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Harmonic Grid Generation for the Tokamak Edge Region

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Abstract

This paper presents a novel method of adaptive grid generation for magnetically confined plasmas, designed both to align the grid with the evolving magnetic field and to concentrate the grid in regions of rapid variation. Special emphasis is placed on the Tokamak edge region.

Adaptive grid generation for computational studies of magnetized plasmas is a novel challenge because of their extreme degree of anisotropy.[1, 2] The nature of magnetic confinement is to restrict the motion of particles across but not along the magnetic field. For relatively collisional plasmas described by fluid equations, this results in a ratio of parallel to transverse thermal conductivities $\chi_\parallel/\chi_\perp \sim 10^{10}$. Discretization errors which cause a small amount of the large parallel heat flux to "leak" into the transverse direction can fatally compromise the validity of the numerical results. Anisotropy also strongly affects the propagation of magnetohydrodynamic (MHD) waves. Waves propagating normal to the magnetic field have much smaller phase velocity than those propagating along the field. Since these slow waves are the most readily destabilized by magnetic field gradients and curvature, accurate representation of small parallel gradients is essential to accurate modeling of linear and nonlinear instabilities. In addition to alignment, we also consider adaptation to large transverse gradients characteristic of resistive instabilities and magnetic reconnection.[3]

The most successful method of dealing with anisotropy has been the use of a flux coordinate system in which at least one coordinate ψ is chosen to be orthogonal to the magnetic field.[4, 5] This has previously been implemented only for static, axisymmetric magnetic fields. As the magnetic field evolves due to currents in the plasma and external coils, the static grid is left behind and loses its alignment. While the initial configuration may be axisymmetric, for which nested flux surfaces are known to exist, the field may evolve into a nonaxisymmetric configuration in which no exact flux surfaces exist, but rather there are regions of multiple small islands and stochasticity. Our approach is to use a variational principle to find a best fit to an aligned grid.[6]

The approach described here makes use of a smooth mapping from a small number of logically rectangular grid blocks to a curvilinear grid which may be aligned with the magnetic field and adapted to regions of rapid variation. The grid may be either occasionally updated or moved continuously with the evolving solution, incorporating the grid motion into a single implicit time step.

The concept of harmonic grid generation originated with the recognition that the solution to Laplace's equation in a Euclidean domain with arbitrary boundaries provides a uniquely smooth, well-distributed grid. This was extended to curvilinear manifolds by the incorporation of the metric tensor $\mathbf{g}$ describing the curvature of the manifold.[6, 7] The novel feature of the present approach is the recognition that $\mathbf{g}$ can be freely chosen to give the grid additional desired properties rather than choosing it to describe a given curvilinear manifold. The job of constructing the grid thus rests entirely on the choice of $\mathbf{g}$.

Our method of harmonic grid generation is based on the variational principle

$$\mathscr{L} = \frac{1}{2}\int_\Omega \frac{1}{\sqrt{g}} \nabla\xi^i \cdot \mathbf{g} \cdot \nabla\xi^i d\mathbf{x}, \quad \frac{\delta\mathscr{L}}{\delta\xi^i} = 0 \tag{1}$$

where $\mathbf{g}(\mathbf{x})$ is a contravariant metric tensor; g is its determinant; the ξ^i are the coordinates in the logical domain Ξ, with repeated indices implying summation; and Ω is the parametric domain, parameterized by coordinates x^i. The Euler-Lagrange equation for an extremum of $\mathscr{L}$ is Beltrami's equation,

$$\nabla \cdot \left(\frac{1}{\sqrt{g}} \mathbf{g} \cdot \nabla\xi^i \right) = 0. \tag{2}$$

We use this equation to generate the inverse mapping $\mathbf{x}(\xi)$ from the logical to the parametric domain, then use it to map a uniform Cartesian grid in the logical domain into an adapted grid in the parametric domain. To obtain the inverse mapping, we express the divergence in the logical coordinates ξ^j,

$$\frac{1}{\mathscr{J}} \frac{\partial}{\partial\xi^j} \left(\frac{\mathscr{J}}{\sqrt{g}} g^{kl} \frac{\partial\xi^i}{\partial x^k} \frac{\partial\xi^j}{\partial x^l} \right) = 0, \tag{3}$$

with $\mathscr{J} \equiv \det \partial\mathbf{x}/\partial\xi$ the Jacobian of the transformation $\mathbf{x}(\xi)$ and with the derivatives $\partial\xi^i/\partial x^j$ expressed in terms of the inverse derivatives $\partial x^i/\partial\xi^j$. Boundary conditions are imposed such that the boundary of the parametric domain is held fixed and the coordinates are orthogonal at the boundary.

