# Stellarator scaling considering uncertainties in machine variables

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The International Stellarator Confinement Database (ISCDB$^{(1)}$) is examined to study the impact of uncertainties on scaling expressions of the energy confinement time. Uncertainties for a subset of ISCDB are discussed and a probabilistic approach, i.e. Bayesian inference, is implemented. Results of the study indicate the relevance of incorporating the uncertainties properly.

$^{(1)}$: URL of ISCDB: http://www.ipp.mpg.de/ISS and http://iscdb.nifs.ac.jp/

## Introduction

Neglecting the uncertainties of data in the regression method employed to derive a scaling relation is equivalent to assigning same weight to every entry in a database. However, the diagnostics which measure these data, i.e. machine variables, may perform differently among the fusion devices, and, moreover, even within a given machine the conditions may change over the years. This should be given credit to in the analysis of the data. A second reason for the consideration of the uncertainties originates from the fact that the uncertainties of some machine variables, e.g. minor radius or absorbed heating power, are of comparable size to the uncertainty of the quantity of interest, e.g. plasma energy content. However, ordinary least squares fitting fails in this case, because it is focuses only on the deviations between response variable and model value but does not incorporate uncertainties of the input variables. An approach to overcome this problem has been performed for tokamak scaling by an errors in variables technique [1, 2]. For the stellarators a discussion of the uncertainties of the $\bar{\iota} = 1/3$ data of W7-AS was performed within a probability theoretical approach, i.e. Bayesian inference [3, 4]. With the upcoming International Stellarator Confinement Database the discussion of the uncertainties of the machine variables is now expanded to all stellarators entering the database. It is ongoing matter not only to acquire representative shots of the fusion machines but also to dig for the uncertainties of those entries to the database. This study aims at an inclusion of those uncertainties and a discussion of relevance on scaling laws.

**Confinement time scaling**

It is common practice to consider the decadic logarithm of the confinement time in order to linearize the usual power law ansatz

$$\log \tau_i = \vec{\alpha} \cdot \vec{x}_i \ , \tag{1}$$

where the index $i$ denotes a single data point. For convenience we use vector notation with the regression parameters $\vec{\alpha}^T = (\alpha_a, \alpha_R, \alpha_P, \alpha_n, \alpha_B, \alpha_{ι}, \alpha_c)$ and the logarithm of the machine variables $\vec{x}_i^T = (\log a_i, \log R_i, \log P_i, \log n_i, \log B_i, \log ι_i, 1)$. Bayesian probability theory provides a straightforward and unique way for the problem of uncertainties in input and response variables in the regression for deriving the scaling exponents. Explicit solutions are available for the linear case and lead to the following likelihood function for data point $i$

$$p(\vec{x}_i, \log \tau_i | \vec{\alpha}, \vec{\sigma}_i) = \frac{1}{(2\pi)^{\frac{7}{2}} \sigma_{\log \tau_i} \prod_{k=1}^{6} \sigma_{ki}} \exp\left\{ -\frac{1}{2} \frac{\left(\log \tau_i - \vec{\alpha} \cdot \vec{x}_i\right)^2}{\sigma_{\tau_i}^2 + \sum_{k=1}^{6} \alpha_k^2 \sigma_{ki}^2} \right\} \ , \tag{2}$$

with the uncertainties $\vec{\sigma}_i^T = (\sigma_{\log a_i}, \sigma_{\log R_i}, \sigma_{\log P_i}, \sigma_{\log n_i}, \sigma_{\log B_i}, \sigma_{\log ι_i})$ of the logarithm of the machine variables. Note that the denominator in the argument of the exponent makes the difference to ordinary least squares fitting.

