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Unstable Ion-Temperature-Gradient Modes in ITER Geometry

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Abstract

The linear stability of ion-temperature-gradient driven drift modes is investigated in three-dimensional ITER-like configuration. An advanced fluid model is adopted and an eigenvalue equation is derived by using the ballooning mode formalism. The derived eigenvalue equation is solved numerically using standard shooting technique and applying WKB type boundary conditions. The growth rates of the most unstable modes and their eigenfunctions are calculated and visualized. The results are correlated with the geometrical effects such as local magnetic shear, normal curvature, geodesic curvature and magnetic field strength.

The magnetic field configuration and eigenvalue problem

The ion-temperature-gradient (ITG) mode is considered to be the most promising candidate for explaining anomalous transport in tokamak plasmas. It is therefore of interest to study the stability of this mode in order to limit energy confinement in the future tokamak reactor ITER. In the present paper, the linear stability of the ITG modes is studied in the three dimensional ITER-like geometry. The VMEC code is used to obtain the three-dimensional equilibrium.

The magnetic field can be expressed in terms of the Boozer coordinates (s, θ, ζ) as $\mathbf{B} = \boldsymbol{\nabla}\alpha \times \boldsymbol{\nabla}\psi = \dot{\psi}\boldsymbol{\nabla}\alpha \times \boldsymbol{\nabla}s$, with $\dot{\psi} \equiv d\psi/ds = B_o\bar{a}^2/2q$ where $\alpha = \zeta - q\theta$ is the field line label, ζ is the generalized toroidal angle, θ is the generalized poloidal angle, q is the safety factor, $s = 2\pi\psi/\psi_p$ is the normalized poloidal flux and serves as the radial coordinate. Here $2\pi\psi$ is the poloidal magnetic flux bounded by the magnetic axis and ψ = constant surface, $\psi_p = \pi B_o\bar{a}^2/q$ is the total poloidal magnetic flux, where B_o is the magnetic field at the magnetic axis, $\bar{a}$ is the average minor radius. The field line curvature $\boldsymbol{\kappa}(\equiv \mathbf{e}_{||} \cdot \boldsymbol{\nabla}\mathbf{e}_{||})$ can be expressed as:

$$\boldsymbol{\kappa} = \frac{\kappa_n}{\sqrt{g^{ss}}}\boldsymbol{\nabla} s + \frac{\kappa_g}{\sqrt{g^{ss}}}\left(\frac{\dot{\psi} g^{ss}}{B}\right)(\boldsymbol{\nabla}\alpha - \wedge\boldsymbol{\nabla} s), \quad (1)$$

where κ_n is the normal component and κ_g is the geodesic component of the field line curvature and are defined as, $\kappa_n = \boldsymbol{\kappa}\cdot\widehat{s}$, $\kappa_g = \boldsymbol{\kappa}\cdot\widehat{s}\times\mathbf{e}_{||}$, where $\mathbf{e}_{||} = \mathbf{B}/B$ is a unit vector along the magnetic field, $\widehat{s} \equiv \boldsymbol{\nabla} s/|\boldsymbol{\nabla} s|$, $\wedge = g^{s\alpha}/g^{ss}$ is the local magnetic shear integrated along the field line(ILMS) and $g^{ij} = \boldsymbol{\nabla} i\cdot\boldsymbol{\nabla} j$ is the dot product of metric coefficients of the flux coordinates. The local magnetic shear[1] is defined as $S = \left(\widehat{s}\times\mathbf{e}_{||}\right)\cdot\boldsymbol{\nabla}\times\left(\widehat{s}\times\ \mathbf{e}_{||}\right) = \left(\mathbf{e}_{||}\cdot\boldsymbol{\nabla}\right)\wedge$.

