32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-4.045 (2005)

Trajectory structures in turbulent plasmas

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In the magnetized plasmas a component of the test particle motion is the stochastic electric drift determined by the electric field of the turbulence and the confining magnetic field. For a static electric field, this drift determines closed trajectories that wind around the contour lines of the potential (trapping effect or eddy motion). For a slowly varying potential the trapping still exists as a transitory process. Typical trajectories show a random sequence of trapping events, during which the motion is approximately closed on contour of various sizes, and long jumps. Trapping characterizes stochastic velocity fields with Kubo numbers $K > 1$. This parameter is defined by $K = V\tau_c/\lambda_c$, where V is the amplitude of the stochastic velocity, τ_c is the correlation time and λ_c is the correlation length.

Test particle transport in the presence of this trapping or eddying process was studied recently by developing new statistical methods (the decorrelation trajectory [1] and the nested subensemble [2] methods). These results apply to Gaussian stochastic fields and determine the statistics of the trajectories (in particular the correlation of the Lagrangian velocity and the time dependent diffusion coefficient) for given Eulerian correlation (or spectrum) of the potential. It was shown that the presence of trapping determines memory effects (represented by a long tail of the correlation of the Lagrangian velocity). The trapping strongly reduces the diffusion coefficients and changes the scaling in the parameters of the turbulence. In the presence of a decorrelation mechanism (collisions, an average velocity, the parallel motion, etc.) a rich class of anomalous transport regimes is obtained due to trapping [3]. In these regimes the dependence of the diffusion coefficient D on the parameters describing the decorrelation is *inverted* due to trapping in the sense that a decrease of D appearing in the quasilinear regime is transformed into an "anomalous" increase of D in the nonlinear regime. Trapping influences not only the values of the diffusion coefficients but also their scaling laws. In particular, the diffusion coefficient depends on the shape of the Eulerian correlation and not only on the global parameters that define the Kubo number [4]. A strong influence has the space-dependence of the Eulerian correlation at large distances, i.e. the small k components of the spectrum.

Apart the strong modification of the transport, trapping also determines coherence in the stochastic motion in the sense that bundles of neighboring trajectories form localized structures similar with fluid vortices [2]. The statistical behavior of the trapped trajectories is completely different from that of the free trajectories. The trapped trajectories have a quasi-coherent behavior. The average, dispersion and probability distribution function for these trajectories and for the distance between two trajectories saturate. A very strong anomalous clump effect characterizes neighboring trapped trajectories, which have clump life-times much longer than the time of flight. The evolution of the distance between two trajectories is slower than Richardson law and depends on the Eulerian correlation of the stochastic potential. The trajectories contained in such structures do not contribute to the large time diffusion coefficient and to the mean square displacement. The latter are determined by the free trajectories which have a continuously growing average displacement and dispersion. The probability distribution function for both types of trajectories are non-Gaussian.

The general conclusion of these test particles studies is that there is a very important qualitative difference between the quasilinear regime appearing in a turbulence with small Kubo numbers $K < 1$ and the nonlinear regime which is characterized by the presence of trapping and appears for $K > 1$.

Test particle trajectories are strongly related to plasma turbulence. The turbulence is described by first order partial derivative equations, that have the solutions represented in terms of characteristics, which are particle trajectories in the stochastic field of the turbulence. The dynamics of the plasma basically results from the Vlasov-Maxwell system of equations, which represents the conservation laws for the distribution functions along particle trajectories. Studies of plasma turbulence based on trajectories were initiated by Dupree [5]. The resonance broadening theory and the clump renormalization were much used especially in the years seventy [6]. These methods do not account for trajectory trapping and thus they apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has to be understand is the effect of this special statistical behaviour of the test particle trajectories on the evolution of the instabilities and on turbulence in magnetized plasmas.

We extend the analysis of instabilities based on test trajectories to the nonlinear regime characterized by trapping. We study linear test modes on a turbulent plasma for the drift instability in a slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (non-adiabatic response) and finite Larmor radius of the ions destabilizes the drift waves. We consider first a turbulent state of the plasma with known statistical characteristics of the electrostatic potential. The perturbations of the electron and ion distribution functions are obtained from the gyrokinetic equation as integrals along test particle trajectories of the source terms determined by the density gradient. The background turbulence produces two modifications of the equation for the linear modes. One consists in the stochastic electric drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity. Both effects are important for ions while the response of the electrons is approximately the same as in a quiet plasma. The average propagator for a mode with wave number $\mathbf{k}$ and frequency ω is defined by

$$P\left(\mathbf{x},z,t\right)=\exp\left(-i\mathbf{k}\cdot\mathbf{x}\right)\int_{-\infty}^{t}d\tau\left\langle\exp\left(i\mathbf{k}\cdot\mathbf{x}^{i}(\tau)\right)\right\rangle\exp\left(i\omega(t-\tau)\right) \tag{1}$$

where $\mathbf{x}^i(\tau)$ are the ion trajectories that start from the point $\mathbf{x}$ at time t and go backward in time, and $\langle ... \rangle$ is the average over the ensemble of these trajectories. For a turbulence with small Kubo number the trajectories are Gaussian with $\langle x_i^2(\tau)\rangle = 2D_{ql}\tau$ and the well known result of Dupree is obtained. If the turbulence has $K > 1$ a part of the trajectories are temporary trapped and vortical trajectory structures appear around the maxima and minima of the stochastic potential. The mean square displacement has a slow time increase at times smaller than τ_c and becomes diffusive at $t > \tau_c$ but only due to a part of the trajectories that are not trapped. The average in Eq.

