Particle interactions with solitary waves in magnetized plasmas[1]

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Wave-particle interactions is one of the most well studied subjects in the physics literature with numerous applications in plasma physics, accelerators, microwave sources, lasers and other branches of physics. Also, particle dynamics under the presence of electrostatic or electromagnetic waves has been one of the main paradigms, on which the modern theory of nonlinear Hamiltonian dynamics and chaos has been applied [1, 2]. However, all previous studies of wave particle interactions from the point of view of Hamiltonian dynamics have been focused in waves having discrete spectrum, namely periodic waves. On the other hand, in many interesting applications of wave-particle interactions a localized wave, with a continuous spectrum, such as a Solitary Wave (SW) has to be considered. Among them we can refer to the RF plasma heating [1] as well as to the investigation of the damping of localized waves in plasmas [3], since the transit time particle acceleration has been considered as the principal dissipation mechanism for the Langmuir soliton collapse. Also, particle dynamics in the case of interactions of short laser pulses with plasmas has several applications to plasma heating, current drive and diagnostics in fusion devices, while other relevant applications refer to pulse propagation in space and astrophysical plasmas [4]. Several of these works are based on the discretization of the spectrum of the wavepackets involved [5], while others treat the particle dynamics on the basis of the direct perturbation approach [6,3]. Kinetic-theoretical approaches have also been employed and the Vlasov equation has been mainly used [7], in limiting cases for the ratio of the modulation time scale with the transit time of the particle through the SW. Other studies, within the context of Hamiltonian approach, have also been based on the adiabaticity assumption [8].

In this work we study charged particle dynamics in the presence of one or more electrostatic SWs having different phase and group velocities and propagating in the absence of magnetic field or along a uniform magnetic field B_0, in a magnetized medium. The forms of the electric field considered have continuous spectra and range from ordinary wavepackets and solitons to ultrashort few-cycle and sub-cycle transient pulses. It is worth mentioning that for the latter, the assumption of adiabaticity for the amplitude modulation, adopted in the aforementioned previous works, does not hold. The particle dynamics are analyzed on the basis of the Canonical Perturbation Method (CPM) [9] which allows us to construct approximate invariants of the motion for the nonintegrable Hamiltonian system. However, the aperiodic character of the Hamiltonian perturbation necessitates a modification of the CPM, in agreement with recent extensions of the KAM theorem for aperiodic perturbations [10]. The resulting invariants contain all the essential information for the phase space of the system, which is strongly inhomogeneous and is studied in terms of appropriate Poincare surfaces of section. Moreover, it is shown that the aperiodic character of the SW results in chaotic transient momentum variation which depends strongly on the initial particle momentum, in terms of a resonant condition, and also there is a complex dependency on the initial particle position: Neighbor particles having the same initial momentum, end up with different momenta after their transition through the SW.

Particle dynamics under the presence of a set of electrostatic SW can be described by the following Hamiltonian:

$$H = \frac{p_z^2}{2} + \sum_n \Phi_n(z - v_{g_n} t)\sin\left[k_n(z - v_{p_n} t)\right] \qquad (1)$$

where Φ_n is the profile of the electrostatic potential and v_{g_n}, v_{p_n} are the group and phase

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velocities of the SW, respectively. Considering the potential part of the Hamiltonian as a perturbation to the free particle motion, and following the standard procedure, according to the CPM [9], we seek a near-identity canonical transformation, to new variables $(\bar{p}_z,\bar{z})$ for which the new Hamiltonian $\bar{H}$ is a function of the momentum p_z alone. To the first order of perturbation, the transformation can be obtained from the equation:

$$\frac{\partial S_1}{\partial t}+v_z\frac{\partial S_1}{\partial z}=-H_1 \qquad (2)$$

where S_1 is the first order term of the generating function $S(\bar{p}_z,z)$, v_z is the particle velocity, and H_1 is the potential part of the Hamiltonian. In order to solve (2), instead of using the usual Fourier series method [9], which apply for periodic perturbations, the Fourier transform is used, yielding

$$S_1=\sum_n\frac{e^{i(k_nz-\omega_nt)}}{v_z-v_{g_n}}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\bar{\Phi}_n(k)}{k+k_na_n}e^{ikz}dk+c.c. \qquad (3)$$

where $\bar{\Phi}_n$ is the Fourier transform pair of Φ_n and

$$a_n=\frac{v_z-v_{p_n}}{v_z-v_{g_n}} \qquad (4)$$

Using the convolution property of the Fourier transform and taking properly into account the contribution of the pole of the integrand, S_1 can be written as follows

$$S_1=\sum_n i\frac{e^{i[k_n(1-a_n)z-\omega_nt]}}{v_z-v_{g_n}}\int_{-\infty}^{z}\Phi_n(\zeta)e^{ik_na_n\zeta}d\zeta+c.c. \qquad (5)$$

