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Equilibrium reconstruction of tokamak discharges with toroidal variation

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Equilibrium reconstruction is the essential tool for determining the field structure and current density in a tokamak discharge. For performance reasons [1], tokamaks are designed to provide a field structure that is close to being axisymmetric around the torus axis. Due to the limited number of toroidal field coils, there are however small deviations from the axisymmetry, the magnetic field ripple. We present a generally applicable algorithm for the calculation of the three-dimensional perturbed field due to the ripple.

The Tore Supra tokamak is equipped with N=18 supra-conducting toroidal field coils, shown in figure 1. The vacuum field generated by N toroidal coils in cylindrical coordinates (R,φ,Z) is

$$\begin{aligned} B_T^{vac} &= \frac{B_0 R_0}{R}(1-\delta(R,Z)\cos N\varphi) \\ \mathbf{B}_p^{vac} &= -\frac{B_0 R_0}{N}\nabla\delta(R,Z)\sin N\varphi \end{aligned} \qquad (1)$$

where δ describes the deviation from axisymmetry, and is calculated from the known geometry of the toroidal field coils. An analytic form for Tore Supra is given in [2]. The function δ achieves its maximum close to the limiter on the low field side with values up to 7%. Since there is no compensation for the generated field ripple, it has to be taken into account for equilibrium calculations. There are various options to account for the ripple field in a method for the reconstruction of plasma equilibria. The most elaborate is the code V3FIT [3], which is a combination of the code EFIT [4] with the 3D inverse equilibrium code VMEC. Another option is to neglect curvature effects in the toroidal direction and treat the toroidal variation with a coordinate transformation, as done in the plasma boundary code DPOLO used at Tore Supra [5]. In this paper, we treat the toroidal variation by an expansion of the toroidal current in the Grad-Shafranov equation [6]. We assume isotropy of the plasma, and negligible flow, such that the equilibrium is described by the force balance $\nabla p = \mathbf{J}\times\mathbf{B}$,

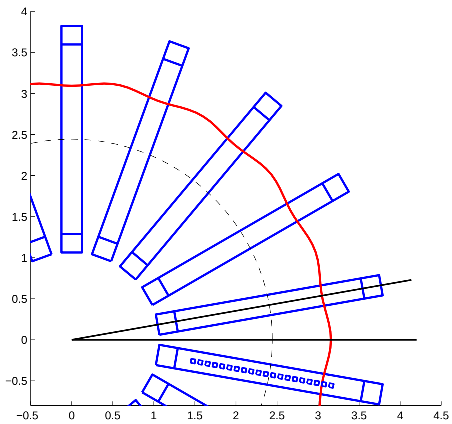

Figure 1: Tore Supra tokamak with 18 toroidal field coils, seen from above. The red line shows a field line with variation due to the ripple.

and Maxwell's equations $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ and $\nabla \cdot \mathbf{B} = 0$. All quantities are decomposed in their axisymmetric part, e.g. $\bar{\mathbf{B}} = \int \mathbf{B} d\varphi/(2\pi)$, and the zero-average part $\tilde{\mathbf{B}} = \mathbf{B} - \bar{\mathbf{B}}$. The equations are then expanded with a small parameter $\varepsilon = O(B_p/B_T)$. It is shown [6], that the axisymmetric part of the lowest nontrivial order leads to the Grad-Shafranov equation. The zero-average component of the expansion gives a correction term for the poloidal current density

$$\tilde{\mathbf{J}}_p^{(1)} = \frac{R\bar{J}_T^{(0)}}{B_0 R_0} \tilde{\mathbf{B}}_p^{(0)} \tag{2}$$

The lowest order oscillating term $\tilde{\mathbf{B}}_p^{(0)}$ is identified as the poloidal component of the ripple field $\mathbf{B}_p^{vac}$ given by equation (1). The toroidal current density is obtained by $\nabla \cdot \tilde{\mathbf{J}} = 0$. The sinusoidal form of the ripple field allows for calculating the magnetic field generated by $\tilde{\mathbf{J}}$ with a poloidal flux function $\tilde{\psi}(R, \varphi, Z)$ alone. With the ripple field from equation (1), we arrive at

$$-\Delta^* \tilde{\psi} = \mu_0 R^2 \nabla \cdot (R J_T \nabla \delta) \frac{\cos N\varphi}{N^2} =: \mu_0 R \mathscr{D}\{J_T\} \cos N\varphi \tag{3}$$

with the definition $\Delta^* := \frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2}$. Note that the resulting field $\tilde{\mathbf{B}}^{plasma}$ has only a poloidal field component. With the correction field given by equation (3), we modify the existing equilibrium reconstruction code EFIT [4]. The code EFIT parameterises the unknown toroidal current in the Grad-Shafranov equation as a linear superposition of test functions $J_k(\Psi, R)$. The flux has to be evaluated at the position of the magnetic sensors that are located under a toroidal field coil ($\cos N\varphi = +1$), and between two coils ($\cos N\varphi = -1$), where it is needed for all other diagnostics, and to determine the last closed flux surface by contact with the limiter. The resulting poloidal flux under the coil (+) and between coils (−) is then given by

$$-\Delta^* \Psi^{\pm} = \mu_0 R \sum_k c_k (1 \pm R\mathscr{D})\{J_k\} \tag{4}$$

