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A model for the evolution of current-driven ELMs

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Basis of the model

ELMs are an important factor in determining tokamak performance [1, 2]. Much theoretical research investigates the role of peeling and balloning modes [3], and systematic integrated modelling codes are also being developed [4]-[7]. In this paper we outline an approach to modelling ELMs in which we envisage edge current density ($j_{\parallel}$) driven ideal toroidal 'peeling' modes initiating and facilitating Taylor relaxation [8] of a tokamak outer region plasma. The model therefore assumes that the plasma is *below* the 'ballooning' stability limit, and is perhaps best suited to describe 'type III' ELMs [4, 9].

The stability criterion for ideal toroidal peeling modes (in the large aspect ratio approximation) is given by [10]

$$\alpha \left\{ \frac{r}{R_0}\left(1-\frac{1}{q^2}\right) + s\frac{d\Delta_{sh}}{dr} - f_t\frac{sR_0}{2r} \right\} \quad > \quad sqR_0\frac{j_{\parallel}}{B_0}\ , \tag{1}$$

where $\alpha = -2(\mu_0 R_0 q^2/B_0^2)\, dp/dr$ (with p the pressure profile), $s = (r/q)dq/dr$ is the magnetic shear (with q the safety factor), Δ_{sh} the Shafranov shift and f_t the fraction of trapped particles. As Eq. (1) shows, the pressure gradient term comprises of (stabilising) Mercier, Pfirsch-Schlüter and (destabilising) bootstrap components.

We assume that when the peeling stability boundary is crossed, a rapid process of energy release occurs producing the above mentioned post-ELM Taylor state. This force-free state in a tokamak has a flattened toroidal current profile, and we assume that the pressure in the edge is entirely lost. At first sight this would appear to generate an even more unstable situation for peeling modes, as the stabilising LHS of Eq. (1) disappears with the pressure, and the destabilising RHS would in general increase, as a flattening of a conventional current density profile would increase the edge $j_{\parallel}$. However, it is known [11] that assuming the relaxation process occurs quickly compared to global diffusion times, a relaxed plasma-vacuum system generally possesses a skin current distribution at its interfaces. We show that current sheets generated in edge tokamak relaxations generally have a stabilising effect on peeling modes. In fact, providing the relaxation region is of sufficient radial extent, this stabilising effect can balance the destabilisation produced by the increase in $j_{\parallel}$, and profiles that are marginally stable to peeling modes can be produced. This balance provides a means of calculating the model ELM width.

Outline of the analysis

Once edge pressure gradients are removed, the peeling mode can be treated in the cylindrical approximation (at least in the large aspect ratio limit). The controlling equation is marginal (zero torque) force balance in the plasma

$$\frac{d}{dr}\left(r\frac{d\psi}{dr}\right)-\frac{m^2\psi}{r} \;=\; \frac{m}{F}\mu_0\frac{dJ}{dr}\psi\,, \tag{2}$$

where ψ is the perturbed poloidal flux, J the equilibrium current density, and $F=(B_\theta/r)(m-nq)$ with m,n the poloidal and toroidal wave numbers. Equation (2) holds everywhere in the plasma, but is subject to boundary conditions both at the plasma/vacuum ($\mathscr{P}/\mathscr{V}$, $r=a$) interface and at the internal plasma radius that defines the extent of the relaxed region, $r=r_E$. We now introduce four quantities that represent physically relevant equilibrium and perturbation quantities at the interface: a) $\Delta_a \;=\; (1/q_{\mathscr{V}}-n/m)$, a dimensionless measure of the 'distance' between the $\mathscr{P}/\mathscr{V}$ interface and the resonance where $m=nq$, b) $\mu_0 J \;=\; (B_0/R_0)(1/r)\, d/dr\left(r^2/q\right)$, relating the toroidal current density to the safety factor q, c) $\mathscr{K}_a \;=\; R_0/(aB_0)\mu_0\mathscr{I}_{as} \;=\; [[1/q]]_{\mathscr{P}}^{\mathscr{V}}$, where $\mathscr{I}_{as}$ is the surface skin current density, and d) $\Delta_a' \;=\; [[(r/\psi)\,d\psi/dr]]_{\mathscr{P}}^{\mathscr{V}}$ the jump in the perturbed poloidal flux radial derivative which is central to MHD stability analysis. The boundary conditions to be applied at $r=a$ and r_E correspond to demanding that the radial field on the perturbed flux surfaces be continuous and that the tangential stress also be continuous. After some algebra, the boundary conditions produced by the sheet currents at $r=a$ and r_E are found to be