The new momentum $\bar{p}_z$ is given to first order by

$$\bar{p}_z=p_z-\frac{\partial S_1}{\partial z} \qquad (6)$$

The function S_1 can be evaluated in terms of (p_z,z) (within the first order approximation). Since the transformed Hamiltonian is not a function of the new position $\bar{z}$, then the new momentum is an approximate (to first order) invariant of the motion for the perturbed system. Utilizing the Lie transforms method [9] it is easy to construct higher order approximations of the invariant of the motion. However, in the following it will be shown that, even at this first order approximation, the invariant (6) provides useful information for the structure of the phase space of the system. It is remarkable that the calculation of the first order invariant does not prerequisites any assumption neither on the scale of the argument (adiabaticity) nor the form of Φ_n, provided only that the corresponding Fourier integral is well defined. Thus, in the context of the invariant (6), particle interactions with both slowly modulated fields and sub-cycle pulses can be studied as well as different kinds of amplitude profiles.

Firstly, we investigate particle dynamics under interaction with one SW, having a Gaussian profile of the form

$$\Phi_n(x)=A_ne^{-x^2/(2\sigma_n^2)},n=1 \qquad (7)$$

Using (5), one obtains

$$S_1=-\sqrt{\frac{\pi}{2}}\frac{\sigma_1A_1}{v_z-v_{g_1}}e^{-\sigma^2(k_1a_1)^2/2}e^{i[k_1(1-a_1)z-\omega_1t]}\left[1+\mathrm{erf}\left(\frac{z-v_{g_1}t-ia_1k_1\sigma^2}{\sqrt{2}\sigma}\right)\right]+c.c. \quad (8)$$

The form of S_1 implies two consequences for the particle dynamics: (a) the effective strength of the perturbation is proportional to the product of the field amplitude A_1 and the field width $sigma_1$, which is intuitively expected and in agreement with the time scaling property [8] of the Hamiltonian (1); (b) the presence of the SW affects strongly particles with initial velocities around the resonant velocity given by $a_1=0$ or $v_z=v_{p_1}$, within an area, the width of which is determined by the product σ_1k_1, as indicated by the exponential term of (8). The width σ_1 of the SW determines its bandwidth and correspondingly the resonant velocity spectrum. In Fig. 1, numerically and analytically obtained Poincare surfaces of section, in the extended phase space, are shown. The numerical results are obtained through direct numerical integration of the ordinary differential equations describing the dynamical system. For each value of initial velocity 500 initial positions equally distributed in a range $[-12\sigma_1, -12\sigma_1 + 2\pi]$ were considered, and (z, v_z) values were recorded at times $t_i = 2\pi i/\omega_1$ $(i = 0, 1, ...)$. The analytical results were obtained as contour plots of the approximate invariant of the motion through (6) and (8). In all cases, there is remarkable agreement between the numerical and analytical results. The phase space is strongly inhomogeneous and, within the resonant areas, particle velocity changes during particle transition through the SW. The particles, after exiting SW, acquire different constant velocities, the values of which depend strongly on their initial position. Outside the resonant areas, particle velocities change slightly during transition. They return to their initial values, when the particles exit from the SW. Moreover, the SW width, σ_1, is shown to affect both the perturbation strength and the width of the resonant velocities area. For ultrashort SW, such as the sub-cycle SW shown in Fig. 1 (top), the resonant area is not centered on the phase velocity v_{p_1}, since there is not enough number of cycles of the carrier wave to affect the particles and the phase velocity effect take place. The increase of the SW width is shown to result in the centering of the resonant area around the phase velocity, as well as the localization of the area of strong interactions.

(a) (b) (c) / (d) (e) (f)

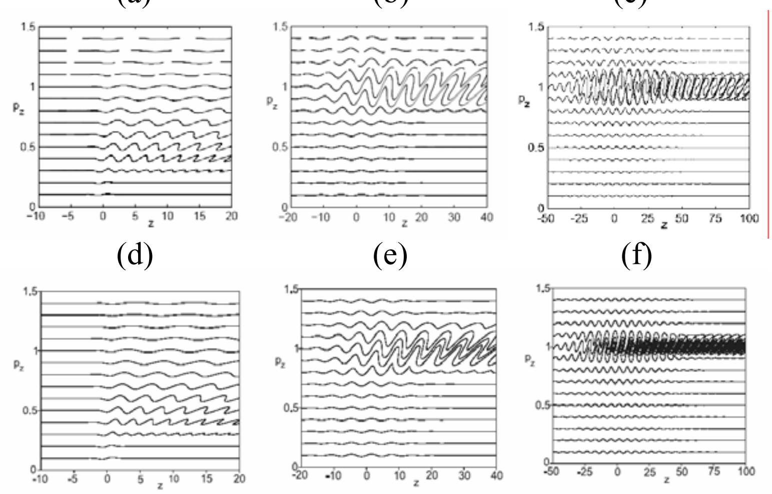

Figure 1: Numerically (a,b,c) and analytically (d,e,f) obtained Poincare surfaces of section for interaction with a SW having $A_1 = 0.005$, $\sigma_1 = 1, 10, 30$ (left to right) $\omega_1 = 1$, $v_{p_1} = 1$, $v_{g_1} = 0$.

