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MHD equilibrium reconstruction for TEXTOR: from static to stationary equilibria

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Introduction

Various experiments clearly demonstrate that the plasma inside a tokamak is not free from plasma instabilities, including magneto-hydrodynamical (MHD) instabilities. To investigate those instabilities, one has first to reconstruct the MHD equilibrium of a particular shot. That reconstruction has to been done with sufficient accuracy to perform a reliable MHD spectral analysis.

In the present paper we focus on the reconstruction of MHD equilibrium of TEXTOR shot 95022. In this shot the plasma rotates in the toroidal direction. Two reconstructions are presented. One assumes a static equilibrium and the other one takes the toroidal rotation into account. In particular, we investigate the influence of the toroidal flow on the different equilibrium quantities, like the pressure profile and the Shafranov shift.

MHD equilibrium

The plasma inside a tokamak is modeled by making use of the ideal MHD equations. For considering an equilibrium, two assumptions are made, viz. that it is time-independent and axisymmetric. The time-independence of the equilibrium can only be justified when MHD waves and instabilities are on a timescale much shorter than the dynamical timescale of the equilibrium. The axisymmetry is justified by the fact that the tokamak itself is axisymmetric. The symmetry of the equilibrium allows us to use cylindrical coordinates (R, Z, φ) (in this order!), where R is the distance to the symmetry axis.

For a static or stationary equilibrium (with purely toroidal or poloidal and toroidal flows), the magnetic field can be expressed in terms of the poloidal magnetic flux function ψ:

$$\mathbf{B} = \frac{1}{R}\mathbf{e}_\varphi \times \nabla\psi + B_\varphi \mathbf{e}_\varphi. \tag{1}$$

For a static equilibrium the ideal MHD equations reduce to the well known Grad-Shafranov equation [1], [2],

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial Z^2} = -\frac{1}{2}\frac{\mathrm{d}I^2}{\mathrm{d}\psi} - R^2\frac{\mathrm{d}p}{\mathrm{d}\psi}, \tag{2}$$

where the pressure p and the function $I \equiv RB_\varphi$ are both readily shown to be flux functions.

If one includes purely toroidal flow, one has to solve an equation very similar to the Grad-Shafranov equation,

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial Z^2} = -\frac{1}{2}\frac{\mathrm{d}I^2}{\mathrm{d}\psi} - R^2\frac{\partial p}{\partial\psi}, \tag{3}$$

where the only difference is that the pressure p is no longer a flux function. Along the poloidal magnetic field lines one has also to solve another equation,

$$\frac{1}{\rho}\frac{\partial p}{\partial R^2}\bigg|_\psi = \frac{1}{2}\Omega^2(\psi), \tag{4}$$

where ρ is the density and the rotation frequency $\Omega = v_\varphi/R$ is a flux function. This equation can be solved analytically if one assumes that either the temperature T, the entropy S, or the density ρ is a flux function [3]. The temperature can be assumed constant on a flux surface on the transport time scale due to the high thermal conductivity along field lines. That transport time scale is long compared to the Alfvén time. The assumption of isentropic flux surfaces has the advantage that it permits a natural extension to poloidal flows, where the entropy has to be a flux function [4],[5]. However, on MHD time scales, the equilibrium is not restricted to flux surfaces of constant temperature or entropy. The other possibility that the density is a flux function will not be exploited in the present paper.

In the toroidally rotating case, the equation for the pressure can best be expressed as

$$p(\psi;R) = \begin{cases} p_{\mathrm{st}}(\psi)\left[(R^2 - R_{\mathrm{M}}^2)\frac{\gamma-1}{\gamma}\Lambda_{\mathrm{S}}(\psi)+1\right]^{\frac{\gamma}{\gamma-1}} & \text{(entropy is flux function)} \\ p_{\mathrm{st}}(\psi)\exp\left[(R^2 - R_{\mathrm{M}}^2)\Lambda_{\mathrm{T}}(\psi)\right] & \text{(temperature is flux function)} \end{cases}, \tag{5}$$

where R_{M} is the radius of the magnetic axis, $\Lambda_{\mathrm{S}} \equiv \Omega^2/(2S\rho_{\mathrm{st}}^{\gamma-1})$, $\Lambda_{\mathrm{T}} \equiv \Omega^2/(2T)$, and p_{st} and ρ_{st} are the pressure and density for the corresponding static equilibrium, respectively. It is clear from this equation that the radial derivative of the pressure at the magnetic axis is larger than zero in the presence of toroidal flow. This means that the maximum pressure shifts radially outward. Similar expressions can also be derived for the density, which leads to the same conclusion for the maximum density.