In this study, we have adopted an advanced fluid model, which is derived in the short wavelength region taking Boltzmann distributed electrons and using ion continuity, energy and parallel momentum equations. The drift wave equation is reduced to an eigenvalue problem by applying ballooning mode formalism and WKB type assumptions in the Boozer coordinates. The resulting eigenvalue equation can be written[2,3] as

$$\frac{d^2\Psi}{d\zeta^2} - \left(\frac{2\chi JB}{\overline{a}\epsilon_n q\bar{R}\dot{\psi}}\right)^2\left[(H^{-1} - \frac{\overline{a}\epsilon_n\Omega_d}{2})\Omega - \left\{H^{-1} + \left(\frac{\chi B_0}{B}\right)^2\left(\hat{\mathbf{k}}_\perp\cdot\hat{\mathbf{k}}_\perp\right)\right\}\Omega^2\right]\Psi = 0, \quad (2)$$

where $\Omega = \omega/\omega_{*e}$, and $\Psi = H\Phi$,

$$H = 1 + \tau^{-1} + \frac{\tau^{-1}\left[(2/3)\,\Omega + (\eta_i - (2/3))\right]}{\Omega + (5/6\tau)\,\overline{a}\epsilon_n\Omega_d}. \quad (3)$$

Here, $\epsilon_n = L_n/\bar{R}$, $L_n^{-1} = -(d\ln n_o/ds)\,\widehat{s}\cdot\boldsymbol{\nabla} s|_{\zeta=0}$, $\chi = (\epsilon\bar{a})^{-1} q\rho_{s0}\partial S/\partial\alpha$, and χ controls the magnitude of the wave vector $\hat{\mathbf{k}}_\perp$ defined as

$$\hat{\mathbf{k}}_\perp = \frac{\bar{a}}{q}\left[\boldsymbol{\nabla}\zeta - q\boldsymbol{\nabla}\theta - \left(\frac{\zeta-\zeta_o}{q} - \theta_k\right)\dot{q}\boldsymbol{\nabla} s\right], \quad (4)$$

$$b = \chi^2\,\hat{\mathbf{k}}_\perp\cdot\hat{\mathbf{k}}_\perp\Big|_{\zeta=\zeta_0}, \quad (5)$$

where $\chi_g \equiv \bar{a}^2 g^{ss}/2q$.

Results

In tokamaks as one moves in the poloidal direction the equilibrium quantities like magnetic field, normal and geodesic curvature and local magnetic shear change periodically. Therefore, the spectrum can also vary with the position of the matching point on the flux surface. This variation is calculated in Fig. 1 by moving the matching point θ_0, and fixing $\zeta_0 = 0$. Since the growth rate has a maximum at $\theta_k = 0$ (in our notation), θ_k is here redefined in such a way that it sets the radial mode number to zero at each matching point and the most unstable modes on the magnetic surface are picked up.

