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Statistical Estimate of Long-Range Time Dependency in HL-1M edge Plasma Turbulence

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Dynamical processes with long-range time correlations have been observed in a variety of subjects, such as magnetically confined plasmas, sand piles and so on. Some different plasma turbulence models have predicted the existence of long-range time dependencies [1-4], which could significantly affect the cross-field turbulent transport in magnetically confined plasmas by means of radial "avalanches" or "heat pulses". There are roughly two types of tools used in assessing the presence of such dependencies in experimental data: (1) spectral scaling exponent α estimate analyses in power spectral density (PSD), such as spectral exponent estimate methods via Fourier transform, or spectral wavelet analysis(SWA). (2) Hurst exponent H assessment methods, such as rescaled range (R/S) technique, structure functions (SF's) technique, detrended fluctuation analysis (DFA), scaled windowed variance analysis, maximum likelihood estimators, and dispersional analysis. The relationship between the spectral exponent α and Hurst exponent H is known to be $\alpha = 2H + 1$ for fractional Brownian motion (FBM) and $\alpha = 2H - 1$ for fractional Gaussian noise (FGN).

For electrostatic fluctuations in ohmically heated HL-1M edge plasma around the zero-velocity flow point, it is measured by Langmuir probes and digitized with frequency 1 MHz, the relevant turbulent particle flux displays long time correlations. SWA[5] is a new and powerful method for studying the fractal properties of a signal. For a stationary stochastic process in discrete time, $\{x(t)\}$, the continuous wavelet transform is defined by

$$W(\tau, b) = \frac{1}{\sqrt{\tau}} \int x(t) \cdot \Psi_{\tau b}(t) dt$$

The original signal can be recovered from its continuous wavelet transform via

$$x(t) = \frac{1}{C_\Psi} \int_0^\infty \frac{d\tau}{\tau^2} \left[\int W(\tau, b) \cdot \Psi_{\tau b}(t) \cdot db \right]$$

Where C_Ψ is a constant depending on the wavelet. Let $S_W(\tau) = W^2(\tau, b)$, which is then the wavelet spectral density function that gives the contribution to the energy at the scale a.

$S_W(\tau) \propto \tau^{\alpha}$, the exponent α is related to the Hurst exponent H in the same way as in the conventional Fourier analysis. Fig.1 shows the spectral exponent α obtained by FFT and SWA analyses.

We use R/S technique[6,7], DFA method and SF's technique to estimate the Hurst exponent *H*. An example of R/S plot is presented in Fig.2(a), and its relevant DFA plot is presented in Fig.2(b). DFA[8] was originally proposed as a technique for quantifying the nature of long range correlations by Peng et al. in 1994. It consists the following steps.

(1) From $\{x(t)\}$ with $1 \le t \le N$, a running summation sequence $y(t)$ is constructed as $y(t) = \sum_{k=1}^{t} x(k)$. The entire sequence is divided into M nonoverlapping blocks $y_m(t)$, each containing k samples. In each block, a least squares line is fit to the data. The y coordinate of the straight line segments is denoted by ys_m. A detrended signal is defined for each block as the difference between the original and the local trend for that block, leading to $y_{m,d}(t) = ys_m - y_m(t)$. The variance of the detrended signal is calculated for each block. $F(k)$ is then defined as the average of the variances over all blocks:

$$F(k) = \frac{1}{M}\sum_{m=1}^{M} \mathrm{var}(y_{m,d}(t))$$

It has been shown by Buldyrev et al. that $F(k)$ varies as a power law in k, i.e., $F(k) \approx k^{\xi}$ for sequences with power law long range correlations such as FGN, with $\xi - 1 = \alpha$, the spectral power law exponent.