The presence of many SWs with different phase velocities, results in corresponding resonant areas of strong interactions in the phase space. Depending on the amplitude, width and phase velocity of each wave, the resonances can be well-separated, weakly or strongly overlapping. In Fig. 2, the particle dynamics under the presence of two SWs is shown. The Poincare surfaces of section, shown in Figs. 2(top), are obtained numerically, with initial conditions chosen as in Fig. 1. The extreme values for exiting particle velocities, $v_{z,out}$, can also be obtained through the first order invariant, and are given by the

$$v_{z,in} = v_{z,out} \pm \sqrt{2\pi}\, \frac{A_1 \sigma_1 k_1 (1 - a_1)}{v_{z,out} - v_{g_1}}\, e^{-\sigma^2 (k_1 a_{1,out})^2 / 2} \qquad (9)$$

In Fig. 2 (bottom) the extreme values of velocity variations are shown $\Delta v_{max,min}$, as well as the average velocity variation with respect to the initial position $< \Delta v >_{z,in}$. The resonance overlap is shown to result in merging of the corresponding resonant areas. It can be remarked that the analytic results also apply directly to cases where particles interact with aperiodic sequences (series) of multiple SW, differing in their initial spatio-temporal positions.

Furthermore, the collective characteristics of particle beams are usually described by the distribution function $F(p_z, z, t)$, which fulfills the Vlasov equation

$$\frac{\partial F}{\partial t} + \left[F, H\right] = 0 \qquad (10)$$

It is well known that, in an integrable system, any function of the invariants of the motion forms a solution of the Vlasov equation. Thus, the approximate invariant of the motion (6) can be used in order to obtain approximate solutions of (10). By setting $\Delta p_z \equiv \partial S_1 / \partial z = O(\varepsilon)$ and Taylor expanding (6) and $F(\overline{p}_z)$, simply yields:

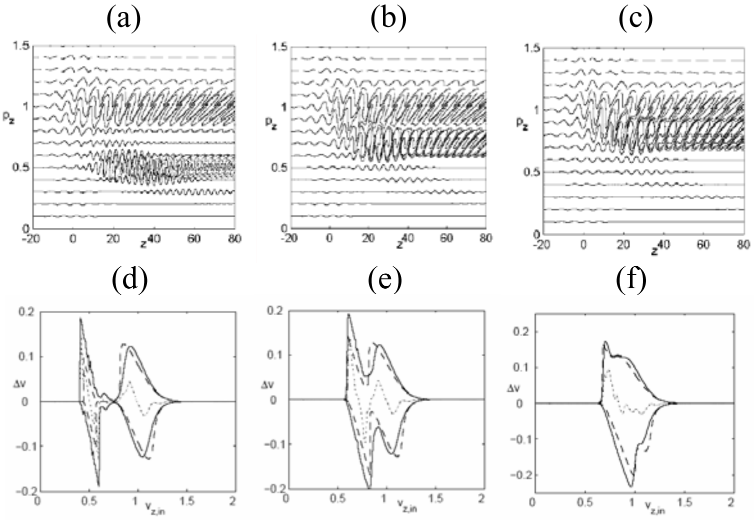

Figure 2: Interaction with two SWs. (a,b,c) Numerically obtained Poincare surface of section; (d,e,f) Numerically (solid) and analytically (dashed) obtained extrema of velocity variation. The averaged velocity variation is also shown (dotted line). The parameters of the SWs are: $A_{1,2} = 0.005$, $\sigma_{1,2} = 10$, $\omega_{1,2} = 1$, $v_{g_1} = 0$, $v_{g_2} = 0.1$, $v_{p_1} = 1$, $v_{p_2} = 0.5, 0.7, 0.8$ (left to right).

$$F(p_z, z, t) = F_0(p_z) - \frac{\partial F_0}{\partial p_z}\Delta p_z + \frac{1}{2}\frac{\partial}{\partial p_z}\left(\Delta p_z^2 \frac{\partial F_0}{\partial p_z}\right) \qquad (11)$$

where an initially (at $z_0 - v_g t_0 \to -\infty$) position-independent (uniform) distribution function is assumed. It is worth mentioning that the simple expression for the approximate distribution function given by (11) and (fS1) has been obtained under no adiabaticity assumption [7], and thus, it is valid for ultrashort pulses as well as solitons and slowly modulated wavepackets. Moreover, it can be used to calculations of certain quantities, such as position-averaged momentum variations for any initial momentum distribution, and it generalizes results related to Madey's theorem (based on the periodicity assumption of the position coordinate [11] which does not hold in our case).

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