32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-1.112 (2005)

Fast ion confinement in tokamak current hole regimes

P. Neururer[1], K. Schoepf[1], V. Yavorskij[1,2], V. Goloborod'ko[1,2]

[1] *Institute for Theoretical Physics, Innsbruck, Austria*

[2] *Institute for Nuclear Research, Kiev, Ukraine*

Introduction

Here we present the establishment of the onfinement domain and a detailed orbit topology description of fast ions in a current hole (CH) tokamak. For that a specific space of constants of motion (COM) is considered, allowing for a clear distinction of possible guiding center orbits. Based on an approximation of the equilibrium poloidal flux function Ψ, analytical expressions for the confinement domain boundaries and its subdivisions are derived.

Constants of motion

Principally, three COM's are sufficient to completely describe the ion motion in an axisymmetric tokamak. Instead of the commonly employed coordinates, i.e. ion energy E, magnetic moment μ and canonical angular momentum p_φ, we take a slightly modified set. The energy variable is replaced by the normalized poloidal gyroradius $d = mcv/e\Psi_a'$, where m and v are the mass and velocity of an ion, c the speed of light, e the elementary charge and Ψ_a' is a parameter determined by the total plasma current and the plasma shape [1]. In lieu of μ we use the normalized magnetic moment $\lambda = \mu B_0/E$ with B_0 denoting the magnitude of $\mathbf{B}$ at the magnetic axis. Our third COM is the maximum radial excursion of a guiding center orbit from the magnetic axis in terms of the plasma radius a, i.e. $x_M = r_M/a$, where r is the radius of the magnetic flux surface and the subscript 'M' denotes the maximum value of a quantity. As a consequence of drift motion, ions are driven continuously to inner magnetic flux surfaces in one half of the torus, whereas, inversely, an outwards drift occurs in the other half. Hence both maxima and minima of an orbit occur in the equatorial plane.

Confinement domain boundaries

The normalized maximum radial coordinate provides a first natural boundary of the confinement domain, since ions with $x_M > 1$ are obviously not confined. As the maxima of co- and counter-going ion orbits appear at different sides of the magnetic axis, these classes are treated separately. With $\mathbf{B}$ pointing out from the poloidal cross section area such that the ions drift vertically downwards, the maxima of co-going orbits are located at the poloidal angle $\chi = 0$, whereas the maxima of counter-going particles occur at $\chi = \pi$. The maxima of trapped particles are situated at $\chi = 0$ and consequently, they are part of the co-going confinement domain.

The second boundary of the confinement domain is formed by the locus of orbits of particles exactly compensating the drift velocity $\mathbf{v_D}$ by the guiding center motion along the helical field line. These drift orbits look like a single point in the poloidal cross section and are called stagnation orbits [2]. Since $\mathbf{v_D}$ is purely vertical, a stagnation orbit can lie in the equatorial plane only, where the poloidal projection of field lines is vertical too. For monotonic current profiles, a stagnation value for ξ (=: ξ_{stg}) is found for a definite range of x_M [3] in the equatorial plane. Note that this is not the case for the CH model, which is based on the poloidal flux function approximation [1]

$$\Psi(x-x_*) = \Psi_a'\psi(x-x_*), \qquad \psi(y) = y\Theta(y), \tag{1}$$

where $x = r/a$ is the normalized flux surface radius, x_* denotes the normalized radial extent of the CH and Θ represents the Heaviside step function. Here, because of the kink in the $\mathbf{\Psi}$-profile, the location $x_M = x_*$ is a degenerate stagnation point where all ξ with $|\xi| \geq |\xi_{stg}|$ fulfil the stagnation condition. These orbits are referred to as stagnation orbits of the 2^{nd} kind. The stagnation conditions, $\dot{x} = 0$ and $\dot{\chi} = 0$, are equivalent to $\partial p/\partial x = 0$ and $\partial p/\partial \chi = 0$, where p represents the normalized angular momentum [4]

$$p = p_\varphi/\Psi_a' = \psi - dAh\xi = \psi - dA\sigma\sqrt{h(h-\lambda)}, \tag{2}$$

with $h = 1 + \frac{x}{A}\cos\chi$, $\sigma = sign(\xi)$ and A denoting the aspect ratio. Performing the partial differentiation above, the stagnation condition reads

$$\xi_{stg}^{(\pm)} = \pm\frac{d}{1+\sqrt{1-d^2}} =: \pm\xi_{stg} \tag{3}$$

for co-going $\left(\xi_{stg}^{+}\right)$ and counter-going $\left(\xi_{stg}^{-}\right)$ stagnation orbits of the 1^{st} kind. Further, one distinguishes stagnation orbits of O-and X-type. O-type orbits are stable, i.e. a small displacement will leave an ion in the vicinity of the stagnation point, whereas X-type orbits are unstable. Checking the stability criterion,

$$\frac{\partial^2 p}{\partial x^2}\frac{\partial^2 p}{\partial \chi^2} - \frac{\partial^2 p}{\partial x \partial \chi}\frac{\partial^2 p}{\partial \chi \partial x} > 0, \tag{4}$$

it turns out that all 2^{nd} kind stagnation orbits are stable, whereas only co-going stagnation orbits of the 1^{st} kind are of O-type.

