Amplitude variation and frequency shift of a reflectometer signal propagating in a time varying plasma

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Reflectometry is now routinely used to study the density fluctuations in fusion plasmas. The level of density fluctuations is usually inferred from the fluctuations of the reflectometry signal phase. However the amplitude variation and the frequency shift of the signal can also provide information on the plasma displacement and on the time evolution of the density gradient length. A simple model based on electromagnetic flux conservation gives the amplitude variation of the reflected wave as a function of the time derivative of the time of flight:

$$E(t) = E_o / \sqrt{1 + \frac{\partial \tau}{\partial t}}$$

where E_o is the incident wave amplitude, τ is the time of flight and *E(t)* is the amplitude of the reflected wave. This effect has been clearly seen during simulation of O-mode and X-mode fast swept reflectometer applied to steady state inhomogeneous plasma on [1]. In the case of a linear density profile the reflected electric field associated to the O-mode Frequency Modulated Continuous Wave reflectometer can be approached by

$$E(t) = E_o / \sqrt{1 + 8 \frac{\partial}{c} \frac{L}{\partial_{pe}^{2}} \frac{\partial \partial}{\partial t}}$$

where L is the density gradient length, ω is the probing pulsation, ω_{pe} is the plasma frequency and ∂ω/∂t is the frequency sweep rate. For typical parameters of tokamak plasma this effect arises when the frequency sweep rate becomes greater than 300 GHz/μs for the O-mode and 100 GHz/μs for the X-mode for typical tokamak plasma parameters. In the meantime the probing electromagnetic field frequency can be also Doppler shifted due to a plasma displacement. In this situation, which can be exemplified with a global displacement of the plasma at constant velocity, the amplitude of the reflected wave remains unchanged for plasma displacement velocity small compared to light velocity. From a full-wave code, one can see the cut-off layer moving in the direction opposite to the plasma displacement. These observations, suggesting a frequency shift $\Delta\omega$ of $2k_oV_o$, have been verified for continuous injected waves as well as for pulses. In general the plasma displacement is slow and the frequency shift can be negligible compared to the beat frequency given by fast frequency sweep reflectometer. But strictly speaking, this Doppler shift induces an error on the cut-off position as all mechanisms inducing frequency shift do. These processes are associated to the time evolution of the density profile. First are considered the cases where the cut-off layer position is fixed whereas the density gradient length varies continuously in time, which can be associated to a plasma expansion or to a density profile relaxation or to the beginning of supersonic gas injection. The model described in reference [2] (equation 4.121) shows the link between the index variation of homogeneous plasma and the induced frequency shift,

$$\frac{\partial\partial}{\partial t} = \frac{1}{2\partial}\frac{\partial\partial_{pe}^{2}}{\partial t}$$

One can see under the assumption of slab model with homogeneous density fluctuation probing by a fixed frequency reflectometer one can notice that the frequency shift width due to the time variation of the plasma density in the vicinity of the cut-off layer. Moreover these density fluctuations induce a time variation of the time of flight, which causes amplitude variation of the reflected wave. An analytical expression can be obtained for a simplified perturbation model corresponding to a step function of width l_f and position x_f that is modulated in time by a function *f(t)*. We use WKB approximation to determine the variation of the time of flight in the case of a linear density profile probed with an O-mode fixed frequency reflectometer. Assuming that the density perturbation induces small variation of the index, the time evolution of the time of flight can be approximated by

$$\delta\tau(t) = f(t)\frac{l_f}{c}\frac{\sqrt{x_c}}{\sqrt{x_c - x_f} + \sqrt{x_c - x_f - l_f}}\frac{x_c}{\sqrt{x_c - x_f}\sqrt{x_c - x_f - l_f}} \quad \text{with } x_c > x_f + l_f$$

where x_c is the cut-off position and c is the light velocity. Then the time variation of the amplitude of the electric field can be approximated by

$$\delta E(t) \sim \frac{1}{2}\frac{\tau f(t)}{\tau t}\frac{l_f}{c}\frac{\sqrt{x_c}}{\sqrt{x_c - x_f} + \sqrt{x_c - x_f - l_f}}\frac{x_c}{\sqrt{x_c - x_f}\sqrt{x_c - x_f - l_f}}.$$

This formula supposes that no secondary cut-off appears during the time evolution of the density perturbation. The situation described here corresponds to a non-zero spatial average density perturbation [3]. The same procedure can be applied to determine the time variation of the reflected electric field in the case of a zero spatial average density perturbation. In this case the time of flight variation implies a change of the reflected wave which can be described by our model as long as the WKB approximation remains valid. If some part of the probing is trapped the expression of the time of flight is no longer valid (see figure 1b).

