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Eigen Modes of a Dielectric Loaded Coaxial Plasma Waveguide

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Abstract: The theoretical studies of High frequency eigen modes of a dielectric loaded coaxial wave guide in the presence of an annular plasma column is presented. The dispersion equation is derived through the application of the appropriate boundary conditions, which result in an eighth order determinant. In the presence of a dielectric layer on the conducting surfaces, the azimuthally symmetric modes have been identified as HE, EH, cyclotron, and space charge modes for coaxial wave guide.

Introduction: There has been growing interest in the plasma filled cylindrical waveguides in recent years [1-4]. A considerable number of microwave sources employ cylindrical waveguides, containing axis encircling electron beams. In these devices the annular plasma column interacts with the modes of empty waveguides, in which case they have been referred to as large-orbit gyrotrons, or with the azimuthally periodic wiggler magnetic field where they are called circular geometry free electron lasers. In either case, frequencies of the generated electromagnetic-electrostatic waves have been shown to have a strong dependence on the radii of coaxial waveguide and on relative positions of the inner and outer radii of the beam [5]. Analysis of the plasma waveguide requires knowledge of its eigen modes. High frequency eigen modes of a magnetized plasma waveguide are characterized in four families, EH and HE waveguide modes, cyclotron modes, and space-charge modes.

The governing equations: Consider an annular plasma column with inner radius R_i and outer radius R_o located inside a dielectric loaded coaxial waveguide is immersed in a uniform static axial magnetic field, $B_\circ \hat{a}_Z$. The coaxial waveguide consists of an inner metallic cylinder with outer radius a and an outer metallic cylinder with inner radius b. The cross section of the dielectric loaded coaxial waveguide with an annular plasma column is shown in Figure 1.

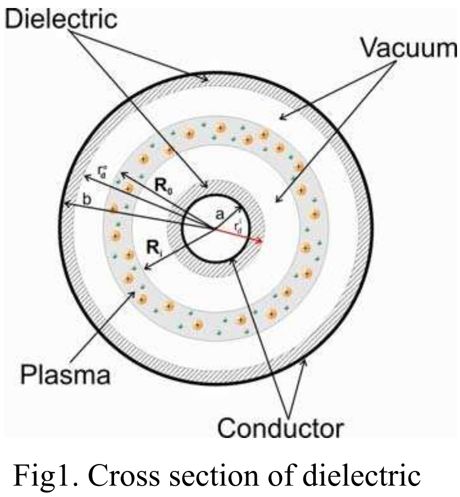

Fig1. Cross section of dielectric loaded plasma waveguide

The inner surface of the outer conductor and the outer surface of the inner conductor are coated with a thin layer of a dielectric material with a dielectric constant of ε. Thus the plasma column only sees a smooth-walled dielectric coaxial waveguide of inner radius r_d^i and outer radius r_d^o. The cold fluid continuity and momentum transfer equations describe the dynamics of the plasma particles in the waveguide. The behavior of electromagnetic fields is governed by Maxwell's equations. Using perturbation technique and upon substitution for $n, \vec{\mathrm{v}}, \vec{E}$, and $\vec{B}$ in aforementioned equations, after some algebra, the linear wave equation for $\vec{E}_{1a}$ within the plasma $\left(R_i < r < R_\mathrm{o}\right)$ can be derived as

$$\begin{pmatrix} \dfrac{\partial^2}{\partial t^2}\left(\dfrac{\partial^2}{\partial t^2}+\omega_p^2\right)\left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}-\nabla^2+\dfrac{\omega_p^2}{c^2}\right)^2+\omega_c^2\left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}-\nabla^2\right)\times \\ \left[\left(\dfrac{\partial^2}{\partial t^2}+\omega_p^2\right)\left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}-\dfrac{\partial^2}{\partial z^2}\right)-\dfrac{\partial^2}{\partial t^2}\nabla_t^2\right] \end{pmatrix} E_{1a}=0 \qquad (1)$$

where ω_p is the plasma frequency, $\omega_p^2=\dfrac{n_0 e^2}{m_0\varepsilon_0}$, and ω_c is the cyclotron frequency, $\omega_c=\dfrac{eB_\mathrm{o}}{m_0}$. Considering a solution of the type

$$E_{1a}(r,\theta,z)=E_{1a}(r)\exp(-i(\omega t-kz-n\theta))$$

leads to the following wave equation

$$\left[\omega^2\omega_1^2\nabla^4-\left(2\omega^2\omega_1^2k_1^2+\omega_c^2\omega_p^2k_2^2\right)\nabla^2+\omega^2\omega_1^2k_1^4+\omega_c^2\omega_p^2\left(k_1^2k_2^2-k^2\frac{\omega_p^2}{c^2}\right)\right]E_{1a}=0 \qquad (2)$$

where $\omega_1^2=\omega^2-\omega_c^2-\omega_p^2$, $\quad k_1^2=k^2-\dfrac{\omega^2-\omega_p^2}{c^2}$, $\quad k_2^2=k^2+\dfrac{\omega^2}{c^2}$

The solutions to the above equation can be expressed as

$$E_{1a}=\left(E_1J_n\left(\overline{K}r\right)+E_2N_n\left(\overline{K}r\right)\right)\exp\left(i(kz+n\theta+\omega t)\right) \qquad (3)$$

where J_n and N_n are the Bessel functions of the first and second kind of integer order n, respectively. The radial wave number $\overline{K}$ can be found from the following quadratic equation,

$$\omega^2\omega_1^2\overline{K}^4+\left(2\omega^2\omega_1^2k_1^2+\omega_c^2\omega_p^2k_2^2\right)\overline{K}^2+\omega^2\omega_1^2k_1^4+\omega_c^2\omega_p^2\left(k_1^2k_2^2-k^2\frac{\omega_p^2}{c^2}\right)=0 \qquad (4)$$

The dispersion equations: The dispersion relation which gives the dependence of ω upon k can be obtained by using equation (4) along with the application of the boundary conditions.