Features and subdivisions of the confinement domain

Inside the confinement domain characteristic regions, bounded by the loci of specific limiting orbits, are to be distinguished. In a CH scenario there will be particles crossing the CH (regions Ia and IIa in fig.1) and those passing outside. There are two limiting orbits both touching the CH, one at the LHS and the other at the RHS, i.e. at $h_m = 1 + x_*/A =: h_*^+$ and $h_m = 1 - x_*/A =: h_*^-$. The functional dependencies $\lambda = \lambda(x_M, d)$ describing the loci of those orbits in COM-space are easily found by introduction of $h = h_*^\pm$ into Eq.(2). In fig.1, where $d = const$ corresponds to constant ion energy and practically steady total plasma current, those loci are labelled λ_{ch}. Similarly we proceed in the determination of the region of trapped particles inside the co-going domain (regions II and IIa in fig.1). Since trapped particles change the sign of ξ along their trajectory, namely at the reflection points, the region of their occurrence (regions II and IIa in fig.1) is bounded by the loci of orbits with $\xi(x_m) = \xi_m = 0$ or, respectively, $\xi(x_*) = 0$ in the CH case, which correspond to particles just not experiencing a change of sign of ξ. Their orbits are D-shaped and form the loci λ_D marked by the black dashed-dotted line in fig.1. Finally, the class

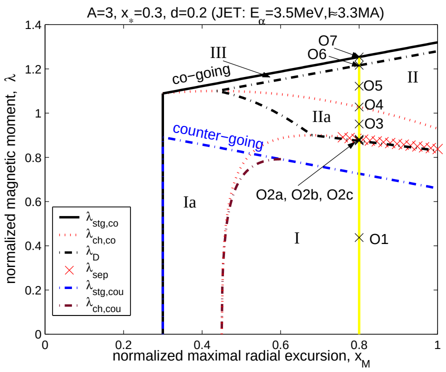

Figure 1: *Confinement domains for co-going and trapped ions (black, red), as well as for counter-going ions (blue).*

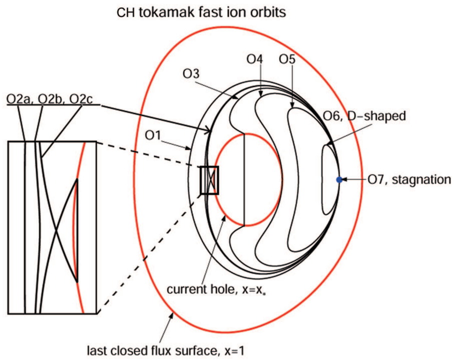

Figure 2: *Fast ion orbits calculated for a current hole tokamak scenario with* $x_* = 0.3$, $d = 0.2$, $A = 3$

of orbits that constitute the only connection between the co- and counter-going confinement domain are the so-called pinch orbits. A pinch orbit, contour O2c in fig.2, is a peculiar trapped ion orbit featuring 3 intersections with the equatorial plane, since its banana tips merge at the mid-plane. The locus of pinch orbits is seen to separate the co-and counter-going confinement domains [4] thus constituting a separatrix labelled λ_{sep} in fig.1. Though a pinch orbit itself may intersect the current hole, both the outer part of the pinch orbit as well as any particle orbit with $\lambda < \lambda_{sep}$ do not cross the CH. Thus, for pinch orbits crossing the CH, λ_{sep} separates orbits through and fully outside the current hole.

Orbit topology in COM-space

Each point in the confinement domain corresponds to a specific orbit. For elucidation of the topology of fast ion orbits in a CH tokamak regime we choose the maximum radial position of an orbit arbitrarily as $x_M = 0.8$ and vary λ. Since in the counter-going confinement domain only regions of orbits outside (region I in fig.1) and inside (region Ia in fig.1) the CH area can be distinguished, we will focus our examination on the co-going domain. The inspection starts at point O1 in region I in fig.1, corresponding with a co-going ion that does not cross the CH. From there we follow the yellow track of increasing λ until reaching the upper confinement domain boundary formed by stagnation orbits. In fig.2 we depict all characteristic orbits along this path. By increasing λ, i.e. decreasing ξ_M, the minimum flux surface coordinate along an orbit must decrease too, since ions with a lower ξ_M can penetrate into regions with lower B only. At point O2a we arrive at a D-shaped orbit, i.e. $\xi_m = 0$. Next we enter the region of trapped ions. Further reduction of ξ_M makes the orbits kidney-shaped, e.g. point O2b, until the pinch orbit is realized at point O2c. Here we enter the domain of trapped ions crossing the CH, until at point O4 the banana orbit only touches on the CH area at the RHS of the magnetic axis. Further increasing λ we arrive at point O5 in region II of trapped ions outside the CH. The subsequent significant point, indicated by O6, represents the second possible D-shaped orbit. Passing region III from O6 to O7 the size of orbits decreases continuously, till the orbit shrinks to a single point identified as a stagnation point.

Conclusion

Based on a simple analytical poloidal flux model [1] for axisymmetric CH tokamak equilibria, we characterized completely the topology of possible fast ion orbits and determined their confinement domains. The trajectorial alterations induced by the presence of a current hole as well as the consequences for fast ion transport became evident.

Acknowledgement

This work has been carried out within the Association EURATOM-OEAW.

References

[1] V.A. Yavorskij et al., Nucl. Fusion **44**, L1-L4 (2004)

[2] J.A. Rome and Y-K.-Peng, Nucl. Fusion **19**, 1194 (1979)

[3] V.A. Yavorskij et al., Nucl. Fusion **43**, 1077 (2003)

[4] P. Neururer, *Fast ion confinement in a current hole tokamak*, Diploma Thesis, University of Innsbruck, (2004)