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MHD-stability studies for a high-β PIES W7-X equilibrium

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I. INTRODUCTION

For a high-β equilibrium representing a W7-X standard high-mirror variant --- which has been obtained with the PIES code in its free-boundary version [1] --- various aspects of MHD stability are investigated. The evaluations of the local ballooning and Mercier criteria complement the global stability studies done with the the present version of the Code for the Analysis of the Stability of 3d equilibria (CAS3D, [2]).

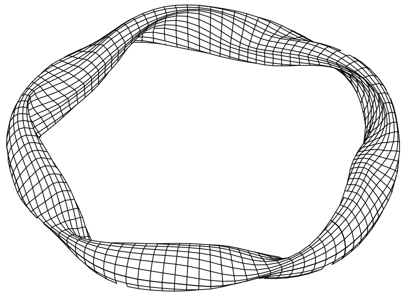


Fig. 1. *LCMS from the PIES code for the W7-X standard high-mirror case at* ⟨β⟩ = 0.041.

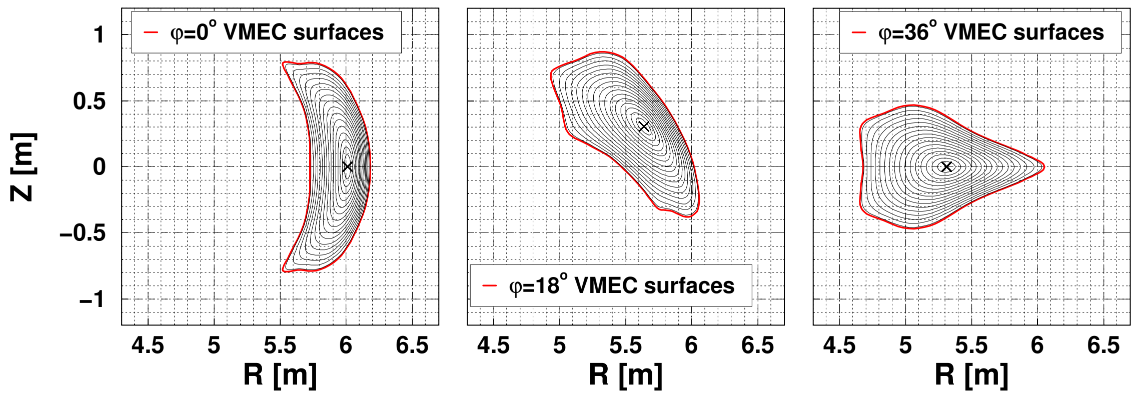
This paper is arranged as follows. Section II describes the 3d MHD equilibrium that is studied here. In Sec. III the MHD stability calculations for this plasma configuration are discussed. Section IV gives a summary.

II. W7-X standard high-mirror variant

The vacuum field of the W7-X standard high-mirror variant is obtained with the current loads given in Fig. 2. The free-boundary PIES code has been used with this vacuum field to compute the equilibrium at ⟨β⟩ = 0.041 [1]. This 'first-principle' equilibrium calculation determines islands and ergodized regions and a last-closed magnetic surface (LCMS, red solid line in the cross-section plots of Fig. 3). Furthermore, the PIES data is post-processed to give a pressure profile (see bottom frame of Fig. 3). Fig. 3 shows the characteristic meridional cross-sections in one of the five field-periods: at $\varphi = 0^o$ (bean-shaped), at $\varphi = 18^o$ (oblique), and at $\varphi = 36^o$ (triangular). The LCMS geometry and the pressure profile are the bridge to the stability study: Together with the condition of vanishing net toroidal current they are used as input for the 3D MHD equilibrium code VMEC [3] in its fixed-boundary version. The resulting magnetic surfaces are shown in the upper row of Fig. 3, the resulting rotational transform profile in the lower row (black solid line), ι(axis) $\approx$ 0.85 and ι(boundary) $\approx$ 0.94. With ⟨β⟩ = 0.041, the mirror field on the magnetic axis is $|B_{\mathrm{max}} - B_{\mathrm{min}}|/(2\overline{B_0}) \approx 0.107$. A number density n(axis) = 3×10^{20} m^{-3}, and the magnetic field B_0(axis) = 2.3 T correspond to a temperature of T(axis) = 2.52 kV for the profile used, i.e. ⟨β⟩ = 0.041 and β(axis) = 0.11.

