Influence of flux surface shape on DALF and ITG edge turbulence

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Introduction

The influence of shaping of magnetic flux surfaces in tokamaks on gyrofluid edge turbulence is studied. Magnetic field structure in tokamaks is mainly due to elongation, triangularity, pressure shift and the presence of a divertor X-point. A series of tokamak configurations with varying elongation and triangularity, and an actual ASDEX Upgrade divertor configuration are obtained with the equilibrium code HELENA and implemented into the gyrofluid turbulence code GEM. Specific effects of flux surface shaping on drift-Alfvén (DALF) and ion temperature gradient (ITG) turbulence in the tokamak edge plasma are analysed and compared.

DALF and ITG edge turbulence models

The dynamical character of cross-field ExB drift wave turbulence in the edge region of a tokamak plasma is governed by electromagnetic and dissipative effects in the parallel response. The most basic drift-Alfvén (DALF) model to capture the drift wave dynamics includes nonlinear evolution equations of three fluctuating fields: the electrostatic potential $\tilde{\phi}$, electromagnetic potential $\tilde{A}_\parallel$, and density $\tilde{n}$. The tokamak edge usually features a more or less pronounced density pedestal, and the dominant contribution to the free energy drive to the turbulence by the inhomogeneous pressure background is thus due to the density gradient. On the other hand, a steep enough ion temperature gradient (ITG) in the edge does not only change the turbulent transport quantitatively, but adds new interchange physics into the dynamics. In addition, more field quantities have to be treated: parallel and perpendicular temperatures $T_\parallel$ and $T_\perp$ and the associated parallel heat fluxes, for a total of six moment variables for each species. Finite Larmor radius effects introduced by warm ions require a gyrofluid description of the turbulence equations. Both the resistive DALF and the ITG models can be covered by using the six-moment electromagnetic gyrofluid model GEM [1], but we will refer for the DALF model to its more economical two-moment version [2].

The edge plasma will be characterised by parameters $C = 5$, $\tilde{\beta} = 1$, $\tilde{\mu} = 5$ and $\tilde{\epsilon} = 18350$ (see [1] for definitions). The perpendicular scale length for the DALF model is set to $L_\perp = L_n$, and for the ITG model to $L_\perp = L_{Ti} = 0.5L_n = 0.5L_{Te}$ so that $\eta_i = L_n/L_{Ti} = 2$. 
Flux surface shaping of tokamaks

The plasma shape of a tokamak enters into confinement time scalings, transport and equilibrium modelling through parameters specifying a vertical elongation $\kappa \geq 1$ and an outboard side triangularity $\delta \geq 0$ that describe the deviation from a simple circular and axisymmetric torus. Tokamak equilibria are here computed with the code HELENA by solving the Grad-Shafranov / Lüst-Schlüter equation [3]. A set of nested flux surfaces in straight field line Hamada coordinates $(V, \theta, \zeta)$ is obtained by specification of given radial profiles of pressure and rotational transform, and of the shape of the bounding last closed flux surface. These Hamada coordinates are then transformed into a field aligned system and re-scaled into local flux tube coordinates $(x, y, z)$ [4]. The tokamak geometry enters the GEM equations via the curvature operator $\mathcal{K} = \mathcal{K}^x \nabla_x + \mathcal{K}^y \nabla_y$, the perpendicular Laplacian $\nabla_\bot^2 = g^{xx} \partial_{xx} + 2 g^{xy} \partial_{xy} + g^{yy} \partial_{yy}$, and the parallel derivative $\nabla_\parallel = b_z \partial_z$. Some metric coefficients $g^{ij}$ that were obtained for elongation $\kappa = 1$ and $\kappa = 2$ with triangularity $\delta = 0$ and $\delta = 0.4$ are shown in Fig. 1. Increasing elongation $\kappa$ specifically rises the local magnetic shear $S = \nabla_\parallel (g^{yy} / g^{xx})$ and reduces $\mathcal{K}^x$ both in the upper and lower regions of the torus that correspond to flux tube coordinates $z = \pm \pi/2$. Local and global magnetic shear have a damping influence on tokamak edge turbulence [6], whereas geodesic curvature acting through $\mathcal{K}^y$ upon $k_y = 0$ modes maintains the coupling for a loss channel from zonal flow energy eventually to turbulent vortices [7, 8]. Both mechanisms reduce the local turbulent $E \times B$ transport. Normal curvature $\mathcal{K}^x$ on the other hand strengthens primarily the interchange forcing of the turbulence ($k_y \neq 0$).

