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Theory of the straight field line mirror

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The straight field line mirror field is a marginally stable, non-axisymmetric magnetic well, which could be applied to a single cell mirror machine of the 2XIIB type. A long thin flux tube confinement region has been modeled with zero plasma beta and a paraxial central magnetic flux line [1,2], and the study has been extended to the first order in the plasma beta [3]. There are important beneficial properties of this vacuum field. The three dimensional marginal minimum B field and the ellipticity of the magnetic flux tube have been obtained analytically. For a given mirror ratio, the maximum ellipticity is smaller than obtained at experimental facilities with a mirror ratio below 5 or so. An additional important result for the marginal minimum B field is that the perpendicular gyro centre drift is zero, and the radial drift component is zero even to the first order in plasma beta and thereby the gyro centre drift surfaces lie on the magnetic flux surfaces, and a locally omnigenious equilibrium is provided. A sloshing ion distribution function for the three dimensional marginal minimum B vacuum magnetic field is constructed theoretically in [4]. The Clebsch coordinates of the gyro center are found to constitute a new pair of motional invariants for the plasma particles gyrating in the marginal minimum B field. A scheme to increase the electron temperature by ions cyclotron resonance heating (ICRH) is suggested in [5], and a recent numerical study to heat minority deuterons at the fundamental resonance in a reactor scale device indicates efficient absorption of the wave.

MHD stable plasma confinement is the first critical issue for a thermonuclear reactor. Our results for the non-axisymmetric open mirror trap predict several beneficial properties of the 'straight field line mirror'.

A minimum B magnetic field (magnetic well) has been shown both theoretically and experimentally to be sufficient to provide an MHD (magnetohydrodynamic) stable confinement of plasmas, since the magnetic field strength increases in all directions. A magnetic well field provides a sufficient criterion on the plasma confining magnetic field to ensure MHD stability

$$\frac{\partial B}{\partial \psi} \geq 0 \qquad (1)$$

where ψ is the radial flux coordinate. A marginal minimum B field corresponds to equality in Eq.1. The unique, marginally stable solution for the vacuum magnetic field $\mathbf{B}_v = \nabla \phi_m$ derived in [1] for a long thin confinement region is

$$\phi_m(z) = cB_0 \left\{ \ln\sqrt{\frac{1+z/c}{1-z/c}} + \frac{x^2}{2c^2}\frac{1-z/c}{\left(1-z^2/c^2\right)^2} - \frac{y^2}{2c^2}\frac{1+z/c}{\left(1-z^2/c^2\right)^2} \right\} + O\left(\varepsilon^4\right), \qquad (2)$$

where $\varepsilon = a/c$, a is the midplane radius and $|c|$ is a longitudinal length scale parameter for the flux tube.

The magnetic flux lines are found to be straight and non-parallel for this field, see Fig.1. A magnetic field line is parameterized by

$$x(z) = (1+\frac{z}{c})x_0 \quad , \quad y(z) = (1-\frac{z}{c})y_0$$

where x_0 and y_0 are the coordinates at the midplane. This corresponds to straight (non parallel) lines in the confining region with 'focal lines' at $|z| = \pm c$.

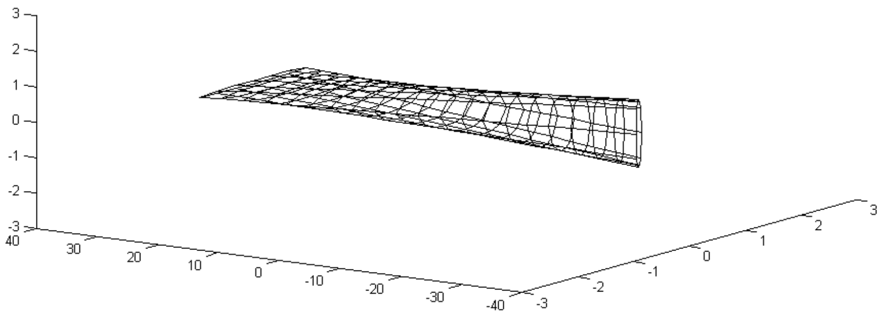

Figure 1. Marginal minimum B magnetic field flux tube surface

The marginal magnetic field strength at the z-axis is

$$\tilde{B}(z) = \frac{B_0}{1 - z^2 / c^2} \tag{3}$$

The local ellipticity of the magnetic flux tube, obtained in [1], is

$$\varepsilon(z) = (1 + |z|)/(1 - |z|) = (\sqrt{R_m(z)} + \sqrt{R_m(z) - 1})^2 \tag{4}$$

