The charged particle distribution function deviation from equilibrium one will linearly depend as on parameters from the factors breaking equilibrium distribution, such, as gradients of density, temperature, velocity etc.

In standard neoclassical theory (SNT) the perturbations connected to a gradient of longitudinal and transverse velocities were not taken into account. Let us remind that in SNT the magnetic field value on a magnetic surface is used. This value is equal to 
\[ B_s = B_0 / (1 + \varepsilon_s \cos \theta) \], where \( B_0 \) is magnetic field on the plasma magnetic axes, \( \varepsilon_s = r_s / R \) is an inverse aspect ratio, \( r_s \) is magnetic surface radius, \( R \) is major tokamak radius, \( \theta \) is poloidal angle.

The present analysis (PA) will carry out for an axisymmetric system (tokamak) with a small inverse aspect ratio \( \varepsilon << 1 \), with a concentric circular flux surfaces for a collisionless (banana-regime) plasma and far from the magnetic axes (\( \Delta r << r_s \), where \( \Delta r \) is the radial drift of the particle from the magnetic surface).

In this paper in contrast to SNT we will use the magnetic field value on a particle drift trajectory \( B = B_0 / (1 + \varepsilon \cos \theta) \). The orbit equation in this approximation is given by [1]

\[ \varepsilon^2 - \varepsilon_s^2 = \zeta (\sigma_v \sqrt{G + \varepsilon \cos \theta} - \sigma_s \sqrt{G + \varepsilon_s \cos \theta_s}) \]  

(1)

where \( \zeta = 2pq / R \), \( \rho \) is the Larmor radius, \( q \) is the safety factor, \( \sigma_v = \pm 1 \) is the velocity sign, \( G = 1 - \mu B_0 / E \), \( \mu \) and \( E \) are the particle magnetic moment and energy, respectively, \( \varepsilon = r / R \) is the local inverse aspect ratio, \( \sigma_s = \pm 1 \) and \( \theta_s \) are the velocity sign and the poloidal angle in the point where the drift trajectory intersect the magnetic surface. For the passing particles we have \( \sigma_v = \sigma_s \).
In SNT the particle trajectory is described with help of three constants-of-motion (COM) which are $E$, $\mu$ and $\varepsilon_s$. In the PA we have five COM, namely $E$, $\mu$, $\varepsilon_s$, $\sigma_s$, and $\theta_s$. So, in our case $\varepsilon$ is the function of COM and $\theta$.

In the SNT the particle longitude velocity is equal to the velocity on the magnetic surface

$$V_{II s} = \sigma_s \sqrt{G + \varepsilon_s \cos \theta} = \sigma_s \sqrt{G} V_{\tilde{V}_{II s}},$$

and in the PA we use the longitudinal velocity on the drift particle trajectory, namely

$$V_\parallel = \sigma_s \sqrt{G + \varepsilon(COM, \theta) \cos \theta} = \sigma_s \sqrt{G} V_{\tilde{V}_\parallel}.$$

In the figure one can see the curves of longitudinal velocity against the poloidal angle for different values of $\sigma_s$. The SNT velocity ($\sigma_s = 0$) is shown dotted, and PA velocities are correspondent to $\sigma_s = \pm 1$.

From the figure one can see that in the SNT the longitudinal velocity is even function and in the PA the last is odd function. It is obvious that radial gradient of longitudinal velocity is equal to zero in SNT ($dV_{II s} / dr = 0$) and is not equal to zero in PA ($dV_{II} / dr \neq 0$).

Let us decompose $\tilde{V}_\parallel$ on even part $\tilde{V}_{II s}$ and its perturbation $\Delta \tilde{V}$

$$\tilde{V}_\parallel = \tilde{V}_{II s} + \Delta \tilde{V} = \tilde{V}_{II s} + (\tilde{V}_\parallel - \tilde{V}_{II s}) = \tilde{V}_{II s} + \frac{\Delta \varepsilon \cos \theta}{2V_{II s}} = \tilde{V}_{II s} + \frac{\partial \tilde{V}_{II}}{\partial \varepsilon} \bigg|_{\varepsilon = \varepsilon_s} \Delta \varepsilon$$

(2)

where $\Delta \varepsilon = \Delta r / R$.