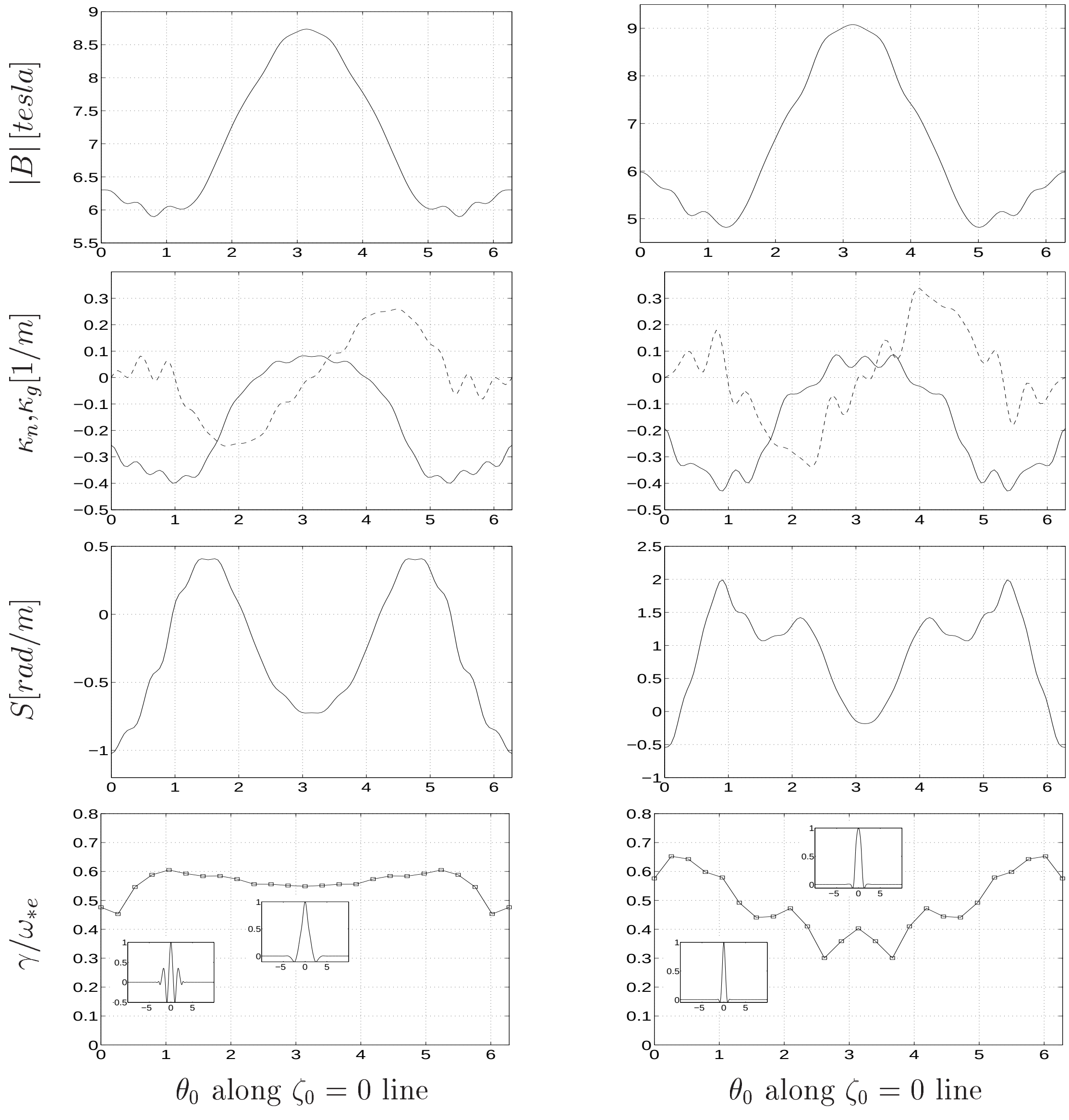

Fig. 1: *Variation in the magnetic field strength $|B|$, the normal curvature κ_n (dotted curve), the geodesic curvature κ_g (solid curve), the local magnetic shear S, and the normalized growth rate γ/ω_{*e} as a function of θ_0 for $s = 0.4$ (left) and $s = 0.7$ (right). The other parameters are: $b = 0.1$, $\tau = 1$, $\epsilon_n = 0.1$, $\eta_i = 3.0$ and $\theta_k = 0$.*

Two magnetic flux surfaces, s=0.4 and s=0.7 are selected for the study. The results in Fig. 1 demonstrates the most unstable modes on the two magnetic surfaces. Some eigenfunctions are also shown as insets. The parameter values used are $\varepsilon_n = 0.1$, $b =$

0.1, $\tau = T_{i0}/T_{e0} = 1$, and $\eta_i = 3$.. The positive local magnetic shear is found to be destabilizing at the magnetic surface, s=0, where global magnetic shear (surface averaged local shear) is reverse. The effect of the magnetic curvature on mode stability at this surface is week, even in some regions with large bad (negative) normal curvature, low growth rate is observed. However, the eigenfunctions are found to be extended at the reverse shear magnetic surface and are therefore more influenced by the average effect rather than the local behaviour of the geometrical quantities.

For positive global magnetic shear surface, s=0.7, the eigenfunctions are more localized and are thus strongly affected by the local geometrical quantities. At this surface, the growth rate has minimum values near the inboard ($\theta_0 = \pi$, $\zeta_0 = 0$) where the normal curvature is favourable (positive) and maximum values in the region where the normal curvature is unfavourable. In the later region, the large value of the positive LMS tneds to stabilize the mode. Further, the modes are more localized in the region of unfavourable curvature and week magnetic field, while the envelope of the eigenfuction becomes broader as it enters the region of favourable curvature and strong magnetic field.

The main concern of the present work is to make a first attempt to investigate the ITG modes and their correlation with geometrical effects in ITER-like geometry. The results have implications on optimization of ITER design specifically with respect to minimization of drift wave turbulence, while this is a considerable challenge which will require considerable work in the future.

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