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Sheath formation in a two-electron temperature plasma

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1 Introduction

Recently Ye et al. [1] have presented a one dimensional fluid model of a sheath that forms in front of a negative electrode that emits electrons. In a recent work [2] we have extended their model by considering the effects of the presence of the hot electrons in the plasma. In this work we further extend the model by taking into account the repulsion of the electrons by the potential drop in the pre-sheath.

2 Model

We consider an infinite plane material surface (collector) in contact with plasma that contains two electron populations and singly charged positive ions that are assumed to be mono-energetic and at rest at a very large distance from the collector. At a large distance from the collector the plasma potential is assumed to be zero $\Phi(x \to \infty) = 0$ and also the electric field there is zero. As we approach the collector the plasma potential gradually decreases, so that a finite electric field exists in the plasma which accelerates positive ions towards the collector. At a certain distance $x = d$ from the collector, the value of the potential is Φ_0 and there the ions have velocity v_0 towards the collector. Plane at $x = d$ is called the sheath edge. There the plasma is still quasineutral, but immediately beyond this point a sheath with large electric field is formed. The potential profile is determined by the Poisson equation:

$$\frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0}\left(n_i(x) - n_1(x) - n_2(x) - n_3(x)\right), \tag{1}$$

where $n_i(x)$ is the ion density, $n_1(x)$ is the density of basic plasma electron population with lower temperature (cool electrons), $n_2(x)$ is the density of the hot electron plasma population and $n_3(x)$ is the density of emitted electrons (called also secondary electrons). Cool and hot electrons are assumed to obey the Boltzmann relation:

$$n_1(x) = n_{1S}\exp\left(\frac{e_0(\Phi(x)-\Phi_0)}{kT_1}\right), \quad n_2(x) = n_{2S}\exp\left(\frac{e_0(\Phi(x)-\Phi_0)}{kT_2}\right). \tag{2}$$

Density of ions and secondary electrons is obtained from the assumption that energy and flux of both particle species are conserved:

$$n_i(x) = \frac{n_S}{\sqrt{1 - \frac{2e_0(\Phi(x)-\Phi_0)}{m_i v_0^2}}}, \quad n_3(x) = n_{3S}\sqrt{\frac{1+\frac{2e_0(\Phi_0-\Phi_C)}{m_e v_C^2}}{1+\frac{2e_0(\Phi(x)-\Phi_C)}{m_e v_C^2}}}. \tag{3}$$

Here n_S, n_{1S}, n_{2S}, and n_{3S} are the respective particle densities at the sheath edge, Φ_C is the collector potential, m_i and m_e are the ion and the electron mass respectively and e_0 is *absolute value* of the elementary charge. The electrons that are emitted from the collector, all leave the collector with the same velocity v_C. We label the ratio between the hot and the cool electron density *at the sheath edge* by β_S: $\beta_S = n_{2S}/n_{1S}$. We also assume that the flux of secondary electrons j_3 from the collector is proportional to the incoming flux of cool and hot electrons in the form: $j_3 = \gamma(j_1 + j_2)$, where γ is the electron emission coefficient and fluxes are given by:

$$j_1 = e_0 n_{1S}\sqrt{\frac{kT_1}{2\pi m_e}}\exp\left(\frac{e_0(\Phi_C-\Phi_0)}{kT_1}\right), \; j_2 = e_0 n_{2S}\sqrt{\frac{kT_2}{2\pi m_e}}\exp\left(\frac{e_0(\Phi_C-\Phi_0)}{kT_2}\right), \tag{4}$$

$$j_3 = e_0 n_{3S} v_C\sqrt{1+\frac{2e_0(\Phi_0-\Phi_C)}{m_e v_C^2}}, \; j_i = e_0 n_S v_0. \tag{5}$$

Using the definition of β_S, the relation between j_1, j_2 and j_3 and the fluxes (4) - (5), the electron densities n_{1S}, n_{2S}, and n_{3S} at the sheath edge can be expressed in terms of the ion density n_S. Those densities are then inserted into (2) and (3) and then (2) and (3) are inserted into (1). We get:

$$\frac{d^2\Psi}{dz^2} = \frac{1}{1+\beta_S+G}\left(\exp(\Psi)+\beta_S\exp\left(\frac{\Psi}{\Theta}\right)+\frac{G}{\sqrt{1-\frac{\Psi}{\Psi_S-\frac{N^2\mu}{2}}}}\right) - \frac{1}{\sqrt{1-\frac{2\Psi}{M^2}}}. \tag{6}$$

The following variables have been introduced:

$$z = \frac{x}{\lambda_D}, \; \Psi = \frac{e_0(\Phi(x)-\Phi_0)}{kT_1}, \; \Psi_S = \frac{e_0(\Phi_C-\Phi_0)}{kT_1}, \; \Psi_0 = \frac{e_0\Phi_0}{kT_1}, \tag{7}$$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT_1}{n_S e_0^2}}, \; \Theta = \frac{T_2}{T_1}, \; \mu = \frac{m_e}{m_i}, \; v_0 = M\sqrt{\frac{kT_1}{m_i}}, \; v_C = N\sqrt{\frac{kT_1}{m_i}}, \tag{8}$$

$$G = \frac{\gamma\left(\exp(\Psi_S)+\beta_S\sqrt{\Theta}\exp\left(\frac{\Psi_S}{\Theta}\right)\right)}{\sqrt{2\pi(N^2\mu-2\Psi_S)}}. \tag{9}$$

