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Topological instability in plasma turbulence model

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In toroidal geometry and for tokamak edge plasmas, the underlying instability of the resistive pressure-gradient-driven turbulence is the so-called resistive ballooning mode. In this paper, we use for the calculations the set of reduced MHD equations in the electrostatic limit described in [1]. The parameters correspond to a medium size tokamak with circular cross section. The numerical scheme, described in [1], is based on the Fourier expansion in the poloidal θ and toroidal ζ angles

$$\Phi = \sum_{m,n} \Phi_{m,n}(\rho,t)\sin(m\theta + n\zeta)$$

$$p = \sum_{m,n} p_{m,n}(\rho,t)\cos(m\theta + n\zeta), \qquad (1)$$

where p is the pressure, and Φ is the velocity stream function.

As discussed in [2], the solution of the set of equations depends critically on the value of the parameter β_0. For a fairly small value of $\beta_0 = 0.003$ there is a time relaxation of all Fourier components to a quasi-stationary regime with only the $n = 25$ mode dominating the spectrum. Because of the toroidal geometry, each toroidal mode has many poloidal components with diferent m values. At higher β_0 several toroidal modes may compete for dominance and the evolution shows a transition to a turbulent state.

To understand transport in this system is necessary to understand the structure of the flows. We use particle tracers to study them. The passive particle (tracers) dynamics is described by the equation

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}_\perp(\mathbf{r},t)\ . \qquad (2)$$

For moderated values of β, trajectories of particle tracers show chaotic behavior with a positive value for the Lyapunov number. For lower values of β, the Lyapunov number is either zero or very small. The filamentary surfaces can result in stochastic jets of particles that cause a “topological instability” [3]. This is the situation we analyze here.

Because of the form of the perpendicular velocity in terms of the stream function, $\mathbf{V}_\perp = -\nabla\Phi \times \mathbf{B}$, the stream function is an effective hamiltonian for the tracers, Therefore, when we refer to iso-surfaces, they are surfaces with a constant stream function Φ.

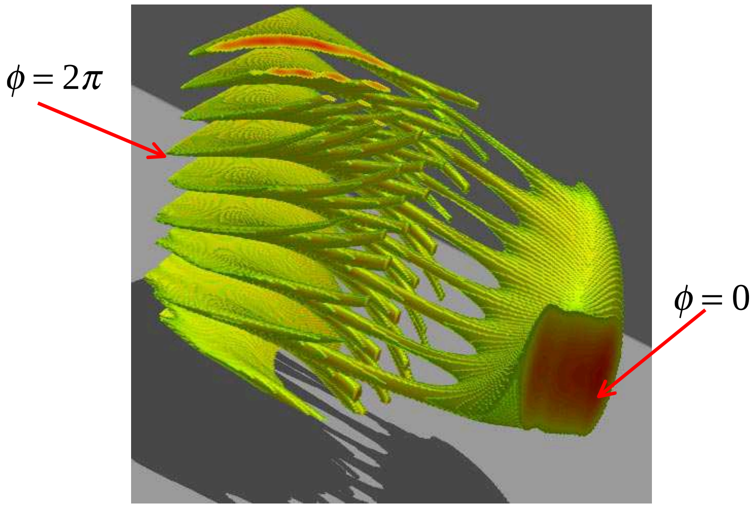

Figure 1: View of an iso-surface and its filamentation going once around the torus.

Even in the case of a single toroidal mode, the structure of the iso-surfaces is very complicated. In the outer region of the torus, the different poloidal components tend to reinforce themselves. They create eddy structures that are elongated in the radial direction (streamers) [4]. However, in the inner region of the torus, there is cancellations between the poloidal components and it results in multiple eddies. All streamers are radially elongated and interconnected through the filaments. Because they wrap on a torus, their direct visualization is not necessarily helpful. To really visualize the three-dimensional character of this structure, we have to unwrap them from the torus. To do so, we plot a Φ isosurface by first considering a single streamer in the $\zeta = 0$ plane and following it along the torus. In the $\zeta = 0$ plane, we draw a box around the streamer, and we extract the data within the box. We do the same type of same thing in each the $\zeta = constant$ planes, but we change the size and shape of the box to fit in it the portion of the constant-Φ surface that is coming from the streamer that we selected in the $\zeta = 0$ plane. The size of the box varies as we move around the torus. Therefore, the figure obtained by this method only provides information on the topology of the structure. Fig. 1 shows one rotation along of the torus. Each filament does not return back to the same streamer but it instead does a twist and returns to another streamer.

To get some statistical properties of the particle tracers, we have implemented several diagnostics. One diagnostic is the so-called ε-separation of trajectories for small ε [3]. Let $d(t)$ be the distance between two trajectories with different initial conditions and let $d(0) \ll R$ with R as a characteristic size of the system. We consider condition

$$d(0) < d(\Delta t) = \varepsilon \ll R \tag{3}$$

where Δt is a time of ε-separation. For different initial pairs of trajectories will be different Δt and one can consider distribution function $P_\varepsilon(\Delta t)$. This function is different from the p.d.f. of Lyapunov exponents since for the definition of Λ one needs a condition $\varepsilon \sim R$.