The need for a field-aligned coordinate system can be appreciated by considering the heat equation in a magnetized plasma,

$$\frac{\partial T}{\partial t} = \nabla \cdot (\chi \cdot \nabla T) = \frac{1}{\mathscr{J}} \frac{\partial}{\partial \xi^i} \left(\mathscr{J} \chi : \nabla \xi^i \nabla \xi^j \frac{\partial T}{\partial \xi^j} \right). \tag{4}$$

By far the largest component of the anisotropic thermal conductivity tensor χ is $\chi_{||}\mathbf{bb}$, with $\mathbf{b}$ the unit vector along the magnetic field $\mathbf{B}$. If $\mathbf{b} \cdot \nabla \xi^i$ is of order unity for all logical coordinates ξ^i, then the much smaller transverse terms in Eq. (4) involve the difference between large terms, resulting in a loss of numerical accuracy. If it vanishes, or nearly so, for all but one coordinate, this inaccuracy can be avoided. Similar considerations hold for other manifestations of magnetic anisotropy.

For simple magnetic fields, such as those in the core region of the tokamak, it is possible to define a flux coordinate ψ labeling the magnetic surfaces, satisfying $\mathbf{B} \cdot \nabla \psi = 0$ exactly. In more complicated cases, such as nonaxisymmetric magnetic fields with multiple islands and regions of stochasticity, this is not possible. For the choice of metric tensor $\mathbf{g} = \mathbf{BB}$, Eq. (1) may be interpreted as a variational principle for minimizing $\mathbf{B} \cdot \nabla \xi^i$ for one of the coordinates and maximizing it for the others. This is too simple, however, because such a metric tensor is singular, $g = \det \mathbf{g} = 0$. In two dimensions, we choose

$$\mathbf{g} = \mathbf{B}_1 \mathbf{B}_1 + \mathbf{B}_2 \mathbf{B}_2 + \varepsilon(\mathbf{x}) \mathsf{I}, \tag{5}$$

where $\mathbf{B}_1 = \mathbf{B}$ is the magnetic field; $\mathbf{B}_2 = k\hat{\mathbf{z}} \times \mathbf{B}$ is orthogonal to the magnetic field, with k an adjustable parameter; and we add a small isotropic term proportional to a specified function $\varepsilon(\mathbf{x})$ which allows the grid to relax to isotropic in the neighborhood of critical points where $\mathbf{B}$ vanishes.

The region at the edge of the tokamak is of critical importance to its performance as a potential fusion reactor.[8] There is an x-point connected to a separatrix dividing the domain into closed field lines in the hot core where the main fusion reactions occur; and open field lines further out, which make contact with material walls. MHD edge-localized modes (ELMs) bridging the pedestal region at the edge of the core and the scrape-off layer outside create a region of turbulence which determine the presence or absence of a transport barrier and thus the quality of confinement in the core. Hot plasma from the core escapes from pedestal region to the scrape-off region and encounters metallic divertor plates, where ionization and recombination occur and the plasma is subjected to impurities. Accurate modeling of this region is essential to an understanding of the tokamak. Adaptive grid generation can make important

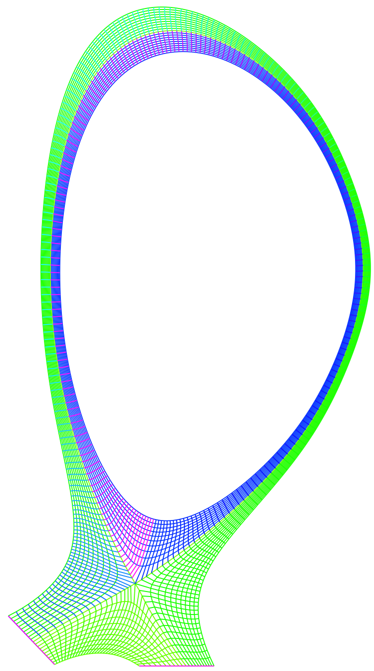

Figure 1: Field-aligned grid for the tokamak edge region.

contributions to the accuracy and efficiency of such a model. Figure 1 shows a grid generated for this region by our method.

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