The tangential component of the electric field and the normal component of magnetic field are continuous at the dielectric-vacuum interfaces. However, due to the presence of surface charge densities and surface current densities on the waveguide surface, the normal component of electric field and the tangential component of magnetic field are not continuous at the conductor-dielectric interfaces. If we assume that the plasma column is stationary and its drift velocity is not appreciable, we can then neglect surface current density on the inner and the outer surfaces of the plasma. Thus, all tangential components of the fields must be continuous at the vacuum-plasma interfaces. Application of the boundary conditions on the interfaces of the conductor-dielectric, dielectric-vacuum, and vacuum-plasma leads to a set of eight linear homogenous equations for the components of the electromagnetic field which can be written in a matrix form.

In order to have a nontrivial solution, the determinant of the 8×8 matrix which will be denoted by "M" must be zero

$$\det M = 0 \qquad (5)$$

Simultaneous solution of the above equation and equation (4) provides the dispersion characteristics of the normal modes of the dielectric loaded plasma waveguide.

The numerical results and discussion: The eigen-modes of a dielectric, filled or empty, coaxial metallic waveguide are generally identified as either purely transverse magnetic (TM) or purely transverse electric (TE). However, the wave fields of a coaxial waveguide containing stationary plasma in the presence of a static, axial magnetic field have both longitudinal electric and longitudinal magnetic components. When the longitudinal electric field component is at least one order of magnitude larger than the longitudinal magnetic component, the waveguide modes are considered as (EH) modes. The (EH) or perturbed (TM) modes are the (TM) modes of an empty waveguide modified by the presence of the magnetized plasma. Similarly, when the longitudinal magnetic component is at least one order of magnitude larger than the longitudinal electric component, the waveguide modes are said to be (HE) or perturbed (TE) modes. In a conventional cylindrical plasma waveguide two more families of modes are identified. The modes of the lower-frequency family are called space-charge modes and those of the higher-frequency family are cyclotron modes.

The eigen-modes of the coaxial, dielectric loaded, plasma waveguide are EH waveguide modes, HE waveguide modes, cyclotron modes and space-charge modes. Each family of waves has an infinite number of modes for each integral value of the azimuthally variation number $l\ (=0,\pm1,\pm2,\dots)$. In order to investigate the characteristics of all four families of eigen-modes, some numerical results will be presented for lowest order $l = 0$, azimuthally symmetric mode of each family.

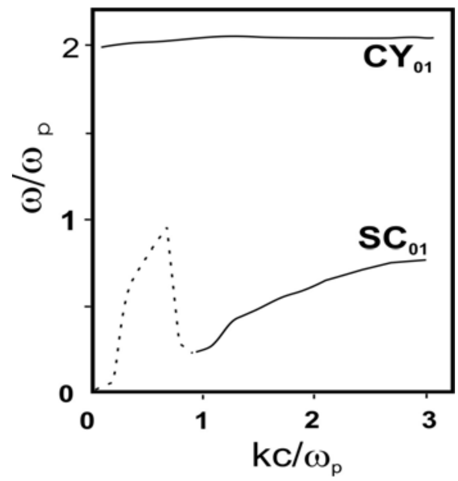

Fig2. Dispersion curves for space-charge and cyclotron modes

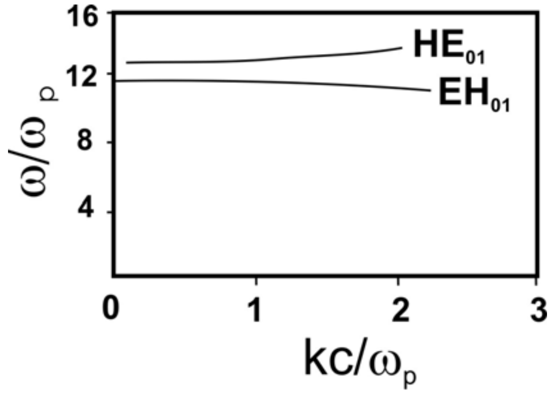

Fig3. Dispersion curves for waveguide modes

Equations (4) and (5) are employed to compute dispersion curves for the azimuthally symmetric eigen-modes of a dielectric loaded coaxial waveguide. The dispersion relations are considered for the case when the plasma column is in an annular form. Figures 2 and 3 show the plots of normalized frequency ω/ω_p as a function of normalized wave number k_c/ω_p for the waveguide and the plasma modes, respectively. The waveguide outer radius is taken to be $b = 1.5$ cm and it is normalized by the factor ω_p/c where ω_p has the value of 1.85×10^{10} rad/sec. The dielectric liners are chosen to be Mylar with the permittivity of $\varepsilon = 3.5$. All the other radii are normalized to the outer waveguide radius. Figure 2 represents the dispersion curves for EH_{01} and HE_{01} waveguide modes for an annular plasma with $R_i/b=0.9$ and $R_o/b=0.55$. In Figure 3, the dispersion curves of the space-charge and cyclotron modes are shown for a normalized cyclotron frequency of $\omega_c/\omega_p=1.86$. Comparison of our results with the case where the dielectric layers are absent indicates that there is a shift in frequency towards higher values [5].

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