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Calculation of Plasma Boundary Using Video Images

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1. Introduction. An important direction of plasma diagnostics in toroidal devices is reconstruction of plasma boundary shape and position using experimental measurements. In the recent past the techniques were mainly based on magnetic measurements and X-rays. Usually reconstruction of the plasma boundary for the whole discharge with traditional techniques requires time and it is not easy to calculate the boundary between discharges during the experimental campaign. In some cases plasma boundary reconstruction is inaccurate, since mathematically the problem is deeply ill-posed [1].

In this paper we propose a new, fast and relatively accurate technique for reconstructing plasma shape and position. It is based on video image processing obtained by a fast CCD camera. Such kind of images are routinely available on a number of tokamaks, e.g. MAST and JET. The main physical effect exploited is higher brightness of the plasma boundary.

The complexity of the problem is determined by several factors. Due to high dynamics of the process it is impossible to change exposition synchronously. So, some frames become indistinct and/or over exposed. The internal surface of the toroidal chamber has usually mirror-like properties and light, reflected from different technological ledges and apertures, mixes with the plasma fluorescence, Fig. 1, A, B.

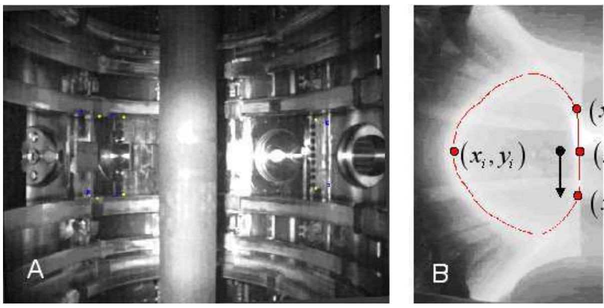

Figure 1: A - photo of the internal surface in MAST. B - photo of the left half of plasma discharge with extracted plasma boundary. The arrow indicates the direction of the grid by-pass.

Several areas with similar brightness can be present. The image is distorted by the optical properties of the camera and the viewing angle which should be corrected for reconstruction of the plasma boundary coordinates.

In spite of the above difficulties the authors managed to create a fast and reliable algorithm for accurate extraction of the cylindrical coordinates (R, Z) of the plasma boundary from the video image. The algorithm is based on the method of dynamic programming. Formulation of the mathematical problem, details of the algorithm and examples of application are presented below.

2. Formulation of the problem. Before proceeding with the image analysis one should remove spherical distortions, caused by the short focus camera. These kinds of distortion make close objects have a larger size in the image. Compensation of spherical distortions is carried out by evaluation of the parameters of the non-linear transformation induced by the lens [2]. Even very strong distortions can be removed with this or other techniques.

It is convenient to formulate the problem in polar coordinates (r, ϕ) in the image plane (u, v). The origin of the system is chosen at some point near the magnetic axis (u_0^*, v_0^*), which can be calculated by analogy with the centre of mass, but using pixel fluorescence intensity I_{pixel} instead of the mass. Introduce uniform grids over radius $r_i = i\Delta r$, $i = 0, \ldots, N_r(j)$ and angle $\phi_j = j\Delta\phi$, $j = 0, \ldots, N_\phi$. Here Δr and $\Delta\phi$ are steps. The number of radial points $N_r(j) + 1$ depends on the angular index j, since the image has a rectangular form. Denote briefly $r_{i(j)}$ - the value of radius r_i on j-the ray $r_{i(j)} \equiv r_i(\phi_j)$.

In order to define the plasma boundary in the image one should choose from the set Ω of all sequences $\{r_i(\phi_j)\}$, $j = 0, \ldots, N_\phi$ of the grid points (r_i, ϕ_j) the one $\{r_i^*(\phi_j)\}$, for which the sum of pixel brightness I_{pixel} with account of constraints is maximum

$$\Phi(\{r_{i(j)}\}) \equiv \sum_{j=0}^{N_\phi} \Big(I_{\text{pixel}}(r_i(\phi_j), \phi_j) + R_{i(j-1),i(j),i(j+1)} \Big) \rightarrow \max_{\{r_{i(j)}\} \in \Omega}, \tag{1}$$

$$R_{i(j-1),i(j),i(j+1)} \equiv \alpha_1 R_{1,i(j)} - \alpha_2 R_{2,i(j-1),i(j)} - \alpha_3 R_{3,i(j-1),i(j),i(j+1)} + \alpha_4 R_{4,i(j-1),i(j),i(j+1)} + \alpha_5 R_{5,i(j-1),i(j),i(j+1)}.$$