$$\Delta_a\left[\Delta_a\Delta_a'+\frac{R_0}{B_0}\mu_0 J_a\right]+\mathscr{K}_a\left[(\mathscr{K}_a-2\Delta_a)\left(\Delta_a'+m-1\right)+2\frac{n}{m}-\frac{R_0}{B_0}\mu_0 J_a\right]=0\,, \tag{3}$$

$$\begin{aligned}&\Delta_{E-}\left[\Delta_{E-}\Delta_E'-\frac{R_0}{B_0}\mu_0\,(J_{E+}-J_{E-})\right]+\\ &+\mathscr{K}_E\left[(\mathscr{K}_E+2\Delta_{E-})\left(\Delta_E'+m+1\right)+2\frac{n}{m}-\frac{R_0}{B_0}\mu_0 J_{E+}\right]=0\,.\end{aligned} \tag{4}$$

(We have shown independently that the LHS of Eq. (3) is directly related to δW, the ideal MHD energy perturbation.) To link Eqs. (3) and (4) and complete the mathematical model, we connect the $\Delta_{a,E}'$ across the relaxed region to give $\Delta_E' \;=\; -2m\left(\Delta_a'+2m\right)/\left(g\Delta_a'+2m\right)$, where $g=1-(r_E/a)^{2m}$.

The relaxed state

The original Taylor relaxation calculation [8] consisted of a constrained minimisation of the magnetic field energy. The relevant conserved quantities for a highly conducting plasma were the total toroidal magnetic flux Ψ_z, and the global helicity K of the magnetic field,

$K = \int_V \mathbf{A}.\mathbf{B}dV$ (with $\mathbf{A}$ the magnetic vector potential $\mathbf{B} = \nabla \times \mathbf{A}$). Within the cylindrical tokamak ordering, Ψ_z conservation is implicit, and the magnetic helicity reduces to $K = \int_{r_E}^{a} (r/q)\left(r^2 - r_E^2\right) dr$. As we are dealing with an *annular* plasma region, and hence two cylindrical boundaries, then it will be necessary to invoke a further invariant of the system to determine a final state. For a highly conducting Tokamak plasma the natural second quantity to be conserved throughout the relaxation process is the annular poloidal magnetic flux $\Psi_\theta = \int_{r_E}^{a} (r/q)\, dr$. Accordingly, our extended relaxation problem can be formulated as finding a minimisation of the poloidal magnetic energy W_θ, subject to conservation of *both* K and Ψ_θ. Formally, we require variations in the functional

$$W_\theta - \lambda_1 K - \lambda_2 \Psi_\theta = \int_{r_E}^{a} \left[\frac{r^3}{q^2} - \lambda_1 \frac{r}{q}\left(r^2 - r_E^2\right) - \lambda_2 \frac{r}{q}\right] dr \qquad (5)$$

to be stationary (with $\lambda_{1,2}$ Lagrangian multipliers). This problem has solution $q^f(r) = r^2/(Cr^2 + D)$ in $r_E < r < a$ (the superscript f denotes the final relaxed profile, C, D are constants to be determined - this q-profile corresponds to uniform toroidal current density). Note that the formalism has no fitted parameters, and gives a uniquely defined final state once the initial state has been specified.