![Fig. 1](image.png)

**Figure 1:** Left: Geometry coefficients entering into the edge turbulence equations. Metric elements $g^{xx}$, $g^{xy}$ (unshifted), normal curvature $\mathcal{K}^y$, geodesic curvature $\mathcal{K}^x$, magnetic field strength $B$ and local magnetic shear $S$ in a tokamak with elongation $\kappa = 1$ and triangularity $\delta = 0$ (blue); with $\kappa = 2$, $\delta = 0.4$ (orange); and an ASDEX Upgrade equilibrium which is additionally shaped by a divertor (black). Right: poloidal cross section of an ASDEX Upgrade configuration shown with a section of the turbulence computation domain.
Computational results

The flux-surface averaged turbulent transport can be characterised by particle transport \( F_n = \langle \tilde{n} \tilde{v}_x \rangle \) and by heat transport \( Q_i = Q_i^{\text{convective}} + Q_i^{\text{conductive}} \) with \( Q_i^{\text{convective}} = \frac{3}{2} \tau_i \langle \tilde{n}_i \tilde{u}_x + \tilde{T}_i \tilde{w}_x \rangle \) and \( Q_i^{\text{conductive}} = \tau_i \langle \left( \frac{1}{2} \tilde{T}_i || + \tilde{T}_i \perp \right) \tilde{u}_x + (\tilde{n}_i + 2 \tilde{T}_i \perp) \tilde{w}_x \rangle \), where \( \tilde{u}_x \) is the gyro-averaged ExB velocity and \( \tilde{w}_x \) is its FLR correction. Normalisation is to standard gyro-Bohm units with the gradient scale length set to unity. We compare particle transport fluxes for the DALF and ITG regimes with ion thermal transport in the ITG case for a series of tokamak equilibria for elongation \( \kappa = 1.00, 1.25, 1.50, 1.75, 2.00 \) and triangularity \( \delta = 0, 0.1, 0.2, 0.3 \) and 0.4. We find in Fig. 2 that all transport fluxes are strongly reduced by elongation. Scaling is by \( \kappa^{-2.1} \) for the DALF model and by \( \kappa^{-2.6} \) for the ITG model: in the latter, also the interchange drive of the ITG instability is reduced by elongation. The main contribution to transport reduction in both regimes is local magnetic shear damping. Shear flow enhancement by the reduced geodesic transfer mechanisms is found to be relatively weak. For an elliptical cross section (\( \kappa = 2 \)) both heat and particle transport are slightly increasing with higher triangularity, while for small \( \kappa \) no influence from triangularity is observed within the fluctuation error bars.

Frequency spectra of the fluctuating electrostatic potential \( \tilde{\phi} \) are shown in Fig. 3 for the DALF and ITG models for \( \kappa = 1 \) and 2. As the drift wave frequency is reduced by \( 1/\kappa \), the spectra are accordingly shifted to lower frequencies for higher elongation. The same shift applies to the geodesic acoustic mode (GAM) frequency. With circular flux surfaces, a GAM peak can be identified in both spectra but is more prominent in the ITG model. The GAM frequency is proportional to \( \sqrt{1 + \tau_i} \) and thus higher for the ITG case with \( \tau_i = 1 \). The zero frequency zonal flow mode is stronger in the DALF model than for ITG turbulence. Concurrent with the more pronounced GAM peak in the ITG case for \( \kappa = 1 \), a lower zonal flow amplitude is there observed in the spectrum than for the \( \kappa = 2 \) case.

Figure 2: Turbulent transport as a function of elongation \( \kappa \) for \( \delta = 0 \) (left), and as a function of triangularity \( \delta \) (right). For both DALF and ITG models, the particle and heat transport scales roughly inversely squared with \( \kappa \) but is nearly independent of triangularity. Fluctuation standard deviation bars are additionally shown on the right figure.
Conclusions

We have presented results for computations of gyrofluid edge turbulence in realistic tokamak geometry for DALF and ITG regimes. It is found that turbulent transport fluxes $F_n$ and $Q_i$ are reduced by increasing flux surface elongation $\kappa$. The influence of triangularity $\delta$ is much weaker and generally also depends on elongation and on a pressure shift. The geodesic acoustic modes (GAM) and the overall turbulence frequency spectra depend on flux surface geometry. An enhancement of zonal flow amplitude in the spectra by elongation and X-point shaping is found to be weak in the DALF case and more pronounced in the ITG case. The reduction of transport observed in our simulations in shaped tokamak geometry is mainly a result of local magnetic shear.

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References