**Discussion of the machine specific uncertainties**

In the following we present the uncertainties of the machine variables of W7-AS, W7-A, CHS and LHD (see Tab. 1). Since the discussion of the uncertainties of all other machines of ISCDB, i.e. ATF, HELE, HELJ and TJ-II is ongoing work, we excluded them from the analysis in this paper. The major radius $R$, toroidal magnetic field $B$ and the rotational transform $ι$ are assumed to have negligible uncertainty in comparison with the other machine variables.

**Effective minor radius:** In W7-AS the range of the uncertainties lies between 2% and 8.9% depending if the plasma was confined by limiters (for configurations with $ι \approx 1/3$) or by magnetic islands (for configurations with $ι \approx 1/2$). In the later case some of the data entries were calculated with an uncertainty of 2%, otherwise the vacuum radii were used and we assume this to be erroneous around 0.01m. In CHS the uncertainty is $\sigma_a = 0.002$m, while for LHD we have $\sigma_a = 0.024$m.

**Absorbed heating power:** To determine the uncertainty of the heating power $P$ we have to take a closer look at the constituents of $P = P_{abs,ECR} + P_{abs,NBI}$. In W7-AS nearly all of

ECR-heating is absorbed and we assume an uncertainty of 5%. The total absorbed NBI-heating power is calculated for few distinct shots distributed among a two dimensional parameter space in $n$ and $a$. Most of the NBI data entries are evaluated from the hyperplane fitted through these points. The uncertainty resulting from the calculations and the fitting procedure is estimated to be 10% of the total value $P_{abs,NBI}$. W7-A has only ECR-heating and we assume 10% for full magnetic field and 20% for half field. For CHS we got an uncertainty estimation of 20% for ECR and 10% for NBI heating. In LHD the uncertainty of the neutral beam deposition corrected for shine through is 8%. Only NBI heating data sets contribute to the database.

| | W7-AS | W7-A | CHS | LHD |
|---|---|---|---|---|
| $N_{data}$ | 712 | 13 | 196 | 162 |
| $a_{min}$ | 0.1123 | 0.0875 | 0.187 | 0.519 |
| $a_{max}$ | 0.1767 | 0.0875 | 0.2 | 0.634 |
| $\sigma_a$ | 2.0-8.9% | 11% | 1% | 2.4% |
| $P_{min}$ | 0.119 | 0.040 | 0.061 | 1.300 |
| $P_{max}$ | 3.218 | 0.130 | 0.945 | 6.516 |
| $\sigma_P$ | 4.5-10% | 10-20% | 10-20% | 8% |
| $n_{min}$ | 0.831 | 0.62 | 0.241 | 0.89 |
| $n_{max}$ | 34.3 | 1.75 | 7.9 | 5.44 |
| $\sigma_n$ | 2.0-8.9% | 11% | 0.7% | 0.21-0.77% |
| $W_{min}$ | 475.5 | 90 | 178.3 | 49650 |
| $W_{max}$ | 29390 | 775 | 3670 | 691300 |
| $\sigma_W$ | 1.4-31% | 10% | 1.3-19% | 2.2-6.8% |

Table 1: Data ranges and uncertainties of the minor radius $a$ [m], the absorbed heating power $P$[MJ], density $n[10^{19}/\mathrm{m}^3]$ and the energy content $W$[J].

**Electron density:** The line integral over the electron density, $\int n_e dl = 2an$, is obtained with very high accuracy (within 2 %) for all machines. The uncertainty in the density $n$ depends therefore mainly from error propagation of the uncertainty in $a$. For CHS the measurement is over a larger distance than the minor radius, reducing the uncertainty to 0.7%. In LHD we face an absolute uncertainty of $6 \cdot 10^{16}\mathrm{m}^{-3}$ due to the mechanical changes in the dimensions of the vessel and a relative uncertainty of 0.1% due to the microwave interferometer diagnostic itself.