CASE	I	#1	#2	#3	#4	#5	A	B
	[MA]	F						
SHM	1.73	1.08	1.05	1	.95	.92	+.1214	-.0405

Fig. 2. *Current loads for the standard high-mirror W7-X studied here. Columns #1 to #5 refer to the five modular-coil types, A and B to the two auxiliary-coil types.*



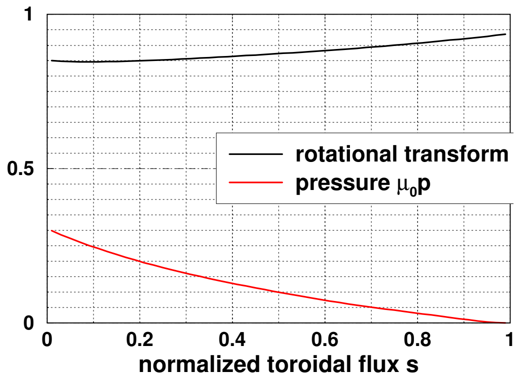


Fig. 3. *W7-X standard high-mirror variant: Top: Meridional cross-sections at the beginning (left), quarter (middle), and half (right) of a field-period. The PIES LCMS (red) is used as input boundary for VMEC giving the finite-β equilibrium (black). Left: Profiles in VMEC calculation at* $\langle\beta\rangle$ = 0.041*: rotational transform (black,* ι*, output), pressure (*$\mu_0 p$*, red, input).*

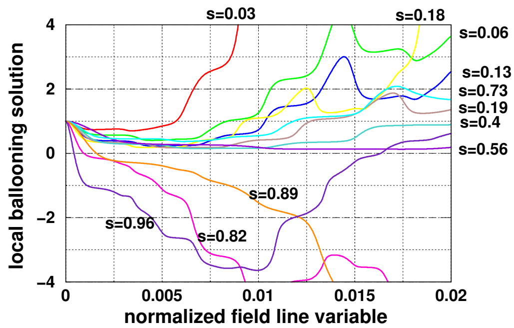


Fig. 4. *Local ballooning solutions versus normalized field line variable* φ *for the W7-X standard high-mirror variant. A change of sign in the solution for* $\varphi < 1/N_\iota \approx 0.01$ *indicates a proper ballooning instability. The field-lines span the surfaces at the indicated* s*-values. Stability prevails inside* $s = 0.82$*.*

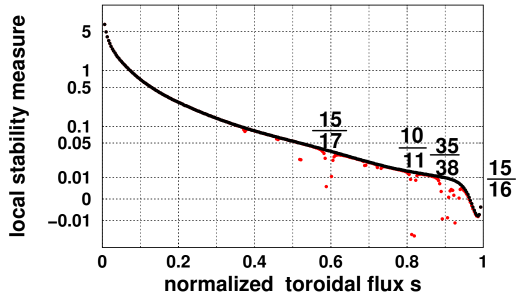
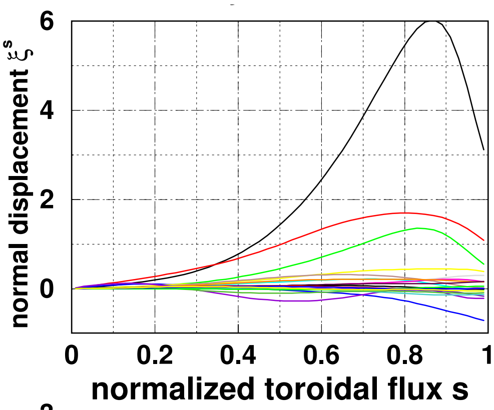
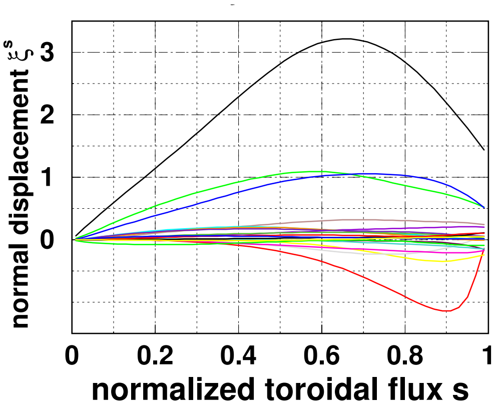


Fig. 5. *Evaluation of the Mercier criterion for the W7-X standard high-mirror case: A negative value indicates instability. Red: Including the diverging parallel current densities at the natural resonances (e.g.* 10/11*). Black: Neglecting these resonances only inside the plasma, stability prevails inside* $s = 0.97$*.*

