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The absorption of gravitational waves in strongly magnetized plasmas

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Introduction

The interaction of gravitational waves (GWs) with plasmas, if it is efficient in transferring energy from the GW to the plasma, might well be the basic mechanism behind the most energetic astrophysical events, such as short Gamma ray bursts (see e.g. [5]). Viewed differently, if the interaction is efficient, then GWs can be absorbed in plasmas, and in this way possibilities for the indirect observation of GWs arise.

In a number of articles (e.g. [6], [4], [3]), it has been shown that GWs excite various kinds of plasma waves, the more efficient, the stronger the background magnetic field is. All these studies are analytical and the equations describing the GW-plasma interaction were linearized. The interaction is though a totally non-linear effect, and there is so-far no conclusive answer to the question of how much energy can be absorbed by a plasma from a GW.

Here, we study the GW-plasma interaction for the first time in its full non-linearity, solving the non-linear system of equations numerically. Our main interest is in the amount of energy absorbed by the plasma from the GW.

Basic equations

The GW is considered as a small amplitude perturbation of the otherwise flat spacetime, and we assume it to be $+$ polarized and to propagate along the z-direction, so that the metric has the form $g_{ab} = \mathrm{diag}(-1, 1+h, 1-h, 1)$, with $h(z,t) << 1$ the amplitude of the GW [1]. In order to express the equations in the observable quantities (electric field $\vec{E}$, magnetic field $\vec{B}$, and 3-velocity $\vec{V}$ of the fluid), we use an orthonormal frame (ONF) [1]. Indices of quantities in the ONF carry a hat in the following. In the ONF, 4-vectors and tensors take the same form as in flat space-time.

We assume an ideal conducting fluid, so that the electric field is given by the ideal Ohm's law, which in the ONF takes the usual form, $0 = \hat{\gamma}\left(\vec{E} + \frac{1}{c}\vec{V} \times \vec{B}\right)$, with $\hat{\gamma} = 1/\sqrt{1 - \vec{V}^2/c^2}$. The evolution of the magnetic field is determined by the Maxwell's equation [2],

$$F_{\hat{a}\hat{b};\hat{c}} + F_{\hat{b}\hat{c};\hat{a}} + F_{\hat{c}\hat{a};\hat{b}} = 0 \qquad (1)$$

with $F^{\hat{a}\hat{b}}$ Faraday's field tensor The electromagnetic energy momentum tensor is defined as $T^{\hat{a}\hat{b}}_{(EM)} = \frac{c^2}{4\pi}\left(F^{\hat{a}\hat{c}}F^{\hat{b}}_{\ \hat{c}} - \frac{1}{4}\eta^{\hat{a}\hat{b}}F^{\hat{c}\hat{d}}F_{\hat{c}\hat{d}}\right)$ and for the fluid, we have the energy momentum tensor

$T^{\hat{a}\hat{b}}_{(fl)} = Hu^{\hat{a}}u^{\hat{b}} + \eta^{\hat{a}\hat{b}}pc^2$, where H is the enthalpy and $u^{\hat{a}}$ the 4-velocity, $u^{\hat{a}} = \hat{\gamma}(c, V_x, V_y, V_z)$ [2]. We assume an ideal and adiabatic fluid, so that $H = \rho c^2 + \frac{p}{\Gamma-1} + p$, with Γ the adiabatic index, ρ the matter density, and p the pressure. The total energy momentum tensor $T^{\hat{a}\hat{b}} = T^{\hat{a}\hat{b}}_{(fl)} + T^{\hat{a}\hat{b}}_{(EM)}$ yields the momentum and energy equations [2]

$$T^{\hat{a}\hat{b}}_{\ ;\hat{b}} = 0. \tag{2}$$

Continuity is expressed by $\left(\rho u^{\hat{a}}\right)_{;\hat{a}} = 0$. The evolution of the GW is determined by the linearized Einstein equation, where we take the back-reaction of the plasma onto the GW into account

$$-\partial_{tt}h + c^2\vec{\nabla}^2 h = -\frac{1}{2}\frac{16\pi G}{c^4}\left(\delta T_{xx} - \delta T_{yy}\right), \tag{3}$$

where δT_{xx}, δT_{yy} are the non-background, fluctuating parts of the components T_{xx}, T_{yy} of the total energy momentum tensor [1]. To close the system of equations, we assume an adiabatic equation of state, $p = K\rho^\Gamma$, with K a constant. The covariant derivatives in the ONF are calculated with use of the Ricci rotation coefficients [2].