Constraints R_k allow extraction of the most "reasonable" boundary: R_1 is the condition of maximum radius, $R_{1,i(j)} = 0.5 r_{i(j)}^2$; R_2 - minimal distance between the neighbouring points, $R_{2,i(j-1),i(j)} = 0.5(r_{i(j)} - r_{i(j-1)})^2$; R_3 - smoothness (minimisation of the second derivative absolute value), $R_{3,i(j-1),i(j),i(j+1)} = 0.5(r_{i(j-1)} - 2r_{i(j)} + r_{i(j+1)})^2$; R_4 - smoothness of the derivative (absence of breaks), $R_{4,i(j-1),i(j),i(j+1)} = (r_{i(j)} - r_{i(j-1)})(r_{i(j+1)} - r_{i(j)})$; R_5 - bulge (point $r_{i(j)}$ lies either on the line through points $r_{i(j-1)}$, $r_{i(j+1)}$, or further from this line with respect to the centre of the polar coordinates system (r, ϕ)), $R_{5,i(j-1),i(j),i(j+1)} = r_{i(j)}(r_{i(j-1)} + r_{i(j+1)}) - 2r_{i(j-1)} r_{i(j+1)} \cos(\Delta\phi)$.

Coefficients $\alpha_1, \ldots, \alpha_5$ determine the contribution of each constraint to the functional $\Phi_{i(j)}$. These coefficients are chosen using test calculations by comparison of the reconstructed boundary with the original. Once obtained they can be used for a wide range of images with comparable plasma size.

3. The algorithm. The solution of the maximisation problem (1) is obtained with the help of dynamic programming. If the number of grid points is not too small then the method always gives some solution. As a result we get parameterization $\{r_i^*(\phi_j)\}$ of the plasma boundary image.

Camera calibration is necessary for reconstructing the cylindrical coordinates (R, Z) of the plasma boundary. Accurate calibration is required because of vibration of the camera during discharges and possible shifts in its position during maintenance. The general approach for camera calibration is presented in, e.g., Refs. [3,4]. However using specificity of plasma imaging in a tokamak the authors developed a fast automatic calibration procedure, which allows application to every frame of the video and avoids manual input of the anchor points. The procedure is not presented here due to space restriction.

The camera is located on the mid-plane and pointing towards the centre column. Assumption of axial symmetry of the plasma boundary allows reconstruction of not just

(R, Z), but all three cylindrical coordinates (R, η, Z) of the 3D curve, projection of which on the poloidal plane $\eta = \text{const}$ gives (R, Z) coordinates of the plasma boundary,

$$R = \frac{R_c|\text{tg}\alpha|}{\sqrt{1+\text{tg}^2\alpha}}, \quad \eta = \begin{cases} 2\pi + \alpha, & \alpha < 0 \\ \pi + \alpha, & \alpha \geq 0 \end{cases}, \quad Z = \frac{v - v_0}{a_v}\frac{R_c}{1+\text{tg}^2\alpha}, \quad \text{tg}\alpha = \frac{u - u_0}{a_u},$$

where $u = r^*(\phi)\cos\phi + u_0^*$ and $v = r^*(\phi)\sin\phi + v_0^*$, α is the angle between the normal line to the image plane at the lens centre and the line through the lens centre and the equatorial projection of point from the surrounding plasma surface (viewing angle), R_c - the value of R at the lens centre, $a_u = f/w$, $a_v = f/h$ - camera calibration parameters with f - the focal length, w and h - camera stretch factors along horizontal and vertical directions appropriately, (u_0, v_0) - centre of the photodetector coordinate system. The values of a_u, a_v, u_0, v_0 are determined in the calibration procedure.

A priori and a posteriori estimation of accuracy of (R, η, Z) reconstruction, caused by the discontinuity of the photodetectors and the final width of the fluorescent layer, gives the value of less than several percent for most images. Processing a 100 Mb video of the plasma discharge requires several minutes on a modest personal computer. The algorithm can reconstruct the plasma boundary even in cases where it is hardly distinguishable by eye.

4. Examples of reconstruction. Fig. 2 presents the reconstructed plasma boundary $\{r_i^*(\phi_j)\}$ in the image plane for MAST. The dashed curve is formed by the projection of the 3D space curve, consisting of tangency points of rays from the centre of the lens to the left half of the surrounding plasma surface (plasma is projected symmetrically). The 3D curve is shown in Fig. 3, where the camera position $(X = 0,\ Y \approx -2,\ Z = 0)$ is marked with the cross. Note that the 3D curve does not lie in the meridional plane.