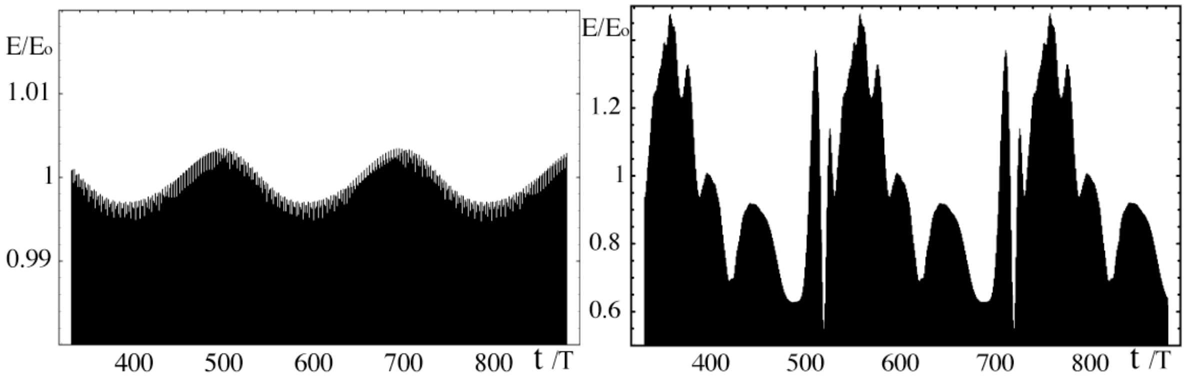

Figure 1: a) Amplitude modulation quasi-sinusoidal induced by an oscillating density perturbation described with gate shape form which can de described by WKB approximation $\Delta n/n_c$ =1% b) same case with high density perturbation $\Delta n/n_c$ =20% inducing secondary cut-offs.

The frequency is also up shifted if the density profile length increases continuously in time. Studying the propagation of pulses we can also notice that the pulse spectrum becomes narrower as the variation velocity of the density gradient length increases. In conclusion, the amplitude of the probing wave is modified only by the change of the density gradient length in the vicinity of the cut-off layer or by all of the processes modifying the time of flight in the plasma only. The displacement of the cut-off layer induces only changes of the frequency of the probing wave.

The destructive interferences are only possible in the 2D case as it is shown [4]. This phenomenon can reduce the reflected signal to very low level on very short time scale due to strict conditions on phase and amplitude in the receiver. In the case of localized oscillating density perturbation (Gaussian perturbation for instance) the response is quite similar to the 1D case if the density perturbation is on the axis of the emitting system. When the density perturbation is put off-axis, the response oscillates at the same frequency but the time evolution is different and the amplitude variation is lower it can be seen on the figure 2 where 5 receivers are used to collect the reflected field.

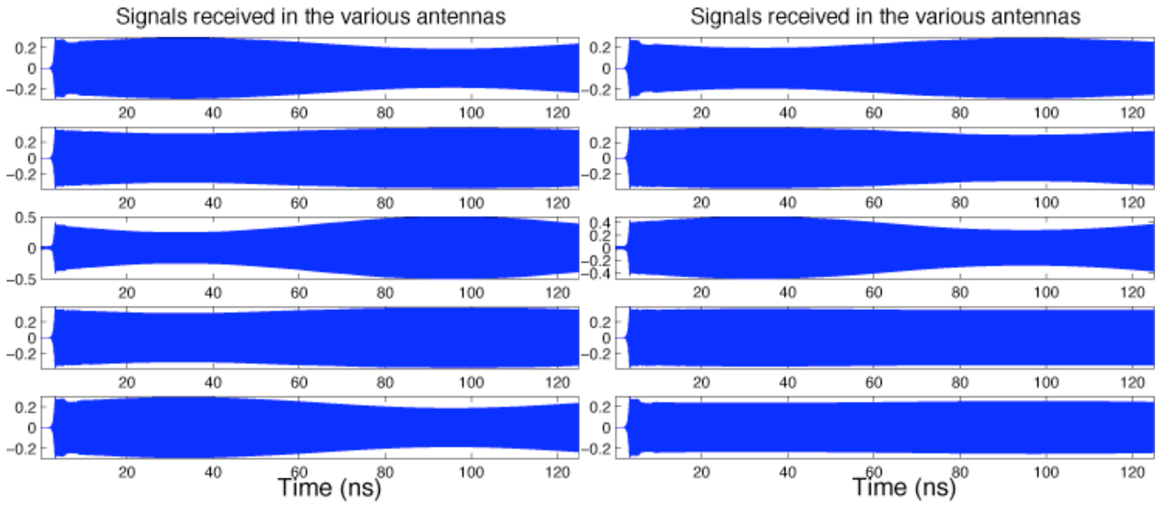

Figure 2: a) 2D full-wave simulations of amplitude modulations in the 5 receivers (separated by 6 cm where the central one corresponds to a mono-static reflectometer) over one period of an oscillating Gaussian (width = 2 vacuum wavelength) density perturbation $\Delta n/n_c$ =10% in front of a fixed frequency reflectometer (40 GHz) b) same case with the density perturbation shifted of 2 .5 cm from the launcher axis.

When the density perturbation in front of the launcher introduces secondary cut-off layer, the amplitude modulation changes dramatically, the probing field is diffracted and the signal is highly reduced in the receiver on the launcher axis. Then when the density perturbation becomes zero, the unperturbed amplitude of the reflected wave is slightly recovered. After that a density hole is formed and the amplitude of the reflected increases and evolutes on different way than the small amplitude case. However a system with multi receivers can follow single events by looking at the amplitude variations but needs 2D full-wave code simulation to interpret it.

References:

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