The model

We focus on the excitation of MHD modes which propagate in the z-direction, parallel to the propagation direction of the GW and perpendicular to the background magnetic field $\vec{B}_0 = B_0\mathbf{e}_{\hat{x}}$. We let consequently $\vec{E}\,||\,\mathbf{e}_{\hat{y}}$ and $\vec{V}\,||\,\mathbf{e}_{\hat{z}}$, and all variables depend spatially only on z. In specifying the general equations to this particular geometry, (i) we express all 4-vector and tensor components through the potentially observable B_x, E_y, and V_z; (ii) we expand the covariant derivatives; (iii) we keep all non-linear terms, no approximations are thus made. In this way, we are led to a system of non-linear, coupled, partial differential equations in a 1-D geometry.

Numerical Solution

We solve the GW-plasma system of equations applying a pseudo-spectral method that is based on Chebyshev polynomials. Time stepping is done with the method of lines, using a fourth order Runge-Kutta method with adaptive step-size control. The one-dimensional grid along the z-direction consists of 256 grid-points and corresponds to a physical domain along the z-axis of length $5.4\,10^7$ cm. The sampling time step Δt is set to $\Delta t = T_{gw}/14$, with $T_{gw} = 1/f_{gw}$.

Parameters, initial and boundary conditions

We assume a background magnetic field B_0 of 10^{15} Gauss, a background density $\rho_0 = 10^{-14}$ gr cm^{-3}, and an adiabatic index $\Gamma = 1.4$. The initial conditions are $B_x(z,0) = B_0$, $V_z(z,0) = 0$, $\rho(z,0) = \rho_0$, and $h(z,0) = 0$. The GW has as boundary condition at the left end z_L of the box $h(z_L,t) = h_0(t)\cos(k_{gw}z_L - \omega_{gw}t)$, so that a a monochromatic plane wave is entering the box,

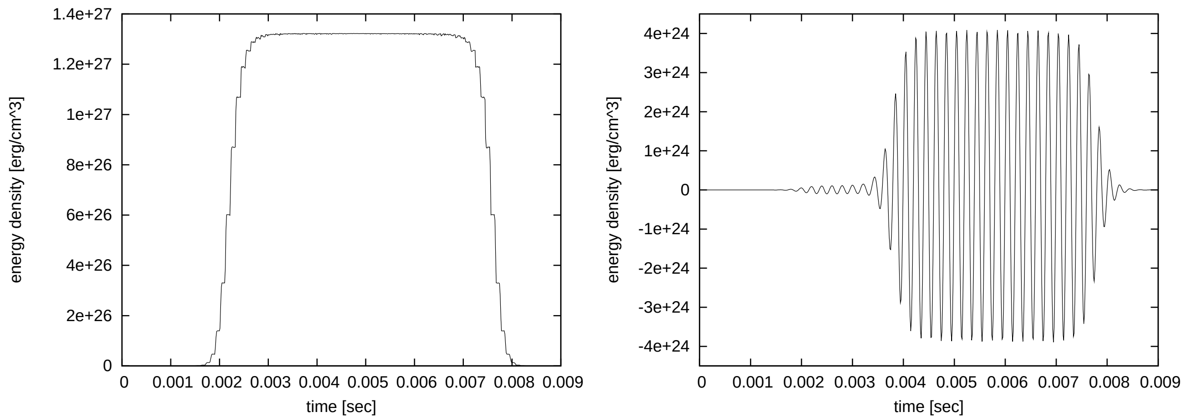

Figure 1: Left: Average energy density of the GW as a function of time. — Right: Mean total energy density of the plasma as a function of time. Negative values appear since the background magnetic energy density is subtracted.

