MHD Stability Analysis and Edge Control of the ATRAS-RFP Plasma
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Abstracts: MHD stabilities for the ATRAS-RFP plasma (major and minor radii of 50, 9cm) are analyzed using energy principle under the assumption of cylindrical symmetry. Many modes of instability that are characteristic of RFP plasma appear around the reversal surface. The structure control of the edge fields is discussed to satisfy Suydam’s stability condition and to have positive magnetic pressure gradient and magnetic well outside of reversed surface.

Experiment and results: The toroidal and poloidal structures of inner magnetic field are observed on the equatorial surface by probe-array inserted into plasma and changed to have cylindrical symmetry for the magnetic axis. The plasma and magnetic pressure profiles are obtained from the fields and shown in fig.1 with the local $\beta$ profile obtained from plasma pressure over magnetic pressure at the same point. Five solid lines for each quantities show the profiles obtained by 200 μsec after the start of discharge. The magnetic pressure has a peak on magnetic axis and decrease monotonically toward the plasma surface, so plasma pressure has a shallow depression and positive gradient around the axis then decrease rapidly toward the plasma surface with large negative gradient. Averaged $\beta$ value is about 0.2 and the device beta defined by volume averaged plasma pressure over magnetic pressure made by external coil on the plasma surface is about thirty. This means that RFP plasma has a high potential as D-D reactor, because strong poloidal field produced by the plasma current confines the plasma itself and the toroidal field made by external coil is very small compared with tokamaks.

MHD stability analysis: Ideal MHD stabilities are analyzed for experimentally observed poloidal and toroidal field configurations by using the energy principle¹). Under the assumption of equilibrium configuration to be cylindrical symmetry, the displacement of plasma is expressed in the form $\xi \exp(im\theta + ikz)$ where $m$ and $k$ are the azimuthal
and axial mode numbers respectively. The energy integral \( W(\xi) \) minimized with respect to \( \xi_{0,z} \) is expressed as
\[
W(\xi) = \frac{\pi}{2\mu_0} \int_0^\infty \left( f \left( \frac{d\xi}{dr} \right)^2 + g\xi^2 \right) dr, \quad f = r \frac{(kB_z + MB_\theta)^2}{m^2 + k^2r^2},
\]
\[
g = \frac{2k^2r^2}{m^2 + k^2r^2} \frac{dP}{dr} + \frac{1}{r} \left( kB_z + MB_\theta \right)^2 \frac{k^2r^2 + m^2 - 1}{m^2 + k^2r^2} \frac{1}{m^2 + k^2r^2} \left( k^2r^2B_z^2 - m^2B_\theta^2 \right),
\]
where \( \xi = \xi_z \) and \( \xi(b) = 0 \). Here, we consider that \( b \) is singular point of Euler Lagrange equation where the condition of \( bkB_z + MB_\theta = 0 \) is satisfied, and displacement \( \xi \) satisfies following conditions \( \xi(b) = 0 \) and \( d\xi/dr = 0 \) in the region of \( 0 < r < d \). For the perturbations satisfying these conditions, the stability depends on the sign of \( g(r) \).

RFP have many rational surfaces resonating with singular points in the plasma, especially around the reversal surface. Therefore, Suydam’s necessary condition for stability is calculated to examine the existence of local instabilities driven by pressure gradient. \( W(r) \) integrated from the magnetic axis to \( r \) and \( g(r) \) are calculated for various mode numbers of \( m \) and \( k \), and shown using blue and green lines with safety factor \( q(r) \) and Suydam’s condition traced by red and yellow lines in followings figures. The value of \( q \) is about 0.12 on the magnetic axis and decrease monotonically toward the plasma surface. It becomes 0 at the reversal surface and changes the sign in the outer region. The Suydam’s condition is not satisfied all over the region where plasma pressure gradient is negative excepting the region of positive pressure gradient.