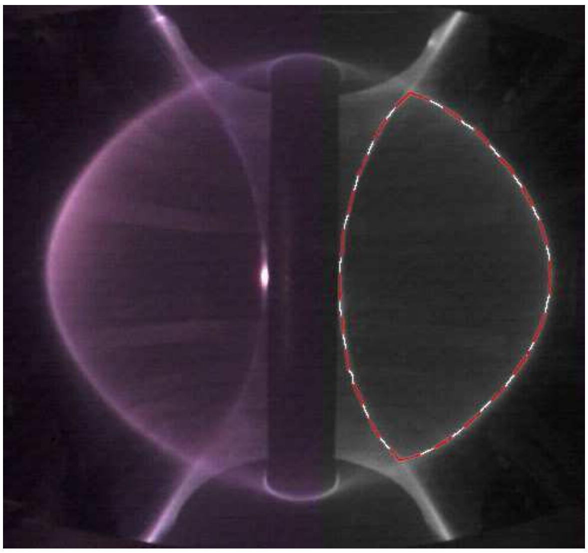

Figure 2: Dashed line in the right part of the photo presents reconstructed plasma boundary in the image plane.

The axisymmetric plasma in (R, Z) coordinates is plotted in Fig 4. The double dot-dashed line gives the scaled plasma boundary in the image plane. One can see that on the outer side the real boundary is a significant distance inside the imaged boundary. This is explained by the influence of the viewing angle α. One should also note that boundary image is up-down and left-right inverted. Comparison of optic and magnetic calculations of the plasma boundary (Fig. 4) shows that both methods give similar results for points close to the Z-axis. However at the outer boundary the optical reconstruction can differ from the magnetic reconstruction and other boundary measurements (Thomson Scattering and D-alpha linear camera) by more than 5%. The cause of this residual is under investigation.

5. Conclusion. The algorithm is implemented in code VIP (**V**ideo **I**mage **P**rocessing), which can interactively evolve (R, Z) coordinates of the boundary synchronously with selected graphs of measured plasma parameters or work in the batch regime as a routine diagnostics for every discharge. The algorithm and the software at present have no analogues. VIP results can be used as an additional constraint for the flux surfaces reconstruction procedures in the inverse option of the SCoPE code [5] or the EFIT code.

The new technique can find in future other impressive applications. The high speed of the algorithm can enable development of a real-time shape control based on video processing, extending the recent success with the linear HOMER camera on MAST for controlling position of the plasma edge [6]. Several cameras can allow reconstruction of the 3D plasma shape and give quantitative information about plasma rotation and axial asymmetry.

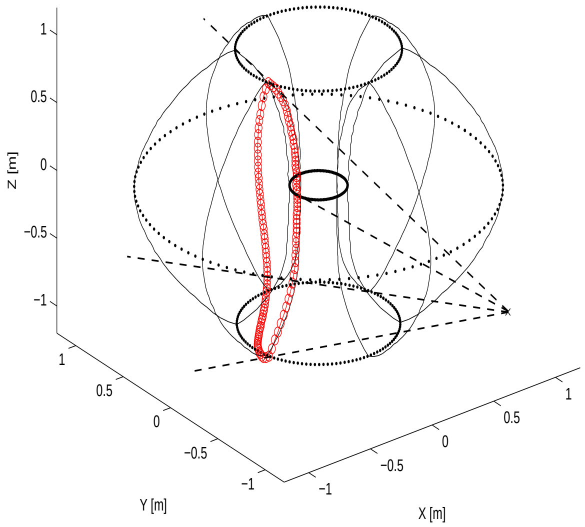

Figure 3: Tokamak Cartesian coordinates. Red circles show 3D curve on the surrounding plasma surface which projects on the photodetector.

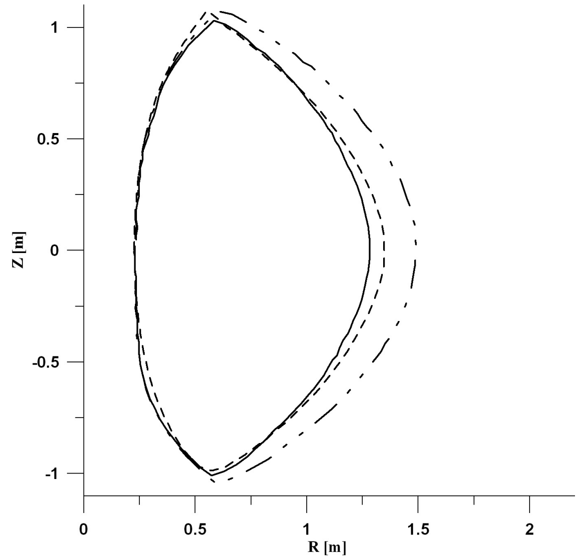

Figure 4: Solid line gives VIP $(R, \eta = \text{const}, Z)$ plasma boundary coordinates for MAST shot 8869. Dashed - EFIT, double dot-dashed - scaled boundary in the image plane.

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