For a small inverse aspect ratio, $\varepsilon \equiv a/R_0 \ll 1$, an analytical expression for the Shafranov shift Δ can be derived. Here, a and R_0 are the width of the plasma and the radius of the last closed flux surface, respectively. For small aspect ratio the Shafranov shift is

$$\frac{\mathrm{d}\Delta}{\mathrm{d}r} = \frac{\varepsilon}{rB_{\mathrm{pol}}^2}\int_0^r \left[rB_{\mathrm{pol}}^2 - r^2\frac{\mathrm{d}}{\mathrm{d}r}\left(2p+\rho v_\varphi^2\right)\right]\mathrm{d}r, \tag{6}$$

where r is the minor radius and B_{pol} is the poloidal magnetic field. From this equation it is clear that the toroidal velocity contributes to increase the Shafranov shift.

MHD equilibrium reconstruction for TEXTOR shot 95022

For the reconstruction of the MHD equilibrium of TEXTOR shot 95022, the numerical code FINESSE [6] has been used. This code uses a finite element method in combination with a Picard iteration scheme to compute the solution of the Grad-Shafranov equation (2) or equation (3). The used elements are cubic elements to ensure sufficient accuracy needed for the MHD spectral analysis. The code can compute static equilibria, equilibria with purely toroidal flow, and equilibria with toroidal and poloidal flow. In this particular TEXTOR shot, the plasma rotates strongly in the toroidal direction while there are strong indications that poloidal flow is negligible. The radius of the last closed flux surface $R_0 = 1.755$m and the width of the plasma $a = 0.455$m.

From experimental data, two reconstructions have been made. One assumes a pure Grad-Shafranov static equilibrium and the other one also takes the toroidal rotation into account. In the case of a toroidally rotating equilibrium, the entropy is assumed to be a flux function. The Shafranov shifts of the static and toroidally rotating case are 6.35cm and 6.46cm, respectively. The numerical results show that the Shafranov shift increases if one takes the toroidal velocity into account. This is in agreement with the analysis in the previous section. Unfortunately, the difference between both shifts is too small to be measured. The small difference between both shifts is due to the fact that the plasma rotates sub-sonically, with a maximum sonic Mach number M_{c_s} of 0.24.

The reconstructed pressure profile together with the experimental data is shown in figure 1. The experimental values of the pressure have been calculated using the experimental values of density and temperature, and the ideal gas law. It is clear from the figure that the reconstruction matches the data in an acceptable fashion with a discrepancy at about $R \simeq 1.7$m. This discrepancy might be due to the diagnostics used or more likely due to some physical process. It is possible that a better match can be obtained by another choice of flux functions. Now, take a closer look at the pressure profile, especially its extremum. The radial locations of the maximum pressure of the static and toroidally rotating case are 1.819m and 1.821m, respectively. Note that the radial location in the latter case is larger than the radius of the magnetic axis of that case. This is in agreement with the statement made in the previous section. Again, the small differences between the two cases are due to the fact that the plasma rotates sub-sonically, and they are too small to be confirmed by present diagnostic means.

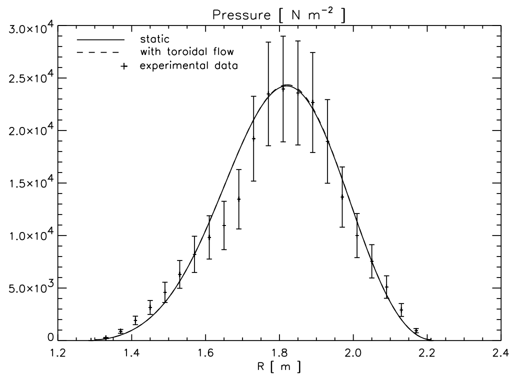

Figure 1: The reconstructed pressure profile for shot 95022. The crosses are the experimental data, while the solid and dashed line are the reconstructed static and toroidally rotating equilibria, which are virtually indistinguishable.

