Waiting-Time Multifractal Statistics of Edge Plasma Turbulence in the T-10 Tokamak and NAGDIS-II Linear Device

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In magnetically confined plasmas, multifractal statistics of turbulence has been detected [1-3] as an evidence of rich property of self-similarity and long-range correlations. The multifractal formalism relies on the fact that the highly nonuniform probability distribution arises from the nonuniformity of the system possessing rich scaling properties and self-similarity. Multifractality is a notation related to the generalization of the classical definition of self-similarity and underlying cascading process. Multifractal nature of the edge plasma turbulence suggests subordinated process in some stochastic time. To investigate a subordinated process of plasma turbulence as Markov one, the waiting-time statistics has to be analysed. The waiting-time is defined as the time interval between two consecutive maxima in burst intensity (Fig.1).

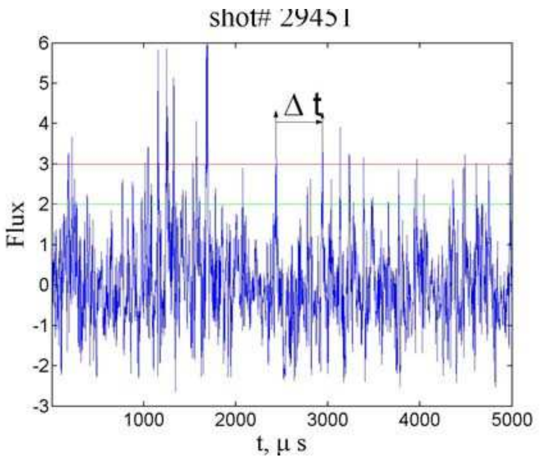


Fig. 1. Example of raw flux signal in T-10 Shot No. 29451. Waiting-time is defined as a period between successive bursts with amplitude above some levels (green and red lines).

If the triggering of the turbulent bursts are not correlated the process must follow a Poisson process, and the probability density function (pdf) of the waiting times, Δt, should be an exponential law: $P(\Delta t)=\gamma^{-1}e^{-\Delta t/\gamma}$, that is consist with self-organized criticality (SOC) hypothesis. However, power law in the waiting-time pdf's observed in some experiments [4] and investigation of statistics in the Refs.[5,6] are the reasons to study dominant underlying dynamics of the edge plasma turbulence in fusion devices.

We have analysed Langmuir probe signals from the T-10 tokamak and the linear divertor plasma simulator NAGDIS-II [7]. The radial range of 29<*r*<34 cm in ohmic discharges with no MHD activity on the T-10 tokamak, and attached and detached helium plasmas in NAGDIS-II steady state discharge have been studied. Density fluctuations *n* are measured at one point, the floating potential φ at two nearby points poloidally separated by distance of Δr, the instantaneous turbulence-induced particle flux across the magnetic field B is estimated as $\Gamma = n\cdot(\varphi_1-\varphi_2)/\Delta rB$. We have analyzed $2\cdot10^5$ points from the T-10 and $5\cdot10^5$ points from NAGDIS-II of the stationary signals sampled at 1 MHZ.

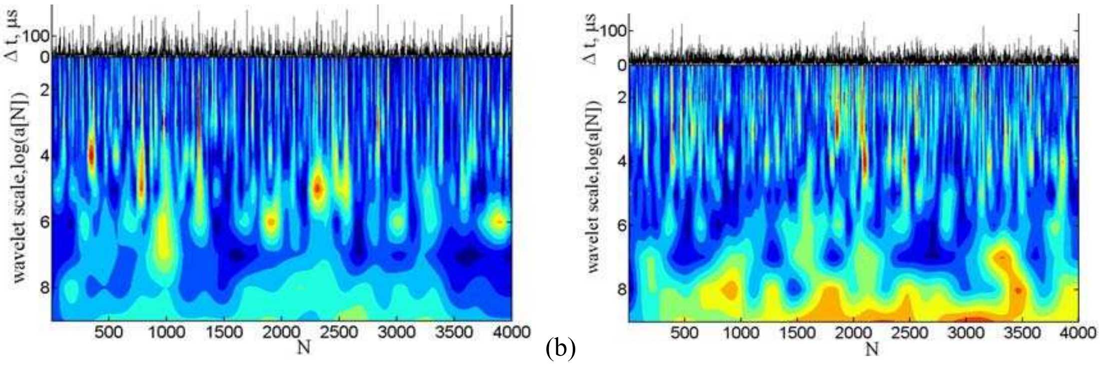
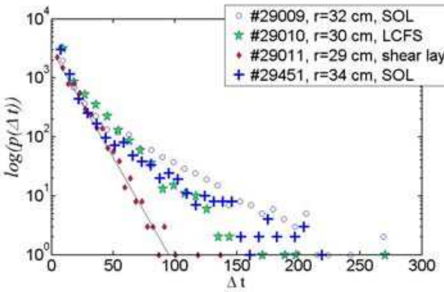
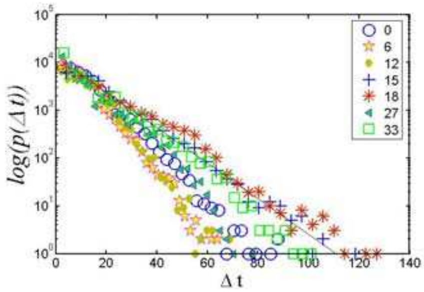