where $R_m(z) = \tilde{B}(z)/B_0$ is the local mirror ratio along the z axis. The maximum ellipticity appears at the mirrors at $|z| = L$. This ellipticity is the smallest possible for a minimum B field at zero plasma beta, assuming that the marginally stable case gives the optimal ellipticity. For a mirror ratio equal to 4, the ellipticity is 13.9, which is substantially smaller than typical ellipticities in the range 20-30 for mirror experiments. Variations of the ellipticity a along a flux tube are shown in Fig 2.

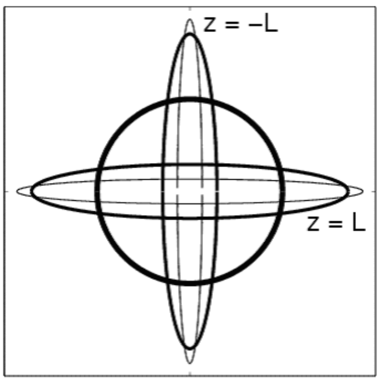

Figure 2 Evolution of the ellipticity along a flux tube. The circle is the boundary at the midplane, and the ellipses are drawn with $R_m = 2$ (thick lines) and $R_m = 4$ (thin lines).

Recent calculations have shown that even with a finite plasma beta, $\beta \leq 0.3$, the magnetic flux tube ellipticity is not strongly modified does not deviate strongly from the zero beta result, the difference is of order 10% (or even less) near the mirrors.

Another advantage of the marginal minimum B field is that the cross-field drift is absent. To the first order in the ratio of the gyro radius to the gradient scale length of the electromagnetic fields, the cross field gyro center drift velocity is

$$\mathbf{v}_{d,\perp} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu}{q}\frac{\mathbf{B}\times\nabla B}{B^2} + \frac{m v_{\parallel}^2}{qB^4}\mathbf{B}\times[(\mathbf{B}\cdot\nabla)\mathbf{B}]$$

where μ is the magnetic moment. The ∇B drift of the gyro centers vanish since $B = B(\phi_m)$. In the zero beta limit, we have $(\mathbf{B}\cdot\nabla)\mathbf{B} = B\nabla B$, and the curvature drifts also vanish [1]. At zero beta we have an omnigeneous equilibrium where each gyro center moves on a single flux surface (or even a straight flux line in our case) and the neoclassical transport is then zero for this vacuum magnetic field. The radial and the angular flux coordinates of the gyro center are constant during the motion. A magnetic field of the form $B = B(\phi_m)$ gives optimal properties with respect to MHD stability, ellipticity and radial drifts.

A nontrivial pair of invariants can be determined for the 'straight field line mirror field'. The gyro centers move on a *single* magnetic field line, and thus the (x_0, y_0) coordinates of the gyro center are constant. By including the gyro motion, it has been shown [3] that this leads to a pair of constants of motion,

$$I_x = x_0 + (1-\bar{s})^2 \frac{\dot{y}_0}{\Omega_0} \quad (5)$$

$$I_y = y_0 - (1+\bar{s})^2 \frac{\dot{x}_0}{\Omega_0} \quad (6)$$

where I_x and I_y are the guiding centre values of the (x_0, y_0) coordinates. The invariance of I_x and I_y explains why there is no perpendicular drift in this particular magnetic field. A distribution function of the form $F(\varepsilon, \mu, I_x, I_y)$ is used in [3] to calculate the diamagnetic drift for the 'straight line mirror field'. To the first order in $\beta = 2\mu_0 P_\perp / B_v^2(s)$, the plasma currents gives rice to the magnetic field

$$\mathbf{B} = (1-\frac{\beta}{2})\nabla(\phi_m + \phi_{m,pl}) \quad (7)$$

$$\phi_{m,pl}(\mathbf{x}) = -\frac{B_0}{8\pi}\int \frac{dV'}{1-\bar{s}'^2}\frac{\partial\beta/\partial s'}{|\mathbf{x}-\mathbf{x}'|} + O(\beta^2) \quad (8)$$

