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Study on Toroidal Ion Temperature Gradient Modes from Reduced Braginskki Equations

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Abstract: In this paper, a dispersion relation for toroidal ion temperature gradient (ITG) modes is derived from reduced Braginskii equations, including the effects of current density. Based on the dispersion relation, the ITG modes are analyzed numerically. The basic properties, obtained by previous works, are reproduced qualitatively. In addition, the new features of ITG modes are revealed. In particular, the results indicate that the large ratio T_i / T_e is very useful for the stabilization of ITG modes.

We start with the reduced Braginskii equations of ions, which can be expressed as [1], respectively,

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i (\mathbf{v}_E + \mathbf{v}_{di} + \mathbf{v}_{pol} + \mathbf{v}_{//i}) = 0 , \qquad (1)$$

$$\frac{dp_i}{dt} + \frac{5}{3} p_i \nabla \cdot (\mathbf{v}_E + \mathbf{v}_{//i} + \mathbf{v}_{pol}) + \frac{5}{3}\frac{c}{e}(\nabla \times \frac{\vec{b}}{B}) \cdot \nabla (p_i T_i) = 0 , \qquad (2)$$

with $d / dt = \partial / \partial t + (\mathbf{v}_E + \mathbf{v}_{pol} + \mathbf{v}_{//i}) \cdot \nabla$. The ion polarization drift (finite Larmor radius effect) can be expressed as,

$$\mathbf{v}_{pol} = \frac{c}{B\omega_{ci}} \frac{d\delta\mathbf{E}}{dt} . \qquad (3)$$

Here the parallel motion of ions is governed by

$$\frac{d\mathbf{v}_{//i}}{dt} = -\frac{e}{m_i} \nabla_{//} \phi - \frac{1}{m_i n} \nabla_{//} p_i . \qquad (4)$$

For an arbitrary vector, e.g., the total fluid velocity $\mathbf{v}_j$, we use $\delta\mathbf{v}_j$ and $\delta(\nabla\cdot\mathbf{v}_j)$ to describe the linearization of $\mathbf{v}_j$ and its divergence while the equilibrium fluid velocity is denoted by $\mathbf{V}_j$. Then, we obtain, respectively, from linearizing Eqs.(1)-(4),

$$\omega\frac{\delta n_i}{n_i}+(\omega_{De}-\omega_{*e})\frac{e\delta\phi}{T_e}-\omega_{Di}\frac{\delta p_i}{P_i}+i\nabla\cdot\mathbf{v}_{pol}-k_{//}\delta v_{//i}=0, \tag{5}$$

$$\begin{aligned}&\omega\frac{\delta p_i}{P_i}+(\tau\omega_{*pi}+\frac{5}{3}\omega_{De})\frac{e\delta\phi}{T_e}-\frac{5}{3}(\omega_{Di}+d_0\omega_J)(2\frac{\delta p_i}{P_i}-\frac{\delta n_i}{n_i})\\&+i\nabla\cdot\mathbf{v}_{pol}-\frac{5}{3}k_{//}\delta v_{//i}=0\end{aligned}, \tag{6}$$

$$\nabla\cdot\mathbf{v}_{pol}=-ib_i(\omega-\omega_{*pi})\frac{e\delta\phi}{T_e}, \tag{7}$$

$$k_{//}\delta v_{//i}=\frac{(k_{//}c_s)^2}{\omega}(\tau\frac{\delta p_i}{P_i}+\frac{e\delta\phi}{T_e}), \tag{8}$$

where $b_i=(k_\perp{}^2+k_r{}^2)\rho_s{}^2$, $\tau=T_i/T_e$, $\rho_s=c(T_e m_i)^{1/2}/eB$, $c_s=(T_e/m_i)^{1/2}$, $\lambda_{Di}=(T_i/4\pi e^2 n_i)^{1/2}$, $d_0=\tau(\rho_i/\lambda_{di})^2(c_s/c)^2$, and ω_J can be expressed as[2]

$$\omega_J=(\omega_{*pi}-\omega_{*pe})(1-q\varepsilon^{-1}k_{//}/k_\perp), \tag{9}$$

with $\varepsilon=r/R<<1$ and q is safety factor. Here ω_J comes from the term $(\nabla\times(\vec{b}/B)$ in the energy equation (2). Thus, the present fluid description, obtained from the reduced Briginkii Equations, includes the effects of η_e on ITG modes because $\omega_{*pe}=(1+\eta_e)\omega_{*e}$. Using the quasi-neutrality $\delta n_i/n_i=\delta n_e/n_e=e\delta\phi/T_e$ and combining the Eqs.(5)-(9), we obtain the following dispersion relation,