Fig.2. Intermittent waiting-time series Δt and its wavelet decomposition. Hierarchy of scales is observed as an evidence of fractality and long-term correlation. (a) T-10 tokamak flux signal, shot No. 29451, r=34 cm. (b) NAGDIS-II, attached plasma, puffing pressure 5mTorr , r=18 mm.

Typical waiting-times series constructed from intervals between successive bursts in the signal is shown in Fig.2. Wavelet technique displays intermittency and the fractal nature of the waiting-time series demonstrating hierarchy of scales in the wavelet decomposition (Fig.2). Fast fluctuations were eliminated by convoluting the signal with a *m*-point smoothing window, *m*=10-20. Typically, the pdf's of these series are not decayed exponentially (Fig.3). Typical power law exponent of the pdf's $P(\Delta t) \sim (\Delta t)^{\varepsilon}$, derived from the slope of the curves in Fig.4, is of $\varepsilon \approx -3 \sim -2$. Power-law behavior of the waiting-time pdf's does not depend strongly on the parameters of the analysis such as threshold of the burst amplitude (Fig.5[a]). Test of the noise influence on the scaling exponent estimation is shown in Fig.5[b]. Multifractal analysis [3] has shown that the waiting-time statistics is multifractal one, i.e. the scaling behavior of structure function $M(q,l)$ of increments, $\delta_l\Delta t=\Delta t(N+l)-\Delta t(N)$, $M(q,l)=\langle|\delta_l\Delta t|^q\rangle \sim l^{\tau(q)}$, has nonlinear scaling and described by a convex function $\tau(q)=qH-\lambda^2q^2$. Multifractal spectrum exhibits a parabolic behavior for data from the T-10 tokamak and the NAGDIS-II (Fig.6) with the multifractality level of $\lambda^2 \approx 0.02 \sim 0.05$ (Fig. 7). Only in the vicinity of the last closed flux surface in T-10, where a shear of poloidal velocity is observed, the multifractality level is low (of < 0.01) exhibiting close to Poisson statistics and monofractality. The lack of an exponential law for pdf's and multifractal statistics of waiting-time allows considering the problem of subordinated process and related issue of the signatures of the SOC paradigm. Our analysis shows a general lack of exponential decay of pdf's for waiting-time in edge plasma

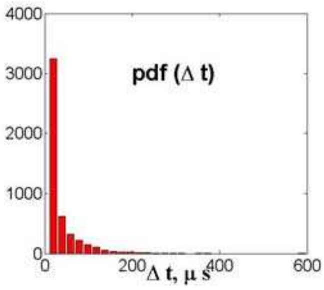


(a)

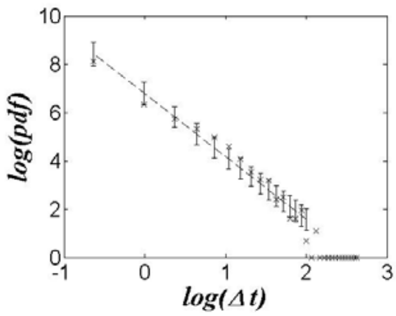


(b)

Fig.3. Pdf's of waiting-time in semi-logarithmic plots don't demonstrate exponential law in many cases. (a) the T-10 density signals from different radial position of probe. Only at r=29 cm in shear layer the pdf exhibits an exponential law. (b) NAGDIS-II radial flux signals, attached plasma.



(a)



(b)

Fig. 4. (a) Probability density function of waiting-time for T-10 shot No. 29451 . (b) in log-log scale the pdf curve is fitted by linear function demonstrating power law region.

turbulent dynamics has raised criticism about the applicability of the SOC concept.

Another conception of the edge plasma turbulence modelling we have to consider is an idea of the stochastic time in the process. The idea of modelling stochastic process as a Brownian motion in a "fractal time" can be found in many approaches. The multifractal nature of the developed turbulence has been modelled [8] by a fractional Brownian motion subordinated with a multifractal stochastic measure. Subordinated processes are Markov processes in a time variable $\mu(t)$ that is itself an (increasing) random process [9]. It can be constructed a stochastic measure $\mu(dt)=e^{w(t)}dt$, $w(t)$ – normal random process, with multifractal spectrum τ defined by $<\mu([0,t])^q>\sim t^{\tau(q)+1}$. In this construction, the subordinated process $S(t)=B(<\mu([0,t])>)$ with stochastic time, where $B(t)$ is the standard Brownian motion. Despite the question of well-definiteness of this construction is still open problem, this example of the construction with multifractal time in the stochastic process may be considered as a model of edge plasma turbulence in fusion devices.