Particle end confinement is critical for mirrors. For a simple mirror with a stationary confining magnetic field, where no time dependent fields are used to enhance confinement, the plasma lifetime is restricted by the ion-ion collision time which at best can give a near marginal energy gain factor unless some mechanism is introduced to increase the confinement time. An ion cyclotron resonance heating scheme, as proposed in [5], is aimed to tilt the pitch angle of the ions under way to escape into the loss cone ions and thus restore their confinement. RF (radio frequency) heating waves could be applied at the region near the mirrors and may be damped before they reach the bulk plasma in the confinement region. By means of this 'magnetic pumping' of the ions, a strong density depletion near the ends may be achieved. This would create a negative electric potential, which enhances the electron confinement, and the electron temperature would increase, compare Fig. 3. This scenario indicates a promising mean to increase the energy gain factor for a single sell mirror machine.

In conclusion, a closed form expression for a marginally MHD stable mirror magnetic field is obtained in both Cartesian coordinates and flux coordinates [1,2]. A marginally stable mirror magnetic field is proved to have quadrupolar symmetry and the magnetic flux lines are straight but non parallel [1]. The ellipticity of the magnetic

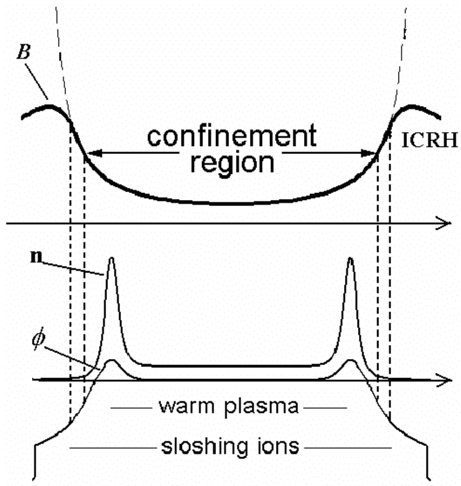

Figure 3 The electric potential obeys a Boltzmann relation in the mayor part of the plasma. The wall potential contributes to electron confinement.

flux tube for the straight field line mirror field is obtained analytically and the values of the ellipticity for typical mirror ratios are smaller than the results obtained by other authors for a superposition of a mirror coil magnetic field and a multipole magnetic field. The perpendicular gyro centre drift is zero for such a field, and the radial drift component is zero even to the first order in plasma beta. Thereby the gyro centre drift surfaces lie on the magnetic flux surfaces, and a locally omnigenious equilibrium is provided. A strong density depletion near the mirrors is suggested in [5] as a mean to create a sufficient potential barrier for the electrons and thereby enhance their longitudinal confinement time and achieve a temperature increase. A criterion on the density ratio of the plasma in the regions near the mirrors and the central cell plasma and a condition to obtain an energy gain factor Q above 10 are given in [5]. It is required that $\varepsilon_{RF} < 0.1$, where ε_{RF} is the ratio of the power, needed to double the energy confinement time, to the power applied to build up the plasma.

The energy and the arc length of the longitudinal bounce position are found to be a convenient pair of invariants in the straight field line mirror. These two invariants together with the gyro center Clebsch coordinate invariants are used to determine the gyro averaged distribution function. A sloshing ion distribution function that is close to a Maxwellian is constructed in [4]. A sketch of the corresponding plasma density profile is shown in Fig.3. A recent more refined numerical study predicts efficient generation of sloshing ions by RF [6].

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