

**THE CHARGED PARTICLE DISTRIBUTION
FUNCTION in a TOKAMAK,
DEPENDENT on a VELOCITY GRADIENT.**

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The particle distribution function deviation from equilibrium one will linearly depends as on parameters from the factors breaking equilibrium distribution, such, as a gradients of density, temperature, velocity etc.

In standard neoclassical theory (SNT) the perturbations connected to a gradient of longitudinal and transverse velocities were not taken into account. Let us remind that in SNT the magnetic field value on a *magnetic* surface is used. This value is equal to $B_s = B_0 / (1 + \varepsilon_s \cos \theta)$, where B_0 is magnetic field on the plasma magnetic axes, $\varepsilon_s = r_s / R$ is an inverse aspect ratio, r_s is magnetic surface radius, R is major tokamak radius, θ is poloidal angle.

The present analysis (PA) will carry out for an axisymmetric system (tokamak) with a small inverse aspect ratio $\varepsilon \ll 1$, with a concentric circular flux surfaces for a collisionless (banana-regime) plasma and far from the magnetic axes ($\Delta r \ll r_s$, where Δr is the radial drift of the particle from the magnetic surface).

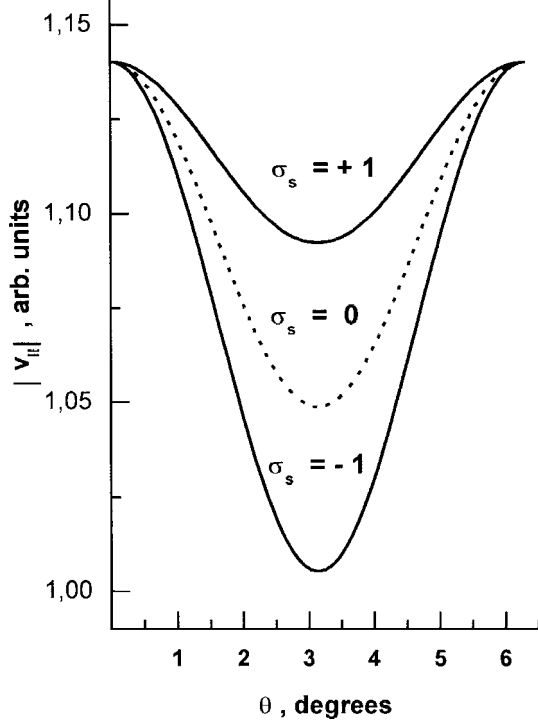
In this paper in contrast to SNT we will use the magnetic field value on a particle *drift* trajectory - $B = B_0 / (1 + \varepsilon \cos \theta)$. The orbit equation in this approximation is given by [1]

$$\varepsilon^2 - \varepsilon_s^2 = \zeta (\sigma_v \sqrt{G + \varepsilon \cos \theta} - \sigma_s \sqrt{G + \varepsilon_s \cos \theta_s}) \quad (1)$$

where $\zeta = 2\rho q / R$, ρ is the Larmor radius, q is the safety factor, $\sigma_v = \pm 1$ is the velocity sign, $G = 1 - \mu B_0 / E$, μ and E are the particle magnetic moment and energy, respectively, $\varepsilon = r / R$ is the local inverse aspect ratio, $\sigma_s = \pm 1$ and θ_s are the velocity sign and the poloidal angle in the point where the drift trajectory intersect the magnetic surface. For the passing particles we have $\sigma_v = \sigma_s$.

In SNT the particle trajectory is described with help of three constants-of-motion (COM) which are E , μ and ε_s . In the PA we have five COM, namely E , μ , ε_s , σ_s , and θ_s . So, in our case ε is the function of COM and θ .

In the SNT the particle longitude velocity is equal to the velocity on the magnetic surface



$$V_{IIs} = \sigma_v V \sqrt{G + \varepsilon_s \cos \theta} = \sigma_v V \tilde{V}_{IIs}, \text{ and}$$

in the PA we use the longitudinal velocity on the drift particle trajectory, namely

$$V_{II} = \sigma_v V \sqrt{G + \varepsilon(\text{COM}, \theta) \cos \theta} = \sigma_v V \tilde{V}_{II}$$

In the figure one can see the curves of longitudinal velocity against the poloidal angle for different values of σ_s . The SNT velocity ($\sigma_s = 0$) is shown dotted, and PA velocities are correspondent to $\sigma_s = \pm 1$.

From the figure one can see that in the SNT the longitudinal velocity is even function and in the PA the last is odd function. It is obvious that radial gradient of longitudinal velocity is equal to zero in SNT ($dV_{IIs} / dr = 0$) and is not equal to zero in PA ($dV_{II} / dr \neq 0$).

Let us decompose \tilde{V}_{II} on even part \tilde{V}_{IIs} and its perturbation $\Delta \tilde{V}$

$$\tilde{V}_{II} = \tilde{V}_{IIs} + \Delta \tilde{V} = \tilde{V}_{IIs} + (\tilde{V}_{II} - \tilde{V}_{IIs}) = \tilde{V}_{IIs} + \frac{\Delta \varepsilon \cos \theta}{2 \tilde{V}_{IIs}} = \tilde{V}_{IIs} + \left. \frac{\partial \tilde{V}_{II}}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta \varepsilon \quad (2)$$

where $\Delta \varepsilon = \Delta r / R$.

