

A practical nonlocal model for electron transport in magnetized laser-plasmas

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Abstract

We present a model of nonlocal transport for multidimensional radiation magneto-hydrodynamic codes. In laser produced plasmas, it is now believed that the heat transport can be strongly modified by the nonlocal nature of the electron conduction. Nevertheless other mechanisms as self generated magnetic fields may affect heat transport too. The model described in this work aims at extending the formula of G. Schurtz, Ph. Nicolai and M. Busquet [1] to magnetized plasmas. A system of nonlocal equations is derived from kinetic equations with self-consistent E and B fields. This equations are analyzed and applied to a physical problem in order to demonstrate the main features of the model.

Introduction

Within the framework of Inertial Confinement Fusion (ICF), the transport of energy by electron conduction is an old problem which remains this day imperfectly solved. The interpretations of experiments require the use of large multidimensional hydro codes which usually deal poorly with electron conduction. The model implemented in the majority of these codes is based on the Spitzer-Härm's theory which tends to over-estimates heat fluxes and has to be reduced to reproduce experimental data. Since the eighties it has been reckoned, thanks to the Fokker-Planck codes [2] and to the experiments [3], that nonlocal effects as well as magnetic fields could significantly modify the heat conduction. However a Fokker-Planck (F-P) code requires a high CPU cost and its coupling to other physical models, as equations of state or radiation transport, is difficult. The aim of this study is to propose a simple and fast model taking into account both processes and which could be introduced in a multidimensional code.

The multidimensional nonlocal model

The reference equation is the Vlasov-Fokker-Planck kinetic equation which determines the evolution of the Electron Distribution Function (EDF) :

$$\partial_t f_e + \mathbf{v} \cdot \nabla f_e - e \left(\frac{\mathbf{E}}{m_e} + \mathbf{v} \times \frac{\mathbf{B}}{m_e c} \right) \cdot \nabla_{\mathbf{v}} f_e = C_{ee} + C_{ei} \quad (1)$$

where e and m_e are the electron charge and mass, c is the speed of the light, C_{ee} and C_{ei} are electron-electron and electron-ion collision operators. \mathbf{E} and \mathbf{B} are the self consistent electric and magnetic fields

In order to simplify the problem, we adopt, as in the F-P codes [4], a first-order Cartesian tensor expansion for the EDF : $f_e(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, v, t) + \frac{\mathbf{v}}{v} \cdot \mathbf{f}_1(\mathbf{r}, v, t)$. Substituting this expansion into the kinetic equation and making use of its orthogonality properties, yields:

$$\frac{v}{3} \nabla \cdot \mathbf{f}_1 - \frac{e\mathbf{E}}{3m_e v^2} \cdot \nabla_v (v^2 \mathbf{f}_1) = C^0(f_0) \quad (2)$$

$$v \nabla f_0 - \frac{e\mathbf{E}}{m_e} \nabla_v (f_0) - \frac{e\mathbf{B}}{m_e c} \times \mathbf{f}_1 = -v_{ei}^* \mathbf{f}_1 \quad (3)$$

where $v_{ei} = [4\pi n_e e^4 Z \Lambda / m_e] / v^3$ is the electron-ion collision frequency which could include electron-electron diffusion to correct the Lorentz gas approximation. Both equations assume a steady state for electrons, *idem est*, that the distribution function adjust itself to the fluid conditions.

The fifth velocity momentum of the anisotropic part of the EDF, \mathbf{f}_1 , gives the heat flux. If we postulate that f_0 is a maxwellian, we recover the classical Braginskii flux. Now, if f_0 is not a maxwellian but remains close to it as in ICF plasmas, we may write $f_0 = f_0^m + \Delta f_0$ and $\mathbf{f}_1 = \mathbf{f}_1^m + \Delta \mathbf{f}_1$. Note that the deformation of the EDF should not be too large, so that the fluid treatment remains valid. Only high order velocity momenta as the heat flux, may be strongly modified by kinetic effects. Density, mean velocity or temperature have to remain correctly evaluated by hydro-codes. Inserting the EDF deformations into kinetic equations, neglecting second order terms, leads to the following anisotropic part :