Now we refer SF's method[9-11] to calculate the Hurst exponent of turbulent particle flux in ohmically heated HL-1M edge plasma. An example is shown in Fig. 3. In this method, the stationary time scale of the raw data, $\tau_1 \le \tau \le \tau_2$, has to be determined first. That is, to observe the $S_{X,q}(\tau)$ vs τ log-log plot, in which the slope $\xi(q)$ is nearly zero within the time range of $\tau_1 \le \tau \le \tau_2$. In Fig. 3(a) the q-th order SF of the raw data, $S_{X,q}(\tau)$, is plotted vs τ for q=1.0, 2.0,3.0, 4.0, 5.0. It can be seen that $S_{X,q}(\tau)$ is approximately constant for time lags between about 100 μs and 2000 μs, indicating that the data are stationary in this range. Then the Hurst exponent, $H(q) = \xi(q)/q$, is determined from the slope $\xi(q)$ of the SF of the integrated data, $S_{W,q}(\tau)$, vs τ log-log plot within the stationary range. In this case, its $S_{W,q}(\tau)$ vs τ log-log plot is presented in Fig. 3(b) for q=1.0, 2.0,3.0, 4.0, 5.0, and its averaged H value is obtained from the slope of relevant $qH(q) \sim q$ plot, as shown in Fig. 3(c). The H(q=2) value obtained by SF method is about 0.74, less than that ($H \sim 0.76$) obtained by R/S technique or that ($H \sim 0.77$) obtained by DFA method. The relationship between the spectral exponent α and Hurst exponent H is known to be $\alpha = 2H + 1$ for fractional Brownian motion(FBM) and $\alpha = 2H - 1$ for fractional Gaussian noise(FGN)[10,11]. By checking if spectral exponent α and Hurst exponent H satisfy such relations, results show that SWA is a proper method to obtain the spectral exponent and SF's method shows advantages to get Hurst exponent. Considering $\alpha = 2H - 1$ relation, SWA and SF's (or R/S, DFA) methods are not independent to each other.

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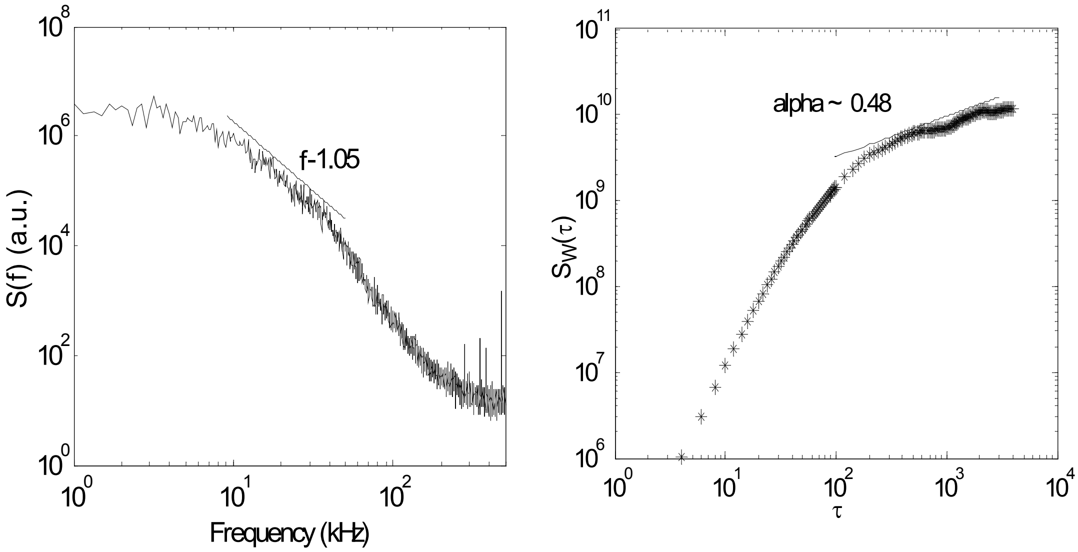

Fig.1 The spectral exponent α obtained by FFT and SWA analyses.