For $\Delta \varepsilon \ll \varepsilon$ one can obtain from (1) that

$$\Delta \varepsilon = \frac{\zeta_T \sqrt{x}}{2 \varepsilon_s} (\sigma_s \tilde{V}_{II s} - \sigma_s \tilde{V}_s)$$

(3)

where $\zeta_T$ is the value of $\zeta$ when $E = T$, $x = E / T(\varepsilon_s)$ is normalized particle energy,

$$\tilde{V}_s = \sqrt{G + \varepsilon_s \cos \theta_s}.$$
\[ E = \frac{mV^2}{2} (\tilde{V}_{II}^2 + 2\tilde{V}_{II} \Delta \tilde{V} + \Delta \tilde{V}^2) + \mu B_s + \mu \frac{\partial B}{\partial e} \bigg|_{e=e_s} \Delta e \]  

(4)

and

\[ E \approx \frac{mV^2 \tilde{V}_{II}^2}{2} + \mu B_s + \left( m\tilde{V}_{II} \Delta \tilde{V} + \mu \frac{\partial B}{\partial e} \bigg|_{e=e_s} \Delta e \right) \]  

(5)

The local Maxwellian function can be presented as

\[ f = f_{NC} \tilde{f}_H \tilde{f}_\perp \]  

(6)

where

\[ f_{NC} = \left( \frac{m}{2\pi T(e)} \right)^{3/2} n(e)e^{-\frac{E_{NC}}{T(e)}} \]  

(7)

\[ \tilde{f}_H = e^{-2x\tilde{V}_{II} \Delta \tilde{V}} \approx 1 - 2x\tilde{V}_{II} \Delta \tilde{V} \]  

(8)

\[ f_{\perp} = e^{\frac{\mu}{T(e_s)} \frac{\partial B}{\partial e}}_{e=e_s} \Delta e \approx 1 + \frac{\mu}{T(e_s)} \frac{\partial B}{\partial e} \bigg|_{e=e_s} \Delta e \]  

(9)

and

\[ E_{NC} = \frac{mV^2 \tilde{V}_{II}^2}{2} + \mu B_s \]  

(10)

Let us consider the situation when \( \partial n / \partial e = 0 \) and \( \partial T / \partial e = 0 \). In this case we have that \( f_{NC} = f_{NC}(e_s) = f_s \), that is in such approximation \( f_{NC} \) coincides with the distribution function on magnetic surface \( f_s \). The function \( f_s \) is isotropic function.

The distribution function \( f \) has been written in the form

\[ f = f_s (1 - 2x\tilde{V}_{II} \Delta \tilde{V} + g_H + \frac{\mu}{T(e_s)} \frac{\partial B}{\partial e} \bigg|_{e=e_s} \Delta e + g_\perp) \]  

(11)

where the functions \( g_H \) and \( g_\perp \) are not depended on poloidal angle.

To find the functions \( g_H \) and \( g_\perp \) we will use the solvability conditions

\[ \int_0^{2\pi} \frac{C(f)}{V_H} d\theta \]  

(12)

for passing particles and
\[
\int_{0_1}^{0_2} \frac{C(f, \sigma_V = +1) + C(f, \sigma_V = -1)}{|V_{II}|} = 0
\]  
(13)

for trapped particles. Here \(0_1\) and \(0_2\) are poloidal angles where the longitudinal velocity changes its sign and \(C\) is a collision operator.

In this paper we have adopted the Lorentz operator

\[
C(f) \equiv 2v_x \tilde{V}_{II} \frac{\partial}{\partial G} \tilde{V}_{II} (1 - G) \frac{\partial f}{\partial G}
\]  
(14)

where \(v\) is a collision frequency.

Using (11), (13) and (14), and neglecting the terms of the second order in \(s\), we have

\[
\tilde{g}_{II} = \frac{1}{2} \int_{e_s} G < \tilde{V}_{II_s} \cos \theta > \, dG
\]  
(15)

and

\[
\tilde{g}_\perp = -\tilde{g}_{II}
\]  
(16)

Here <> means averaging along the poloidal angle, and \(g = \tilde{g} \cdot \sigma_s \zeta_T x^{3/2} / (2e_s)\).

From (2) one can see that perturbations \(\tilde{g}_{II}\) and \(\tilde{g}_\perp\) are determined by the radial gradient of longitudinal velocity which is determined by the radial gradient of the toroidal magnetic field.

So, the longitudinal velocity distribution function is

\[
F_{II} = f_s F_{II} = f_s \left( 1 - \sigma_s \frac{\zeta_T x^{3/2}}{2e_s} (\tilde{V}_{II_s} - \tilde{V}_s) \cos \theta + g_{II} \right)
\]  
(17)

and the perpendicular velocity distribution function is

\[
F_\perp = f_s F_\perp = f_s \left( 1 + \sigma_s \frac{\zeta_T x^{3/2}}{2e_s} (\tilde{V}_{II_s} - \tilde{V}_s) \cos \theta + g_{\perp} \right)
\]  
(18)

A toroidal current calculated with help of the longitudinal velocity \(V_{II}\) and the distribution function \(F_{II}\) is asymmetry current described in [2].