$$\mathbf{f}_1 = \mathbf{f}_1^m - \frac{\lambda_{ei}^*}{1 + u^6 \Omega^2} \left[+ \nabla \Delta f_0 + u^3 \Omega \times \nabla \Delta f_0 + \frac{e \Delta \mathbf{E}}{T_e} f_0^m + \frac{u^3 \Omega \times e \Delta \mathbf{E}}{T_e} f_0^m - \frac{e \mathbf{E}_B}{m_e v_T^2 u} \nabla_u \Delta f_0 - \frac{u^3 \Omega \times e \mathbf{E}_B}{m_e v_T^2 u} \nabla_u \Delta f_0 \right] \quad (4)$$

where \mathbf{f}_1^m is the Braginskii's integrand, $\lambda_{ei} = v/v_{ei}$ the electron mean free path, \mathbf{E}_B the Braginskii's electric field, $u = v/v_T$, $v_T = \frac{\sqrt{2\pi n_e Z e^4 \Lambda}}{m_e^{1/2} T_e^{3/2}}$ is the mean collision frequency and $\Omega = \frac{e\mathbf{B}}{m_e c v_T}$, is the Hall parameter which gives an indication of the magnetic field influence inside the plasma. Eq(4) is combined with a diffusive equation, obtained in the same way, giving the evolution of Δf_0 . The nonlocal and magnetic fields effects on the electron transport are pointed out in the \mathbf{f}_1 expression. The leading effect comes from the term $\nabla \Delta f_0$. This term dominates when the range of heat carrying electrons exceeds the EDF gradient length, which is the usual criterion for nonlocality. This term induces, among in other things, preheat ahead of temperature gradient. A Δf_0 causes a $\Delta \mathbf{E}$ which itself acts on f_0 . This effect appears in the third term. The fifth

one gives the electric field effect on Δf_0 . Magnetic fields produce the three other terms which could be analyzed in the same way. The difference comes from the rotation ($\Omega \times$) due to the Lorentz's force. There are similar, in the nonlocal case, to the Righi-Leduc term. Last, \mathbf{B} fields directly act on the distance travelled by electrons, reducing the mean free path by $1 + u^6 \Omega^2$. It cancels the nonlocal effects when \mathbf{B} becomes large. The formulation presents all the expected properties. As \mathbf{B} tends to zero, Eq.(4) degenerates to the nonlocal formula already shown in the reference [1]. As the \mathbf{B} influence increases, the electron mean free path reduces, crossed components appears and the source term \mathbf{f}_1 is not the Spitzer-Härm integrand any more but becomes the Braginskii one. If \mathbf{B} continues to grow, all the nonlocal effects disappear and one finds the local characteristics of the fluxes. Let us note to finish that the formulation could be simplified. The nonlocal effects come mainly from the spatial gradients. In the absence of \mathbf{B} field, it has been shown that the terms related to the electric fields reduce the nonlocal effects and can be taken into account through a modified source term and electron mean free path limitation [1]. This last limitation is related to the stopping distance induced by the electric field. With magnetic fields, the same way could be followed, using the Braginskii's electric fields which contain the cross component due to the Lorentz's force.

Application

Let us consider a temperature gradient (Fig. 1-a), along the x axis, sharp enough to induce nonlocal effects ($k\lambda_e \simeq 0.8$, where k is the logarithmic temperature gradient). A magnetic field, along the z axis, such as $\Omega \simeq 0.1$, is present inside the x-y plane. The Fig. 1-d shows the fluxes colinear to the gradient while the Fig. 1-c presents those which are perpendicular to the gradient and to \mathbf{B} , *idem est*, along the y axis. Without \mathbf{B} , we get the well-known nonlocal results : a flux smaller than the local flux and a preheat foot ahead of gradient. With \mathbf{B} , various effects appear. Both local and nonlocal fluxes are reduced, noting that the reduction is less important for the nonlocal case due to a decrease of the electron mean free path. The delocalization ahead of gradient decreases for the same reason and because the direction of part of the electrons changes. However the flux remains smaller than the local flux and nonlocal. Consequently, in the perpendicular direction to the gradient, we get, after rotation, a smaller and nonlocal flux relative to the Righi-Leduc component : the electrons ahead of gradient also turn. By construction, our model uses the departure to the Maxwellian to build the heat flows. We present on Fig. 1-b , an example of deformation of the isotropic part of the EDF with or without the magnetic field. The function is calculated at the point marked F on the Fig. 1-a. The deformation of f_0 due to the electrons coming from the hot part of the plasma is strongly reduced by the presence of \mathbf{B} , showing the decrease of the mean free paths and so of the nonlocal effects.