For the \( m = 0 \) mode: The equation of \( g(r) \) becomes as follows,
\[
g(r) = 2 \frac{dP}{dr} + \frac{1}{r} \left( B_z \right)^2 \left( k^2r^2 + 1 \right).
\]
The values of \( g(r) \) become positive in the region of positive pressure gradient and negative in the region of rapid decreasing pressure gradient and small toroidal field. The values of \( W(r) \) are everywhere positive excepting on magnetic axis. The radial profiles of \( g(r) \) and \( W(r) \) are shown for \( k = \pm 10 \) in fig.2 with \( q(r) \) and Suydam’s condition. Sausage instabilities making the magnetic island for resistive plasma on the reversal surface with broadband of \( k \) independent on sign of \( k \) appear around the reversal surface where toroidal fields are small.

For the \( m = 1 \) mode: The current driven modes are marginal unstable around the
magnetic axis for long waves of $-10 < k < 10$ as shown in fig. 3 with resonant mode number of $k$ shown by pink line, even if the stabilizing effect of the positive pressure gradient exist. These modes become once more unstable in the middle region of plasma column then retrieve toward the plasma surface. Therefore, $g(r)$ and $W(r)$ are negative all over the plasma. For short wave length of $k < -40$, current driven modes become stable all over the plasma, and large negative pressure gradient makes plasma unstable in the middle region of plasma column as shown in the bottom of fig.3 for $k = -40$. $W(r)$ changes the sign to positive all over the plasma.

For the $m \geq 2$ modes: The current driven modes of every wave length are stable in the whole area of plasma and negative large pressure gradient make plasma unstable for the short wave length of $k$ around the inside of the reversal surface. These are shown in fig. 4 for the case of $m = 2$, $k = -25$ and $m = 2$, $k = -100$. In the outside of it, plasma is stable for every wave number of $k$ as seen in fig.4, because the negative pressure gradient is small and toroidal field is negative there. On the contrary, for $k > 0$ unstable short modes concentrate in

**fig.3.** $W(\xi)$ integrated from the magnetic axis to $r$, safety factor $q(r)$, Suydam’s condition and $g(r)$ for $m=1$, $k = -10$ (top) and $k = -40$ (bottom).

**fig.4.** $W(\xi)$ integrated from the magnetic axis to $r$, safety factor $q(r)$, Suydam’s condition and $g(r)$ for $m=1$, $k = -25$ (left) and $k = -100$ (right). Pink line show the $k$ value satisfying the $krB_z + mB_\phi = 0$. 
the outer region of the reversal surface because of negative of the toroidal field there. These results are suggesting the existence of dynamo effect needed by the RFP field configuration sustained inductive toroidal electric field.

**Edge controls:** We consider edge control for three conditions such as Suydam’s stable condition, positive gradient of magnetic pressure and magnetic well to make the stable region in the outer region of the reversal surface. The condition for magnetic well is always satisfied in the cylindrical symmetry RFP plasma because of

\[
\delta \int \frac{dl}{B} = -\frac{2\pi R}{B_z} \left( \frac{dB_z}{dr} \right) d\delta r < 0.
\]

The sufficient conditions satisfying Suydam’s and positive gradient of magnetic pressure are calculated under the assumption of linear dependency of poloidal and toroidal field. These dependencies are defined as follows,

\[
\alpha = -\frac{dB_z}{dr} = \text{const.} > 0, \quad \beta = -\frac{dB_z}{dr} = \text{const.} > 0.
\]

These results are shown in fig.5 with experimental results expressed by dots, where the same colored dots show that obtained by the same shot but different time. Suydam’s conditions are satisfied above the decreasing red line of \(\alpha\), and positive magnetic pressure gradient under the increasing blue line of \(\alpha\). The two conditions are satisfied simultaneously in very narrow region.

**Conclusions:** The current driven \(m=1\) kink modes appear in all over the plasma column for long wavelength of \(-20 < k < 0\) and concentrate in the middle region of it as the wavelength becoming shorter. For \(m \geq 2\) modes the current driven modes are stable independently of the \(k\) number. The pressure driven instabilities are dominant for the modes satisfying the following condition of \(m^2 << k^2 r^2\) and concentrate around large negative gradient of plasma pressure. These instabilities suggest the existence of the dynamo action that is needed to sustain RFP configuration.

The edge control for three conditions such as Suydam’s stable condition, positive gradient of magnetic pressure and magnetic well are achieved only in the narrow outer region of the reversal surface. It needs flat profile of the poloidal field and deep \(F\) value.