**Confinement time:** For the calculation of the confinement time we use the total plasma energy as determined by diamagnetic measurements. In W7-AS the uncertainty depends on the magnitude of the ripple from the magnetic field giving $\sigma_W = 200\mathrm{J}\cdot B_t/2.553T$. For some shots a compensating magnetic loop has not been considered and we have to add a relative uncertainty of 10%. For W7-A we assume 10%, while CHS again suffers from the ripple in the magnetic field $\sigma_W = B\cdot$ 40J/T. For LHD the diamagnetic energy is calculated from the sum of toroidal, helical and paramagnetic/diamagnetic fluxes. The total uncertainty results from error propagation of the single uncertainties of those fluxes. Eventually, we get the uncertainty in the confinement time $\tau = W/P$ from error propagation. Note, that all above discussed uncertainties have to be transformed to logarithmic scale.

| | $\alpha_a$ | $\alpha_R$ | $\alpha_P$ | $\alpha_n$ | $\alpha_B$ | $\alpha_{\tau}$ | $\alpha_c$ |
|---|---|---|---|---|---|---|---|
| 1 | 2.21±0.05 | 0.65±0.04 | -0.59±0.02 | 0.51±0.01 | 0.83±0.02 | 0.40±0.05 | -1.10±0.06 |
| 2 | 2.06±0.03 | 1.15±0.03 | -0.60±0.02 | 0.57±0.01 | 0.99±0.02 | -0.03±0.03 | -1.49±0.09 |
| 3 | 2.39±0.04 | 1.22±0.03 | -0.77±0.02 | 0.69±0.02 | 0.88±0.02 | 0.04±0.04 | -1.32±0.10 |

Table 2: Case study results: (1) ISS95 restated for comparison; (2) ISCDB subset with W7-AS, W7-A, CHS and LHD not using uncertainties; (3) same as (2), but using uncertainties of the machine variables.

## Discussion

Tab. 2 shows the ISS95 result [5] in the first row. Note that it was obtained with an configuration dependent parameter. The second row is the result for the small subset of W7-AS, W7-A, CHS and LHD neglecting the uncertainties of the data. The discrepancies in $\alpha_R$ and $\alpha_{\tau}$ in comparison with ISS95 can be explained by the choice of this subset without paying attention to configuration. Eventually, the third row shows the result of the present study. If we compare the results of the subsets in row 2 and 3, the most prominent differences are found in exactly those scaling exponents where the discussion of the uncertainties took place, i.e. $\alpha_a$, $\alpha_P$ and $\alpha_n$. This demonstrates the importance of considering the uncertainties of the machine variables. As can be seen in Fig. 1 the result of the fit $\log \tau_{fit}$ and the measured $\log \tau_{exp}$ agree well within the abscissa error bars, which is a direct measure for the reliability of the prediction.

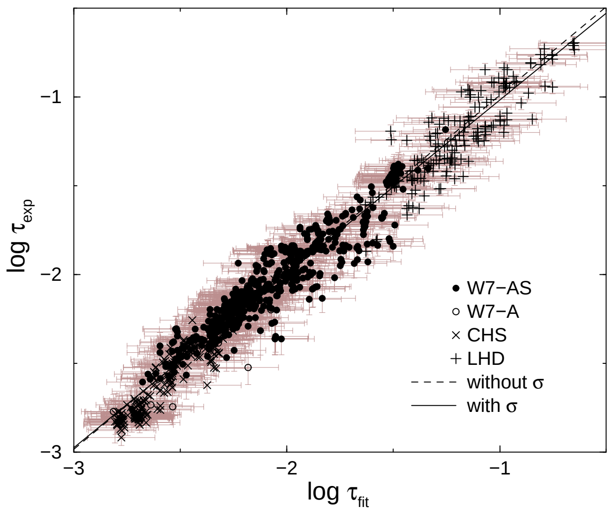


Figure 1: Plot of the confinement time from experiment vs. fit. The dotted line represents the result for the analysis not employing the uncertainties. The error bars are stated for both $\log \tau_{exp}$ and $\log \tau_{fit}$.

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