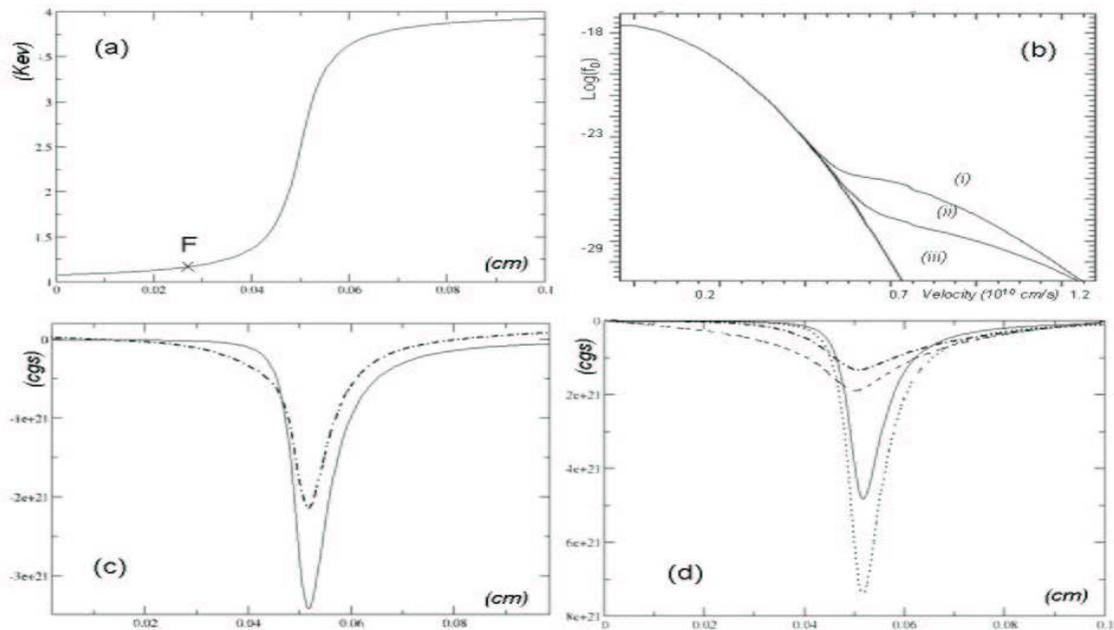


Figure 1: (a) temperature gradient; (b) isotropic part of the EDF : (iii) Maxwellian, (ii) nonlocal with \mathbf{B} and (i) nonlocal w/o \mathbf{B} ; (c) fluxes perpendicular to the gradient ; (d) fluxes colinear to the gradient ; solid lines refer to Braginskii, dotted lines to Spitzer-Harm, dashed line to nonlocal and dash-dotted lines to nonlocal with \mathbf{B}

Conclusion

We developed a multidimensional nonlocal model taking account of the magnetic field effects. Without \mathbf{B} , the model was already introduced into various large hydrodynamics codes dedicated to the ICF as FCI2 at CEA, HYDRA at Livermore, LILAC and ORCHID at Rochester. The model presented in this paper, should improve the prediction and the interpretation of the laser experiments, in particular when the nonlocal model alone does not seem sufficient to reproduce the experimental results. Note that independently of their calculations, the effect of magnetic fields slows down only little the simulation relative to a calculation without \mathbf{B} . The model is developed in the new 2D-Hydrodynamic code, CHIC [5], under development at the CELIA.

References

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