An application

When edge peeling marginality, Eq. (1), is reached and edge pressure gradient is lost, then Eqs. (3)-(4) become the equations governing stability. We then need to find a relaxed state that is marginally stable to *all* possible peeling modes. To illustrate, we investigate an initial parabolic safety factor profile $q^i = q_0 + (q_a - q_0)\, r^2,\ 0 \le r \le 1$. For a given (q_0, q_a), Eqs. (3)-(4) (with $\mathscr{K}_{a,E} = 0$) give a sequence of (m, n) values (concentrating at $m = nq_a$) for which the initial profile is peeling unstable (LHS of Eq. (3) $\sim \delta W < 0$). For each (m, n) value we increase $dE = a - rE$ in Eqs. (3)-(4) until $\delta W = 0$, and peeling marginality is regained. It is then natural to assume that the model ELM width corresponds to the largest $dE = dE(max)$. Next, we may ask how the $dE(max)$ values vary as the initial equilibrium is varied. Figure 1 shows the result of such a calculation and we have plotted $dE(max)$ for $q_0 = 1$ and a range of q_a. Note that a feature of this plot is the 'deterministic' scatter of the results; the chaotic regions are caused by intrinsic variability in the quantity Δ_a. Figure 2 plots the m values for which the maximal mode exists.

For small ELM widths it proves possible to expand the entire set of equations determining the system and derive an analytic expression for dE. The maximal relaxation width within this

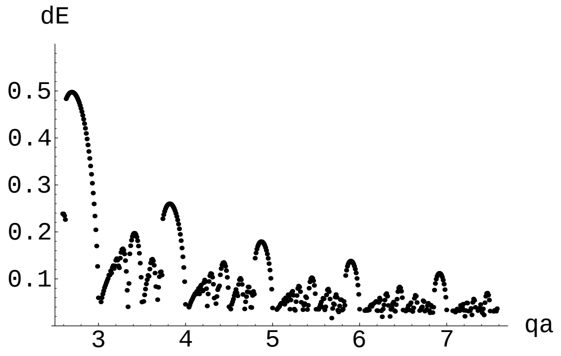

Figure 1: *The maximal marginal dE, plotted against the edge q value.*

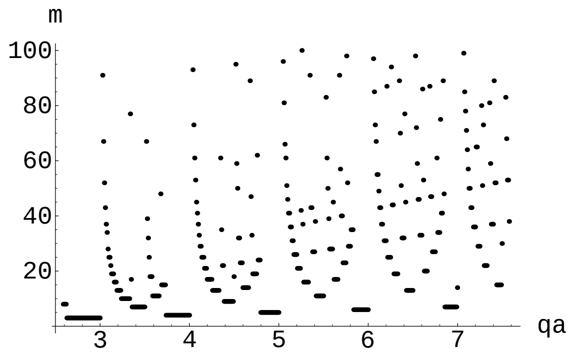

Figure 2: *The poloidal mode number m which gives the maximal marginal dE of Fig. 1.*

expansion is given by

$$(\frac{dE(max)}{a})^2 = -\frac{3}{4n}\frac{I_a^2}{(aI_a')} \qquad (6)$$

(we have put $I_a = (\mu_0 R_0/B_0)J_a$, and $'$ denotes d/dr). If we combine the results of Fig. 1 with the value of the critical pressure gradient as given by Eq. (1), we can calculate the experimentally measured values of ELM energy loss as a fraction of the plasma energy assuming a pressure equal to the pedestal value ($\Delta W_{ELM}/\Delta W_{ped}$). Putting in typical values for a highly collisional ($f_t = 0$) MAST discharge yields $\Delta W_{ELM}/\Delta W_{ped}$ of $\sim$ 1%, in accord with the observations.

Conclusions

We have considered a new model for ELM instabilities that hypothesises an edge Taylor relaxation initiated by peeling modes. The predicted ELM widths, energy losses and natural scatter in the predictions are in general accord with experimental observations. More detailed comparisons will be the subject of future work.

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