In the case of pseudochaos $\Lambda = 0$ and dispersion of trajectories is polynomimal, i.e.

$$\ln d(\Delta t) = \ln \varepsilon \sim \ln \Delta t + \text{const} \tag{4}$$

and we expect a power law

$$P_\varepsilon(\Delta t) = \text{const.}/(\Delta t)^{\gamma_\varepsilon} \tag{5}$$

where γ_ε is some constant exponent for large Δt. We call Δt separation time and $P_\varepsilon(\Delta t)$ p.d.f. of separation time, or simply distribution of separations.

To check the distribution of separations, tracer particle dynamics was considered with equation

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}_\perp(\mathbf{r},t) + V_0\mathbf{b} \tag{6}$$

Here V_0 is a velocity along the magnetic field (compare to (2)). Because of the symmetry of the problem the constant velocity V_0 can be arbitrary. In practice, it should be $V_0 \ll V_\perp$, otherwise many particles will never feel the complicated structure of the velocity field $\mathbf{V}_\perp(\mathbf{r},t)$. The results of simulation are presented in Fig. 2. Particle trajectories stay very close while they travel in tubes of filaments, and they mix (weakly) in the streamers. The narrow tubes have an angular length poloidally of about π. This implies a length toroidally of πpR where q is the so-called safety factor, or rotational number. From this the maximum separation time is

$$\max \Delta t \sim \pi q R/V_0 \; . \tag{7}$$

As V_0 increases, the $\max \Delta t$ decreases and $P_\varepsilon(\Delta t)$ exponentially decays due to randomly distributed initial conditions.

When V_0 is small or zero, the estimate (7) does not work and the random initial conditions play a different role since the mixing is defined by a random walk from one tube to another through the "free" space in streamers. In this case, assuming a pseudochaotic behavior of particles, one can expect the law (4), and that is evident from Fig. 2 for $V_0 = 20$ with

$$\gamma_\varepsilon = 2.18 \pm 0.2 \tag{8}$$

The calculations for this case were done with an initial separation $d(0) = 0.001$ and for $\varepsilon = 0.003$. The statistics vary between 7 and 50 millions of events providing a very reliable result.

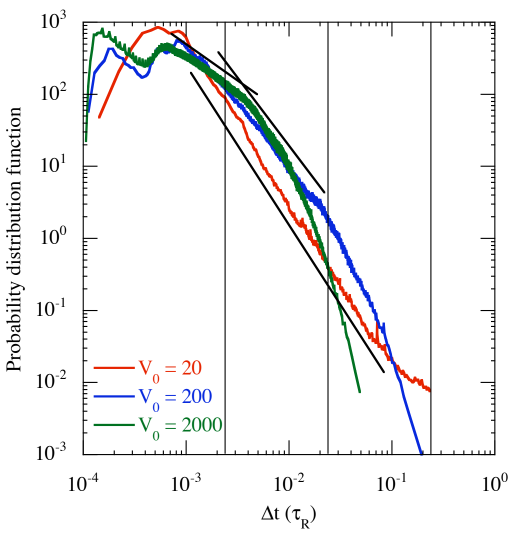

Figure 2: Distribution of separation time for different values of V_0.

We have also measured the distribution of Poincaré recurrences. To do so, we start a bunch of tracer particles in a localized region in the plane $\zeta = 0$. For each particle, we calculate the time to come back to this region. From this measuremets, we calculate the probability distribution. For the results presented here, the region in the $\zeta = 0$ plane is defined by $0.65 < \rho < 0.75$ and $-0.12 < \theta < 0.12$.

In the case of a power law distribution and large t, the asymptotics for p.d.f. $P(t)$ of recurrences to a small domain in phase space at time t coincides with p.d.f. of separations $P_\varepsilon(\Delta t)$ [5]. On this basis one can expect that

$$\gamma_\varepsilon = \gamma_{\text{rec}} \tag{9}$$

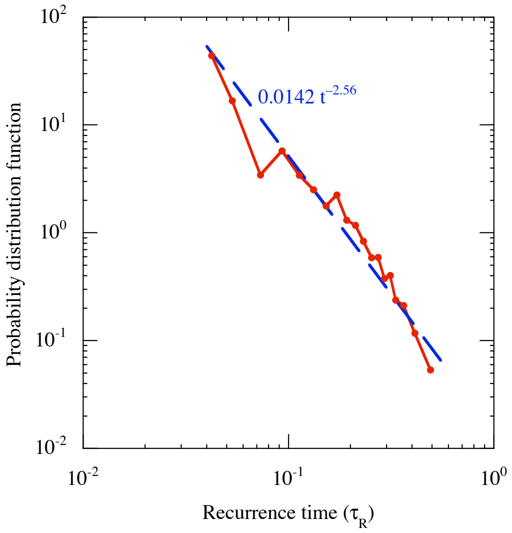

Figure 3: Distribution of Poincaré recurrences to the domain: $0.65 < r < 0.75$; $-0.12 < v < 0.12$, $\zeta = 0$. The statistics correspond to 38878 events.

where γ_{rec} is the exponent of $P(t)$. While it is difficult to get a good statistics for the p.d.f. of recurrence time, the data in Fig. 3 are in a fairly good agreement with (8).

A multiplicity of bars can be used as a model of an iso-surface with filaments [3]. Detailed simulations of a multi-bar-in-square billiard model show that the recurrence exponent γ is 2.15 when the number of bars tends to infinity, in a good agreement with γ_ε in (8).

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