The amplitude h_0 raises within roughly 0.5 ms from 0 to 10^{-4}, at which value it stays constant for 6 ms, where after it decays to 0 again. The GW frequency is $f_{gw} = 5\,\mathrm{kHz}$. B_x, E_y, and v_z have free outflow boundary conditions at both edges of the box.

Numerical Results

The total mean energy density E_{total} in the system at a given time t is determined as

$$E_{total}(t) = \left[\frac{1}{8\pi}\int E_y(z,t)^2\,dz + \frac{1}{8\pi}\int (B_x(z,t)^2 - B_0^2)\,dz + \frac{1}{2}\int \rho(z,t)v_z(z,t)^2\,dz\right]/L \quad (4)$$

with L the size of the system — note that we subtract the magnetic energy that corresponds to the constant background magnetic field B_0. Fig. 1 shows $E_{total}(t)$ and the the mean energy density $E_{gw}(t)$ of the GW as a function of time, where $E_{gw}(t) = \frac{c^2}{32\pi G}\omega_{gw}^2 \bar{h}(t)^2$, with $\bar{h}(t)$ the mean instantaneous amplitude of the GW oscillation. At maximum GW amplitude, the energy density of the GW amounts to $E_{gw} = 1.32\,10^{27}\,\mathrm{erg/cm^3}$. Once the GW enters the system, the plasma starts to absorb energy from the GW, and in roughly 1 ms after the GW has reached its maximum amplitude the absorption has reached its maximum, the energy density in the plasma is roughly $4\,10^{24}\,\mathrm{erg/cm^3}$. When the GW leaves the system, the energy in the plasma decays almost together with the GW amplitude.

Fig. 2 shows the electric field in the box for a fixed time at maximum absorption. The GW obviously excites wave motions in the plasma that travel with the GW. The figure also illustrates that strong electric fields are generated, of increasing intensity towards the right edge of the box, where the electric field assumes a value of $3\,10^{12}$ statvolt/cm ($9\,10^{16}$ V/m).

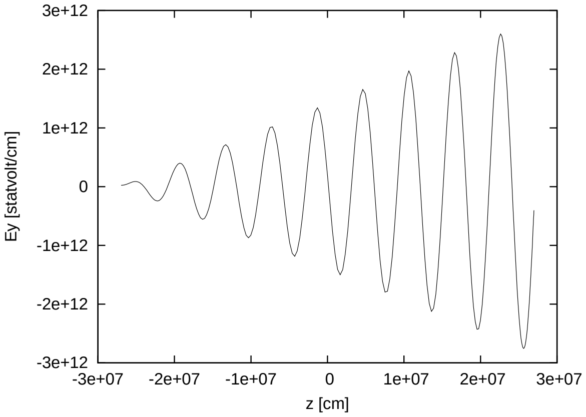

Figure 2: Electric field as a function of z at a fixed time $t = 0.00545$.

Discussion

The numerical results show that, for strong magnetic fields and low densities, a large amount of the energy of the GW is absorbed by the plasma on a short time-scale, which is of the order of 5 GW periods, i.e. in the millisecond range. The energy absorbed by the plasma is thus not proportional to the duration of the GW-plasma interaction if this duration is longer than typically a few GW periods. It also seems that the amount of energy absorbed and, correspondingly, the amplitudes of the field oscillations, are proportional to the box size, i.e. to the size of the region of constant magnetic field. For the box-size considered, the absorbed energy is a fraction 10^{-3} of the GW energy density, so that the back-reaction onto the GW is not important.

Applying these results to a typical magnetar, and assuming constant ρ_0 and B_0 with values as used in this work, then we find that a volume of $10^{27}\,\mathrm{cm}^3$ contains a total energy of 10^{52} erg, which is the energy that is typically released in a short Gamma ray burst (e.g. [5]), and for which to explain the plasma-energization studied here also has a fast enough time-scale.

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