arbitrary. This is a great benefit from the practical viewpoint. In particular, this eliminates a danger of the phase instability when the plasma perturbation 'slips' toroidally with phase reversal, escaping from the action of the applied stabilizing field.

5. Conclusion

In the cases considered, the results strongly depend on the position of the coils inside the vessel: smaller gains are needed to stabilize the RWM for smaller x_c, i.e., when the coils are placed more close to the plasma. This is a natural expected result. Most important may be that the feedback system with internal coils using the proportional algorithm (7) with either radial or poloidal sensors can be operated with constant gains, independent of Γ_m. In both respects, the systems with internal coils look better than similar systems [7, 8] with external coils.

In DIII-D experiments [1-3, 5] the feedback with external coils demonstrated better performance with poloidal probes as compared to that with radial probes, in agreement with numerical [3-5, 10, 11] and analytical [7, 8] results. Our analysis shows that the same should be expected for the feedback with in-vessel coils. The analysis is based on the single mode approximation which may overestimate the efficiency of the feedback [8]. More precise quantitative results will follow from numerical modelling.

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