$$A_0\Omega^3+A_1\Omega^2+A_2\Omega+A_3=0, \tag{10}$$

where

$A_0=(1+b_i)$,

$$A_1 = \omega_{De} - \omega_{*e} - b_i\omega_{*pi} - \frac{10}{3}(b_i + 1)\omega_1 + \frac{5}{3}b_i\omega_{Di},$$

$$A_2 = \omega_{Di}[(\tau^{-1} - \frac{5}{3}b_i)\omega_{*pi}] + \frac{5}{3}\omega_{De} + \frac{5}{3}\omega_1] - \frac{8}{3}(k_{//}c_s)^2,$$

$$A_3 = [\omega_{*pi} + \frac{5}{3}\tau\omega_{*e} + \frac{5}{3}(1+\tau)\omega_1](k_{//}c_s)^2,$$

with $\omega_1 = \omega_{Di} + d_0\omega_J$. If ω_J is taken to be zero, the present dispersion relation reduces to the previous one [3].

In the following numerical analyses on the dispersion relation (10), all frequencies are normalized to ω_{*e}, e.g., $\omega_{De}/\omega_{*e} = 2\varepsilon_n$ and $k_{//}c_s/\omega_{*e} = (k_{//}/k_\perp)(R/\rho_s)\varepsilon_n$. Here the major radius R/ρ_s is taken to a fixed value. Fig.1 shows the growth rate and real frequency as function of $k_p\rho_s = k_\perp\rho_s$ for different η_i. The unstable $k_\perp$ spectrum vanishes when $\eta_i \le 0.89$. Thus, we have a critical threshold $\eta_{ic} = 0.89$ (here the parameters $\tau = 1$, $\varepsilon_n = 0.1$, and $k_{//}\rho_s = 0.0001$). The conclusion roughly coincides with the previous η_i thresholds around a number of order unit obtained by both the kinetic and fluid methods.[4-8]

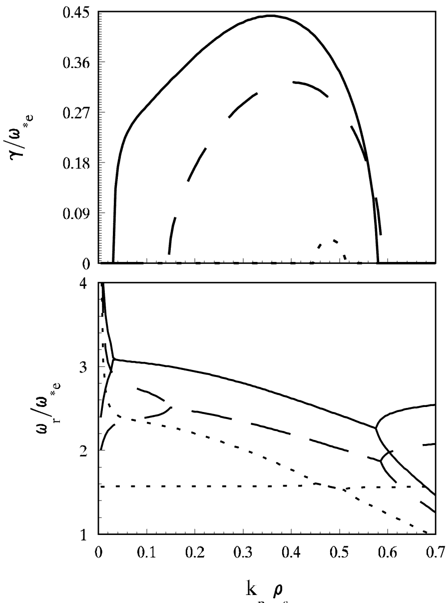

Fig.1 Growth rate and real frequency versus $k_p\rho_s = k_\perp\rho_s$ for $\eta_i = 1.5$ (solid line), 1 (dashed line), and 0.9 (dotted line). $\tau = 1$, $\varepsilon_n = 0.1$, $R/\rho_s = 900$, $\eta_e = 0$, and $k_{//}\rho_s = 0.0001$.

However, it is different from previous results that, in addition to $\eta_{ic} \equiv \eta_{ic1}$, we have another critical η_i stability thresholds η_{ic2}. That is, the mode is also stabilized when $\eta_i \geq \eta_{ic2}$. Fig.2 shows the critical stability threshold η_i as function of ε_n for different value of τ. The curves η_{ic1} and η_{ic2} form a closed and unstable parameter regime. The results indicate that the large τ parameter is very useful for the stabilization of ITG modes no matter whether the parameter ε_n is large or small.

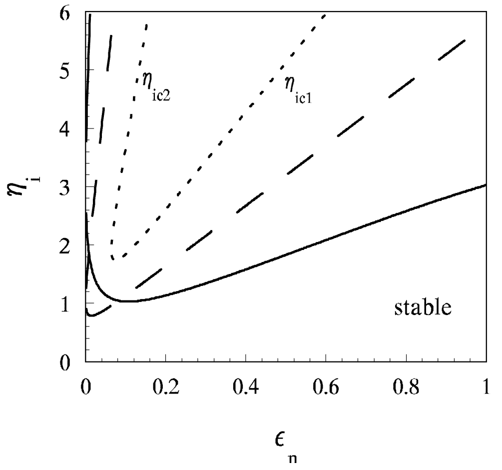

Fig.2 Critical stability threshold η_i versus ε_n for $\tau = 1$ (solid line), 2(dashed line), and 3(dotted line). $k_\perp \rho_s = 0.35$ and the other parameters are the same as Fig.1.

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