Figure 2 shows the reconstructed poloidal magnetic field together with the experimental data. The poloidal magnetic field measurements are done using motional Stark effect (MSE). There is good agreement between the experimental data and the reconstruction. The reconstruction shows that the poloidal magnetic field in the static and toroidally rotating case are almost the same. This is again due to sub-sonic rotation of the plasma.

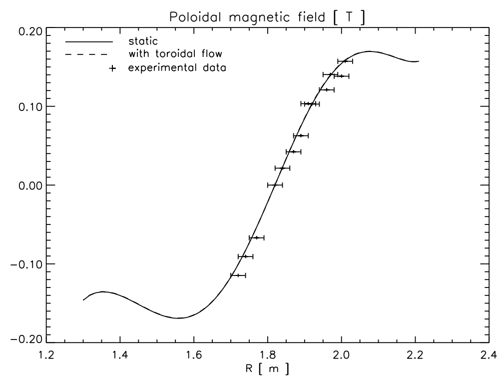

Figure 2: The reconstructed poloidal magnetic field profile for shot 95022. The crosses are the experimental data, while the solid and dashed line are the reconstructed static and toroidally rotating equilibria, which are virtually indistinguishable. The errors in the poloidal magnetic field are not yet known.

In the case of purely toroidal flow, the density and toroidal velocity have been reconstructed. Both reconstructed profiles are plotted in figure 3. Charge exchange recombination spectroscopy (CXRS) has been used to measure the toroidal velocity. This velocity was induced by neutral beam injection (NBI) in TEXTOR. The reconstructed density is in good agreement with the experimental data, while the toroidal velocity is essentially only acceptable in order of magnitude. The reconstruction of the precise flow profile data does not yield a perfect match with the data since, from MHD point of view, the radial location of the extremum of the flow profile should be larger than the radius of the magnetic axis. The data clearly indicates the opposite.

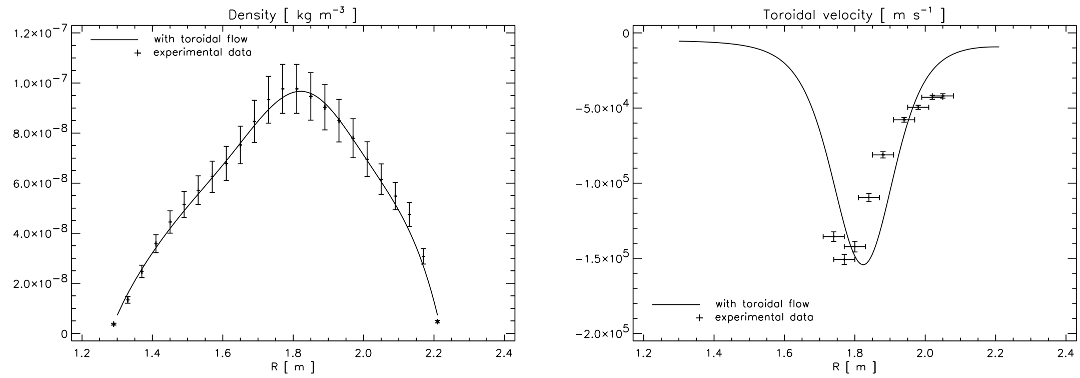

Figure 3: The reconstructed density and toroidal velocity for shot 95022. The crosses are the experimental data, while the solid line is the reconstructed toroidally rotating equilibrium.

Similar results have been found when the temperature instead of the entropy is taken as a flux function.

Conclusions

We have reconstructed the MHD equilibrium of TEXTOR shot 95022 assuming static equilibrium or equilibrium with purely toroidal flow. Overall, there is agreement between the experimental data and the reconstructed static equilibrium. Taking the toroidal flow into account leads to a small correction on the reconstructed static equilibrium. Note that a good reconstruction of the toroidal flow is not possible because the radial location of the extremum of the experimental data is smaller then the radius of the magnetic axis, which conflicts with the ideal MHD model.

If one takes the toroidal flow into account, the Shafranov shift increases and the pressure shifts in the radially outward direction. This is shown analytically as well numerically. However, the effect of the toroidal flow is small because the velocity is sub-sonic.

After clarifying the discrepancies with the experimental data, the reconstructed equilibria will be used to perform a MHD spectral analysis of this shot. In this kind of analysis it is expected that the toroidal flow plays a more important role in terms of its effect on both local and global modes.

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