III. IDEAL MHD STABILITY

For the equilibrium (compare in Sec. II) representing a W7-X high-mirror variant, evaluation of the local stability criteria (local ballooning in Fig. 4 and the Mercier criterion in Fig. 5, [4]) indicates stability in the plasma core. Instability is only found near the plasma boundary: The local ballooning stability prevails for the region with the normalized toroidal flux $s \lesssim 0.82$, i.e. 90% of the normalized minor radius. With the Mercier criterion, stability prevails for $s \lesssim 0.97$ not accounting for the very narrow regions around natural resonances, e.g. $\iota = 10/11$, not resolved in the present PIES equilibrium. In principle, the plasma reacts with pressure flattening at the natural resonances so that the divergence of the parallel current density is suppressed. For this reason, the natural resonances have also been eliminated in the equilibrium data used for the stability study. CAS3D calculations have been done to cover various





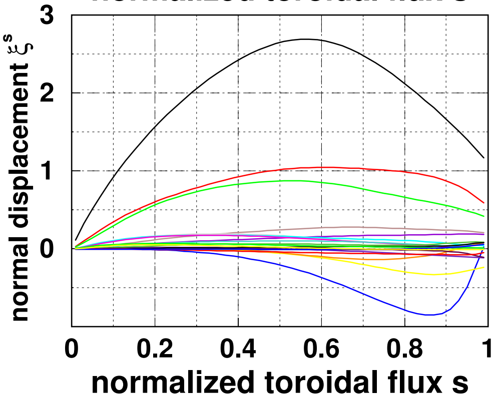


Fig. 6. *Global stability: Normal displacement harmonics for stable low-node-number free-boundary perturbations (each with the 20 largest). Top left: N = 0 mode-family: n = 10: black m = 11, red m = 12, green m = 10. Bottom left: N = 1 mode-family: n = 6: black m = 7, red m = 6, green m = 8. Right: N = 2 mode-family: n = 7: black: m = 8, green: m = 9, blue: m = 7. Computation parameters: 51 radial points, 155 normal displacement harmonics, perturbed-magnetic-field normalization, adiabatic index $\gamma = 5/3$.*

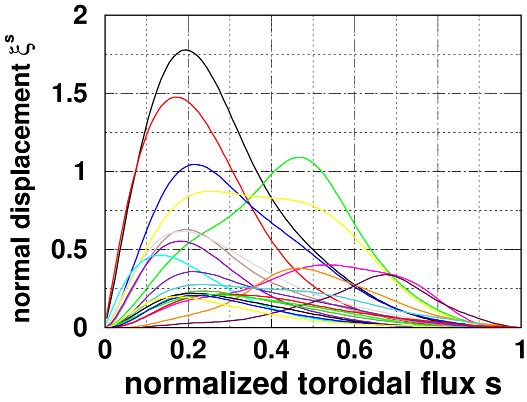
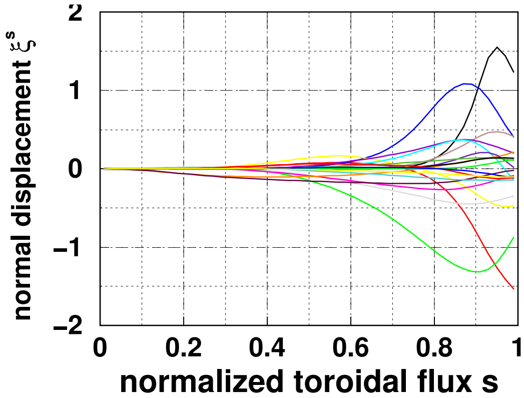
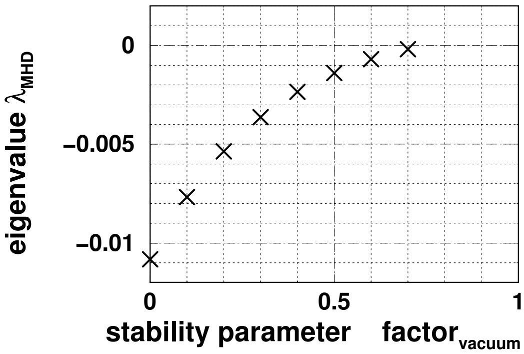