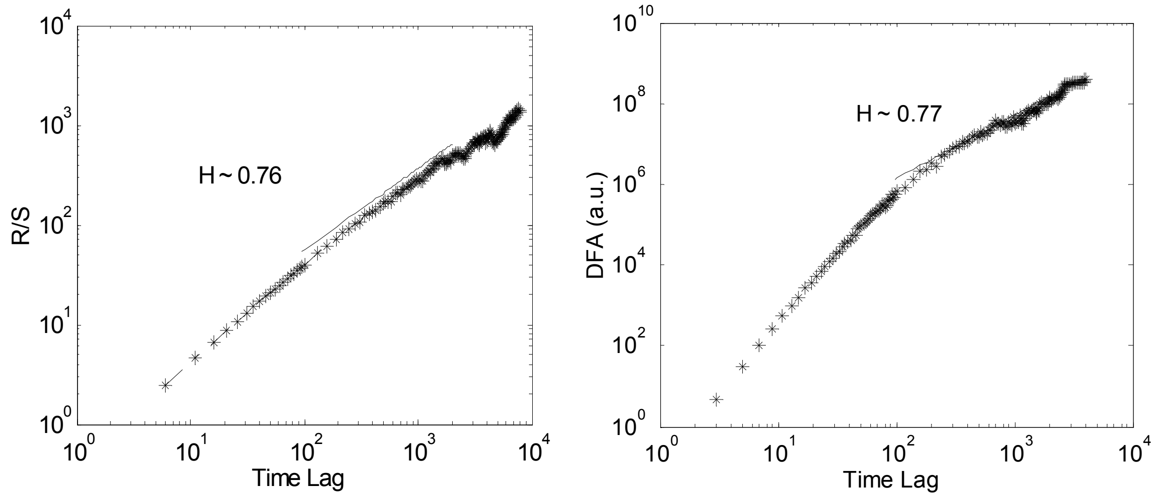

Fig.2 *H* values obtained by R/S and DFA analyses.

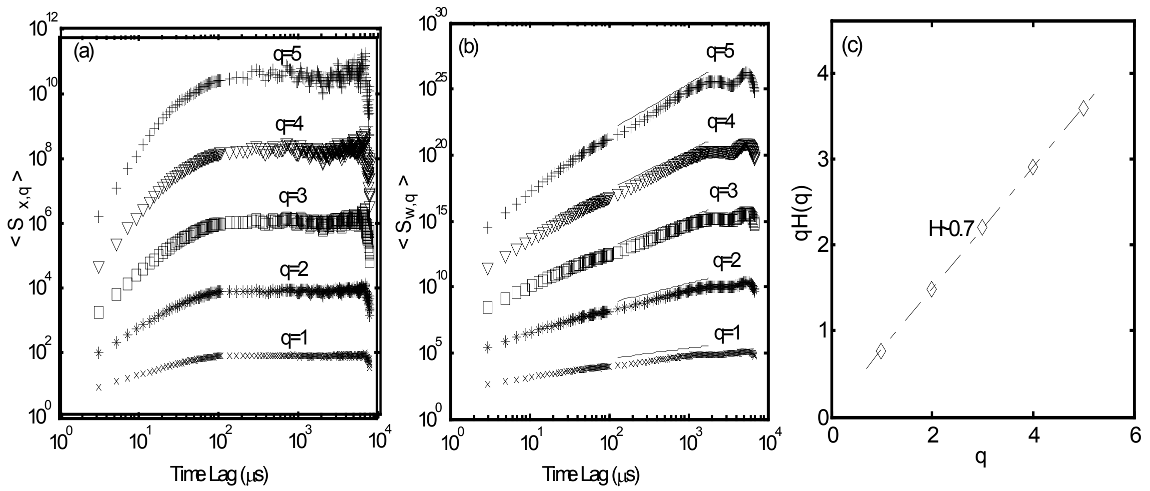

Fig.3 *H* value obtained by SF's method