For $\Delta \varepsilon \ll \varepsilon$ one can obtain from (1) that

$$\Delta \varepsilon = \frac{\zeta_T \sqrt{x}}{2 \varepsilon_s} (\sigma_v \tilde{V}_{IIs} - \sigma_s \tilde{V}_s) \quad (3)$$

where ζ_T is the value of ζ when $E=T$, $x = E/T(\varepsilon_s)$ is normalized particle energy,

$$\tilde{V}_s = \sqrt{G + \varepsilon_s \cos \theta_s}. \text{ As } E = \frac{mV^2}{2} \tilde{V}_{II}^2 + \mu B \text{ we have}$$

$$E = \frac{mV^2}{2} (\tilde{V}_{\text{II}s}^2 + 2\tilde{V}_{\text{II}s}\Delta\tilde{V} + \Delta\tilde{V}^2) + \mu B_s + \mu \left. \frac{\partial B}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta\varepsilon \quad (4)$$

and

$$E \approx \frac{mV^2\tilde{V}_{\text{II}s}^2}{2} + \mu B_s + \left(m\tilde{V}_{\text{II}s}\Delta\tilde{V} + \mu \left. \frac{\partial B}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta\varepsilon \right) \quad (5)$$

The local Maxwellian function can be presented as

$$f = f_{\text{NC}} \tilde{f}_{\text{II}} \tilde{f}_{\perp} \quad (6)$$

where

$$f_{\text{NC}} = \left(\frac{m}{2\pi T(\varepsilon)} \right)^{3/2} n(\varepsilon) e^{-\frac{E_{\text{NC}}}{T(\varepsilon)}} \quad (7)$$

$$\tilde{f}_{\text{II}} = e^{-2x\tilde{V}_{\text{II}s}\Delta\tilde{V}} \approx 1 - 2x\tilde{V}_{\text{II}s}\Delta\tilde{V} \quad (8)$$

$$f_{\perp} = e^{\frac{\mu}{T(\varepsilon_s)} \left. \frac{\partial B}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta\varepsilon} \approx 1 + \frac{\mu}{T(\varepsilon_s)} \left. \frac{\partial B}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta\varepsilon \quad (9)$$

and

$$E_{\text{NC}} = \frac{mV^2\tilde{V}_{\text{II}s}^2}{2} + \mu B_s \quad (10)$$

Let us consider the situation when $\partial n / \partial \varepsilon = 0$ and $\partial T / \partial \varepsilon = 0$. In this case we have that $f_{\text{NC}} = f_{\text{NC}}(\varepsilon_s) = f_s$, that is in such approximation f_{NC} coincides with the distribution function on magnetic surface f_s . The function f_s is isotropic function.

The distribution function f has been written in the form

$$f = f_s (1 - 2x\tilde{V}_{\text{II}s}\Delta\tilde{V} + g_{\text{II}} + \frac{\mu}{T(\varepsilon_s)} \left. \frac{\partial B}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_s} \Delta\varepsilon + g_{\perp}) \quad (11)$$

where the functions g_{II} and g_{\perp} are not depended on poloidal angle.

To find the functions g_{II} and g_{\perp} we will use the solvability conditions

$$\int_0^{2\pi} \frac{C(f)}{V_{\text{II}}} d\theta \quad (12)$$

for passing particles and

$$\int_{\theta_1}^{\theta_2} \frac{C(f, \sigma_v = +1) + C(f, \sigma_v = -1)}{|V_{II}|} = 0 \quad (13)$$

for trapped particles. Here θ_1 and θ_2 are poloidal angles where the longitudinal velocity changes its sign and C is a collision operator.

In this paper we have adopted the Lorentz operator

$$C(f) \cong 2\nu x \tilde{V}_{II} \frac{\partial}{\partial G} \tilde{V}_{II} (1 - G) \frac{\partial f}{\partial G} \quad (14)$$

where ν is a collision frequency.

Using (11), (13) and (14), and neglecting the terms of the second order in ε_s , we have

$$\tilde{g}_{II} = \frac{1}{2} \int_{\varepsilon_s}^G \frac{\langle \tilde{V}_{II s} \cos \theta \rangle}{\langle \tilde{V}_{II s} \rangle} \frac{dG}{\tilde{V}_s} \quad (15)$$

and

$$\tilde{g}_{\perp} = -\tilde{g}_{II} \quad (16)$$

Here $\langle \rangle$ means averaging along the poloidal angle, and $g = \tilde{g} \cdot \sigma_s \zeta_T x^{3/2} / (2\varepsilon_s)$

From (2) one can see that perturbations \tilde{g}_{II} and \tilde{g}_{\perp} are determined by the radial gradient of longitudinal velocity which is determined by the radial gradient of the toroidal magnetic field.

So, the longitudinal velocity distribution function is

$$F_{II} = f_s f_{II} = f_s \left(1 - \sigma_s \frac{\zeta_T x^{3/2}}{2\varepsilon_s} \left((\tilde{V}_{II s} - \tilde{V}_s) \cos \theta + g_{II} \right) \right) \quad (17)$$

and the perpendicular velocity distribution function is

$$F_{\perp} = f_s f_{\perp} = f_s \left(1 + \sigma_s \frac{\zeta_T x^{3/2}}{2\varepsilon_s} \left((\tilde{V}_{II s} - \tilde{V}_s) \cos \theta + g_{II} \right) \right) \quad (18)$$

A toroidal current calculated with help of the longitudinal velocity V_{II} and the distribution function F_{II} is asymmetry current described in [2].

1. Gott Yu.V., Yurchenko E.I. Plasma Physics Reports, v.25, #5, p.363, 1999.
2. Gott Yu.V., Yurchenko E.I. Plasma Physics Reports, v.28, #5, p.382, 2002.