The unknown coefficients are determined by minimising a least square functional

$$\chi^2 = \sum_{m=1}^{N_{meas}} \frac{1}{\sigma_m^2} (F_m^{calc}\{\Psi^+; \mathbf{c}\} - F_m^{meas})^2 + \lambda^2 \Re \tag{5}$$

with the Grad-Shafranov type equation (4) as a constraint. In equation (5), F_m^{meas} is the measured value, σ_m^2 is the estimated uncertainty of the measurement, and F_m^{calc} a functional to recalculate it from the flux function and the coefficients. The regularising term $\lambda^2 \Re$ accounts for the ill-posedness of the reconstruction problem. The innermost flux surface is obtained from the poloidal flux given by (4) plus the contribution from the vacuum ripple field. The poloidal flux function is not obtained directly from (1), but can formally be derived from the perturbation of the axisymmetric poloidal flux caused by the vacuum ripple field $\Psi(\mathbf{r} + d\mathbf{r})$. The coordinate transformation $d\mathbf{r}$ is obtained by integrating the vacuum field (1) along a toroidal flux surface. The resulting poloidal flux is then approximately given by

$$\Psi_{total} = \Psi^{-} - \frac{R^2}{N^2}\nabla\Psi^{-}\cdot\nabla\delta \tag{6}$$

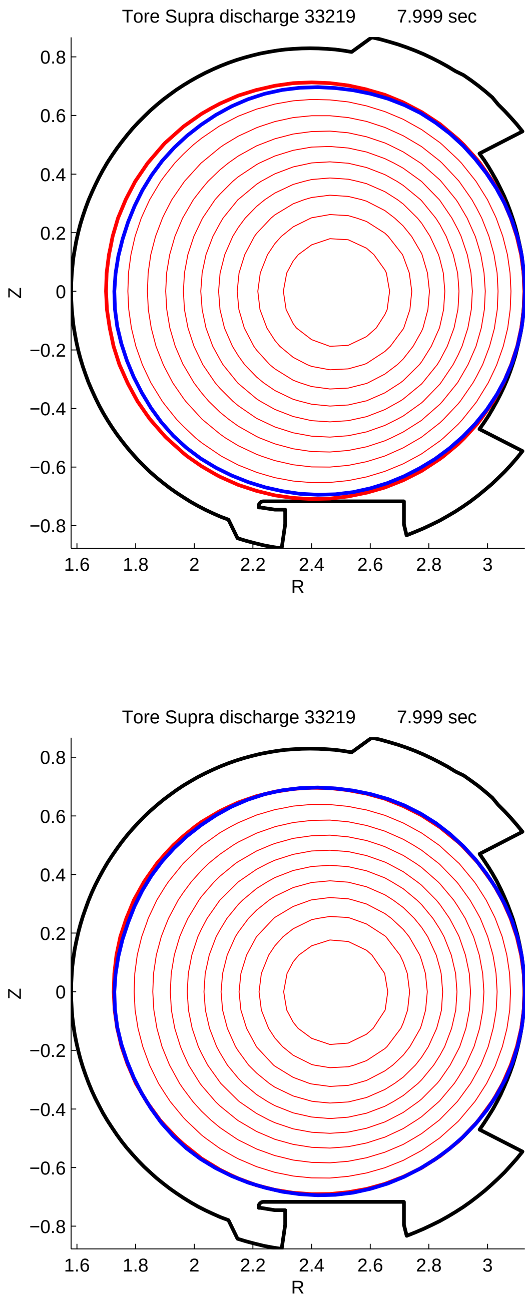

Figure 2: Field lines of Tore Supra discharge 33219 at t=8sec. At this time, the discharge terminates on the antenna protection limiter, where the effect of the toroidal field ripple achieves its maximum.

Top figure: The figure shows a comparison of the boundary code DPOLO (blue) with the equilibrium reconstruction code EFIT WITHOUT ripple correction (red). As expected, there is a large discrepancy of the position of the boundary, up to 5 cm at the high field side.

Bottom figure: Here, we apply the ripple correction for EFIT as described in this paper. There is good agreement between both codes, within an accuracy of 1 cm.

The modifications were implemented into the real-time EFIT version that is used at Tore Supra [7]. The algorithm with the modifications described above preserves the convergence behaviour of the code, using roughly the same number of iterations. The algorithm requires more inversions of the Grad-Shafranov operator. Since the Grad-Shafranov solver of the code

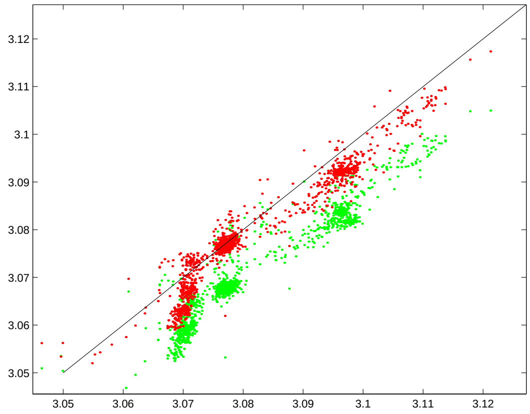

Figure 3: Position of plasma boundary close to ICRH antenna protection limiter, comparison with boundary code DPOLO. The points represent selected time slices from 51 shots, where green points stand for EFIT without ripple correction, and red those with ripple correction, giving far better agreement.

is optimised, the computational cost is only slightly increased, for a typical case from 40 msec per timeslice to 50 msec. The results are compared with the real-time boundary code DPOLO that uses coordinate transformation for the ripple correction [5]. A typical result is shown in figure (2). Both methods that use completely different algorithms agree on the position of the plasma boundary within a few millimetres. A systematic comparison of a range of shots proves good agreement of the plasma boundary, as shown in figure (3). The code therefore now serves as routine equilibrium analysis code for Tore Supra discharges. The algorithm described in the paper is applicable to any tokamak once the function δ describing the ripple field is known. It can be extended to work also for a superposition of modes. This is applicable to magnetic field ripple caused by ripple compensation with ferromagnetic inserts, as proposed for the ITER device [8] .

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