Fig. 7. *Global stability: Stable medium-node-number fixed-boundary perturbation in the N = 2 mode-family. The dominant harmonic has Fourier indices m = 26 (poloidal) and n = 22 (toroidal). Computation parameters: 51 radial points, 280 normal displacement harmonics, perturbed-magnetic-field normalization, adiabatic index $\gamma = 5/3$, phase-factor transform.*

regions of the perturbation Fourier space, e.g. low-node-number perturbations and medium-node-number perturbations. In theory, the only decoupling in 3D linear MHD stability comes from the discrete symmetry of the plasma equilibrium, i.e. the finite number of field-periods (five for W7-X), and gives decoupled mode families. In practice, however, within such a mode family low-node-number and medium-node-number perturbations are only weakly coupled by high-order, and therefore, small equilibrium harmonics, so that they may be studied separately. When using the perturbed-magnetic-energy normalization stable equilibria are characterized by positive, i.e. stable, minimum eigenvalues [2]. Fig. 6 shows the normal displacement harmonics of stable low-node-number free-boundary perturbations in each of the 3 mode families existing in a five-periodic device, that have been determined using this normalization. All of them are radially extended and the dominant harmonic is accompanied by tokamak-type side-bands. Because they are non-resonant in a close neighbourhood of the plasma boundary, ι(boundary) = 0.95, they all have their maximum amplitudes inside the plasma. Only those harmonics that are resonant on the plasma boundary, i.e. for which $m\iota + n = 0$, do not feel the stabilizing influence





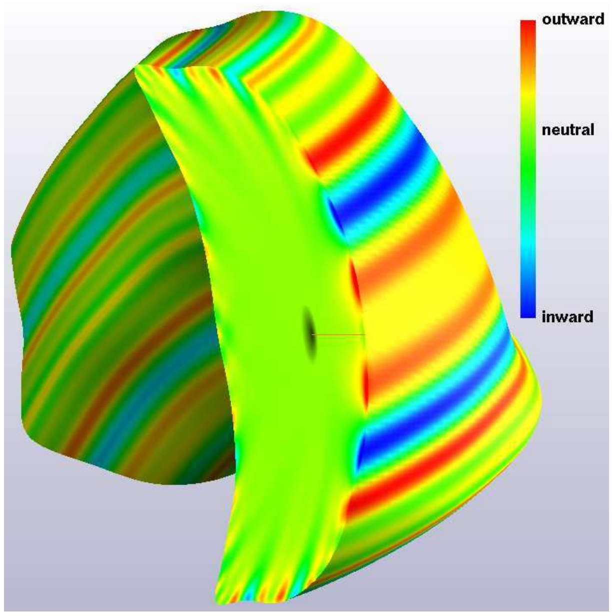


Fig. 8. *Medium-node-number free-boundary perturbation. Top left: Normal displacement harmonics for the perturbation at f_{vac} = 0.3. This perturbation is unstable because the vacuum contribution was decreased by a factor of 0.3. Bottom left: CAS3D eigenvalues (unstable, if negative) in a sequence in which the, in principle stabilizing, vacuum contribution is artificially decreased. The full (physical) problem is obtained at unity stability parameter. Right: Normal displacement contours for the perturbation of the top left frame.*

of the vacuum contribution. The fixed-boundary medium-node-number perturbation shown in Fig. 7 has been computed using the phase-factor transform, thay is offered in the CAS3D code [5]. With many harmonics of comparable amplitudes the perturbation is a pronounced global ballooning-mode. Medium-node-number free-boundary perturbations have been studied in a stability sequence using a factor applied to the, in principle stabilizing, vacuum contribution as a sequence parameter (see Fig. 8). For vanishing sequence parameter the vacuum contribution is omitted, the perturbations are unstable. For unity sequence parameter the full vacuum contribution is used, the physical problem is obtained, and the perturbations are stable. With the dominant harmonics having m = 27, n = 25 and m = 16, n = 15, and the maximum amplitudes near the plasma boundary (see Fig. 8), the perturbations are of the peeling-mode type.

IV. SUMMARY

A high-mirror W7-X variant generated by a high-β PIES run shows favourable MHD stability properties. Apart from a region near the boundary the local stability criteria indicate stability. Various types of global ideal MHD eigenfunctions have been determined and have all proven stable: kink-type external perturbations ($m \sim$ 10), ballooning-type internal perturbations ($m \sim$ 25), and peeling-type external perturbations ($m \sim$ 25). It remains to be studied when the latter ones become unstable admitting higher harmonics in the perturbation.

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