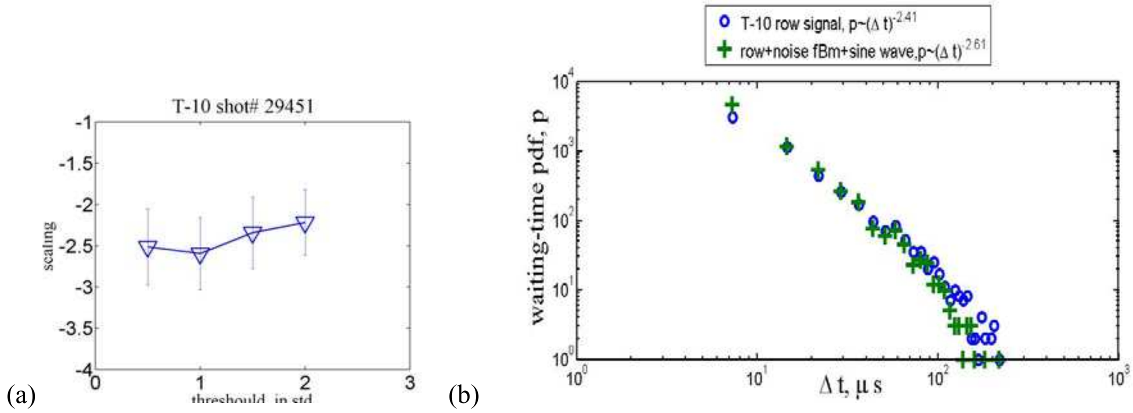


Fig.5. (a) Power-law exponent vs. threshold of the burst's amplitude-in constructing of waiting-time series. There is no strong dependence on the threshold. (b) Test of the noise influence on the scaling exponent estimation. Up to 30% of simulated noise (fractional Brownian motion test signal and sine wave) added to the raw experimental signal does not affect on the power-law behavior of the waiting-time pdf's.

In conclusion, experimental analysis of data from the T-10 and NAGDIS-II has shown that a number of the experimental flux and density signals demonstrate waiting-time multifractal statistics. Stochastic process with multifractal time has to be examined as a model of the edge plasma turbulence.

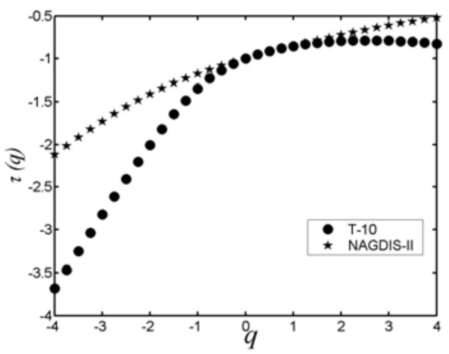


Fig.6. Examples of multifractal spectra for waiting-time series in the T-10 and NAGDIS-II. Parabolic spectra demonstrate multifractal statistics of waiting-time.

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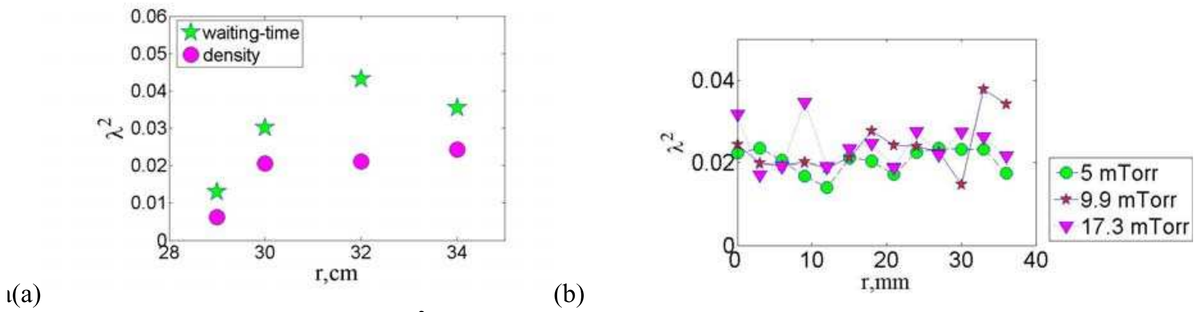

Fig.7.Parameter of multifractality λ^2 for edge plasma turbulence. (a) Radial dependence of λ^2 in the T-10 for raw density and waiting-time series , in the shear region at r=29 cm it is decreased. (b) waiting time for flux signals from the NAGDIS-II, attached (5mTorr), detached (17.3 mTorr) and attached-detached transition (9.9 mTorr).