(1) can be calculated numerically using the probability distribution of the stochastic trajectories obtained with the nested subensemble method [2]. In the second cumulant approximation a simple analytical expression is obtained:

$$P(\mathbf{x},t)=\frac{i}{\omega}\exp\left(-\frac{1}{2}k_i^2S_i(K)\right)\left[1-\frac{ik_i^2\mathcal{D}_i}{\omega+ik_i^2\mathcal{D}_i}\exp(i\omega\tau_c)\right] \tag{2}$$

where $S_i(K))$ is the square of the average size of the trajectory structures along direction i and $\mathcal{D}_i$ is the diffusion coefficient in the nonlinear regime. $S_i(K))$ is zero for turbulence with $K << 1$ and it increases algebraically at $K > 1$, with an exponent that depends on the correlation of the stochastic potential.

The dispersion relation is obtained from the neutrality condition. It has the solution

$$\omega=\omega_{*e}\frac{\Gamma_0\left(\frac{k_\perp^2\rho_L^2}{2}\right)\exp\left(-\frac{1}{2}k_i^2S_i\right)}{2-\Gamma_0\left(\frac{k_\perp^2\rho_L^2}{2}\right)} \tag{3}$$

$$\gamma=\sqrt{\pi}\frac{\omega\left(\omega_{*e}-\omega\right)}{2-\Gamma_0}\frac{1}{|k_z|\,v_{Te}}-k_i^2\mathcal{D}_i\cos(\omega\tau_c)+k_ik_jR_{ij}\omega/k_2 \tag{4}$$

where $\omega_{*e} = k_2V_{*e}$ is the diamagnetic frequency, ρ_L is the ion Larmor radius and $\Gamma_0(b) = exp(-b)I_0(b)$. The tensor R_{ij} has the dimension of a length and is defined by

$$R_{ji}(\tau,t)\equiv\int_\tau^t d\theta'\int_{-\infty}^{\tau-\theta'}d\theta M_{ji}(|\theta|) \tag{5}$$

where M_{ij} is the Lagrangian correlation

$$M_{ji}(|\theta'-\theta|)\equiv\left\langle v_j\left(\mathbf{x}^i(\theta'),z,\theta'\right)\ \partial_2v_i\left(\mathbf{x}^i(\theta),z,\theta\right)\right\rangle, \tag{6}$$

and v_j is the ExB drift velocity.

The background turbulence with large Kubo number has a complex influence on the mode. The ion trajectory structures (the quasi-coherent component of their motion) determines the exponential factor in the frequency. Its effect is the displacement of the unstable k-range toward small values. The random component in the ion motion determines a damping term in the growth rate that produces the stabilization of the large wave numbers. It is similar with the effect appearing in a turbulence with small K, but with the diffusion coefficient influenced by trapping. The fluctuations of the diamagnetic velocity determines the last term in the growth rate (4). The tensor R_{ij} contributes to the growth of the modes.

The results obtained for the linear modes on a turbulent plasma can be used for determining the self-consistent evolution of the turbulence. A kind of initial value problem is studied by an iterative calculation of growth rates and frequencies of the test modes and of the correlation determined by these modes.

Starting from a large spectrum of modes with very small amplitudes (thermal bath) the growth rates can be determined for each mode. In this quasilinear regime only the diffusion of the ions will influence the modes producing the damping of the modes with $k_\perp\rho_L \gg 1$. The modes with $k_\perp\rho_L \sim 1$ increase as linear modes. Thus the amplitude of the stochastic potential increases continuously while the correlation length remains comparable with ρ_L and the correlation time is $\tau_c = 1/\omega_{*e} \sim \rho_L/V_{*e}$.

When K becomes larger than 1, the trapping generates a coherent motion for a part of the ions and S begins to grow. Small structures are formed and persists during the correlation time of the potential. This ordered motion of the ions acts similarly with the cyclotron gyration: it decreases the frequency of the modes and displace the value $\omega \cong \omega_{*e}/2$ (where is the maximum of the growth rate) toward smaller k. Thus the correlation length of the turbulence is increasing. In this regime the amplitude of the ExB velocity remain approximately constant, while the correlation length and the correlation time are increasing. The diffusion coefficient also increases because of λ_c ($\mathcal{D} = V\lambda_c K^{-\alpha}$). The unstable domain is continuously displaced toward small wave numbers and narrowed due to the increase of the diffusion coefficient. The potential cells becomes elongated along the direction of the gradient because $k_2 > k_1$ in the unstable domain of the test modes. The evolution is much slower than at $K < 1$ due to the small values of the growth rate obtained in this stage. Thus the energy taken by the instability from the electrons produces a motion of the ions with increasing coherent component. This motion is reflected in the turbulence that looses the random aspect: large ordered cells are produced.

In the same time, as the Kubo number increases above one, the term determined by the fluctuation of the diamagnetic velocity is growing due to the fact that the turbulence becomes anisotropic. The evolution at this stage strongly depends on the tensor R_{ji}, on the amplitude of the potential and also on the shape of the correlation (normal for such a strongly nonlinear state). We expect that the toroidal aspects have a strong influence on this tensor.

In conclusion, we have shown that a strong qualitative difference appears between the test modes on a turbulence with large Kubo numbers compared to the case of $K < 1$. In the nonlinear case quasi-coherent structures of trajectories appear due to trapping. They are clearly associated with order and structure formation in turbulent plasma. The evolution of the drift turbulence toward large scales (inverse cascade) is determined by the eddying motion of the ions, which displaces the unstable domain to small k values.

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