Following the procedure, described in the previous paper [2] the Poisson equation (6) is integrated once over Ψ and expanded in Taylor series. The first and second order terms cancel each other out and the third order terms give the modification of the Bohm criterion due to the presence of the hot and of the emitted electrons in the system:

$$M = \sqrt{\frac{1+\beta_S+G}{\left(1+\frac{\beta_S}{\Theta}\right)+\frac{G}{2\left(\Psi_S-\frac{N^2\mu}{2}\right)}}}. \tag{10}$$

If emission of the secondary electrons from the collector increases, eventually the density of secondary electrons and consequently negative space charge in front of the collector becomes so high, that the electric field at the collector becomes zero. This is called the critical emission and the corresponding emission coefficient γ_c is the critical emission coefficient. Following the procedure, described in the previous paper [2] the condition for the zero electric field at the collector is obtained:

$$0 = \frac{1}{1+\beta_S+G}\cdot\left[\begin{array}{c}(\exp(\Psi_S)-1)+\beta_S\Theta\left(\exp\left(\frac{\Psi_S}{\Theta}\right)-1\right)+ \\ +2G\left(\Psi_S-\frac{N^2\mu}{2}\right)\left(1-\sqrt{1-\frac{\Psi_S}{\Psi_S-\frac{N^2\mu}{2}}}\right)\end{array}\right]- \tag{11}$$

$$-\frac{1+\beta_S+G}{\left(1+\frac{\beta_S}{\Theta}\right)+\frac{G}{2\left(\Psi_S-\frac{N^2\mu}{2}\right)}}\left(1-\sqrt{1-\frac{2\Psi_S\left(\left(1+\frac{\beta_S}{\Theta}\right)+\frac{G}{2\left(\Psi_S-\frac{N^2\mu}{2}\right)}\right)}{1+\beta_S+G}}\right).$$

When the collector is floating the total current J_{tot} to the collector is zero:

$$J_{tot} = \frac{j_i+j_3-j_1-j_2}{e_0 n_S\sqrt{\frac{kT_1}{m_i}}} = \sqrt{\frac{1+\beta_S+G}{\left(1+\frac{\beta_S}{\Theta}\right)+\frac{G}{2\left(\Psi_S-\frac{N^2\mu}{2}\right)}}}+ \tag{12}$$

$$+\frac{1}{1+\beta_S+G}\left[G\sqrt{N^2-\frac{2\Psi_S}{\mu}}-\frac{1}{\sqrt{2\pi\mu}}\left(\exp(\Psi_S)+\beta_S\sqrt{\Theta}\exp\left(\frac{\Psi_S}{\Theta}\right)\right)\right]=0.$$

Note that β_S is the hot to cool electron density ratio *at the sheath edge*. One may expect that this ratio is not the same at the sheath edge and in the unperturbed plasma very far away from the collector. By the assumption of our model the plasma potential in the pre-sheath decreases monotonically from $\Phi(x) = 0$ at $x \to \infty$ to $\Phi(x) = \Phi_0$ at $x = d$. Therefore part of the cool and of the hot electrons that move towards the collector are repelled away from it back into the plasma by this potential drop. We assume that in the pre-sheath region the hot and the cool electrons obey the Boltzmann relation. So the cool and the hot electron densities n_{1S} and n_{2S} at the sheath edge and the respective electron densities n_{10} and n_{20} at $x \to \infty$ are related by: $n_{1S} = n_{10} \exp(\Psi_0)$, $n_{2S} = n_{20} \exp(\Psi_0/\Theta)$. From the assumption of energy conservation in the pre-sheath it is straightforward to calculate the potential drop in the pre-sheath: $\Psi_0 = -M^2/2$. The hot to cool electron density ratio β_0 at $x \to \infty$ is defined by: $\beta_0 = n_{20}/n_{10}$. When n_{1S}, n_{2S}, Ψ_0 and β_0 are inserted into the definition of β_S, one gets:

$$\beta_S = \beta_0 \exp\left(\frac{M^2(\Theta-1)}{2\Theta}\right). \tag{13}$$

When (13) is inserted into (10) we find:

$$M = \sqrt{\frac{1+\beta_0 \exp\left(\frac{M^2(\Theta-1)}{2\Theta}\right)+G}{\left(1+\frac{\beta_0}{\Theta}\exp\left(\frac{M^2(\Theta-1)}{2\Theta}\right)\right)+\frac{G}{2\left(\Psi_S-\frac{N^2\mu}{2}\right)}}}. \tag{14}$$

The model presented in this section can be summarized in the following way. Equations (11), (12), (13) and (14) form a system of 4 equations for 4 unknown parameters: Ψ_S, G, β_S and M if all the other parameters μ, β_0, Θ and N are given. When Ψ_S, G, β_S and M are found, all the other parameters like the critical emission coefficient γ_c, the potential drop in the pre-sheath Ψ_0 and the densities of all 3 groups of electrons at the sheath edge n_{1S}, n_{2S}, and n_{3S} can be found. In some cases the system of equations (11) - (14) can have even 5 solutions. An example of such a solution for the following parameters: μ = 1/1836, Θ = 40, N = 60 and β_0 = 0.1685 is given in Table 1.

References

1) M. Ye, S. Takamura, *Phys. Plasmas*, **7**, (2000), 3457

2) T. Gyergyek, M. Čerček, *Contrib. Plasma Phys.*, **45**, (2005), 89

Ψ_S	M	β_S	γ_c
-5.38057	1.26187	0.366204	0.942517
-9.77484	1.21764	0.347139	0.938446
-13.2491	1.20057	0.340223	0.934172
-108.103	2.85168	8.878	0.559657
-32.751	6.91091	$2.17978 \cdot 10^9$	0.838573

Table 1: