32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-1.089 (2005)

Hilbert Spectrum Analysis of Mirnov Signals

I.I. Orlovskiy, A.M. Kakurin

Nuclear Fusion Institute, RRC "Kurchatov Institute", Moscow, Russia

Introduction.

Signals of magnetic probes in tokamaks (Mirnov signals) represent evolution of MHD-perturbations. Often the signals are nonstationary, i.e. their amplitude and frequency vary in time, sometimes during a period of oscillation. Analysis of such signals by widely used methods such as spectrograms and wavelet analysis shows good efficiency but sometimes their time-frequency resolution is insufficient due to the indeterminacy principle [1].

Hilbert spectrum analysis (HSA) is recently developed technique for determining amplitude and frequency evolution of nonstationary signals [2]. It is used in conjunction with e*mpirical mode decomposition* (EMD) method which decomposes signal to a set of *monocomponent* oscillations of variable amplitude and frequency [3, 4]. Combination of EMD and HSA, also known as Hilbert-Huang transform, allows analyzing multicomponent nonstationary signals with resolution limited only by sampling rate of the signals.

Hilbert spectrum analysis and empirical mode decomposition.

The main idea of HSA is to construct complex signal for the analyzed real signal. The positive part of the spectrum of the initial real signal $x(t)$ is multiplied by two and the negative part is set to zero. Such spectrum corresponds to complex signal $z(t)$ whose imaginary part is equal to the Hilbert transform of the real part that equals to $x(t)$ (1, 2). For complex signals amplitude $A(t)$, phase $\psi(t)$ and instantaneous frequency $\omega(t)$ are uniquely defined (3, 4, 5). Using these definitions one can calculate time-frequency distribution of the signal as two-variable function $A(t, \omega(t))$ which is called Hilbert spectrum.

$$z(t) = x(t) + iy(t), \tag{1}$$

$$y(t) = \mathrm{H}[x(t)] = v.p.\int_{-\infty}^{+\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau, \tag{2}$$

$$A(t) = |z(t)| = \sqrt{x^2(t) + y^2(t)}, \tag{3}$$

$$\psi(t) = \arg z(t) = \arctan\frac{y(t)}{x(t)}, \tag{4}$$

$$\omega(t) = \frac{d\psi(t)}{dt} = \frac{x(t)y'(t) - x'(t)y(t)}{x^2(t) + y^2(t)}. \tag{5}$$

Though expressions (1) – (5) are defined for any function $x(t)$ which satisfies existence conditions for the integral (2), the physical meaning of parameters (3)-(5) is obvious only if $x(t)$ belongs to the class of *monocomponent* functions, i.e. the number of its extremes and the number of zero-crossings differ at most by 1 and the mean between the upper and lower envelopes equals to zero [4]. In practice, most of the signals are not monocomponent so EMD method proposed by Huang et al [3] significantly expanded applying of HSA in various fields. Iteration procedure of EMD represents signal $x(t)$ as a sum of amplitude-and-frequency modulated components $f_i(t)$ which are almost monocomponent functions and monotonic trend $r_n(t)$:

$$x(t) = \sum_{i=0}^{n-1} f_i(t) + r_n(t). \qquad (6)$$

Although uniqueness of the decomposition is still not proved, the method turned out to be very effective [5]. First, initial signal is decomposed to a set of components, and then HSA is applied to each of them to determine amplitude and frequency dependencies (fig. 1a). Finally, Hilbert spectrum calculated as the sum of Hilbert spectra of each component (fig. 1b) shows significantly higher resolution than spectrogram and wavelet transform (fig.1cd).

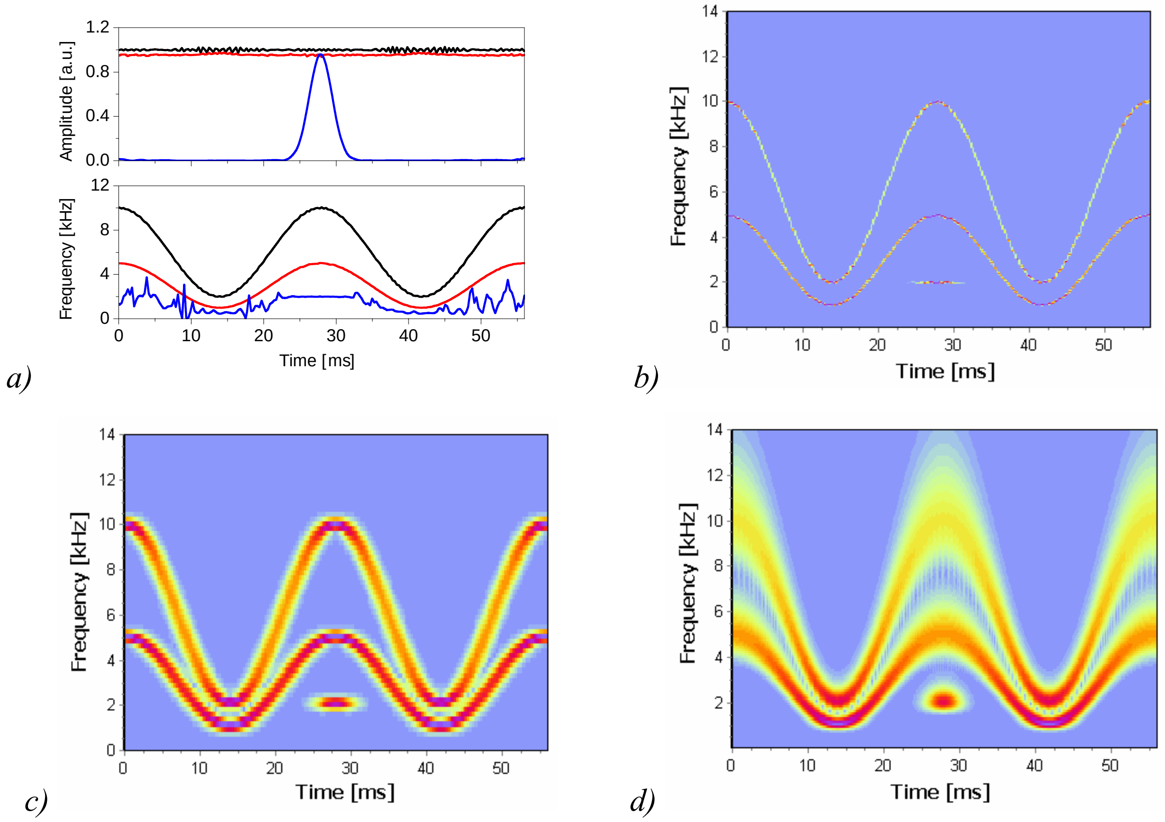

Fig. 1. 3-component signal is sum of 2 sinusoidal FM components and 1 Gaussian wavepacket: a) amplitudes and frequencies of components obtained by EMD/HSA; b) Hilbert spectrum; c) spectrogram; d) wavelet transform.

HSA of experimental data.

EMD has been applied to experimental data from T-10 tokamak (fig. 2). In the discharge shown two tearing-modes with poloidal m=3 and m=2 developed simultaneously. Signals M3 and M2 are cylindrical Fourier-harmonics. The signal P1 from a single magnetic probe was processed by EMD that resulted in two components P1_0 and P1_1. One can see that the component P1_0 looks similar to M3 and P1_1 is almost alike M2 that demonstrates effectiveness of EMD. Then HSA was used to calculate amplitudes and frequencies of P1_0 and P1_1. Rapid changes in frequency of magnetic island rotation during a period of oscillation (the so-called intrawave modulation) produce oscillations in instantaneous frequencies that are clearly seen in fig 3. The same dependencies represented on time-frequency plane as Hilbert spectrum are shown in fig. 4.

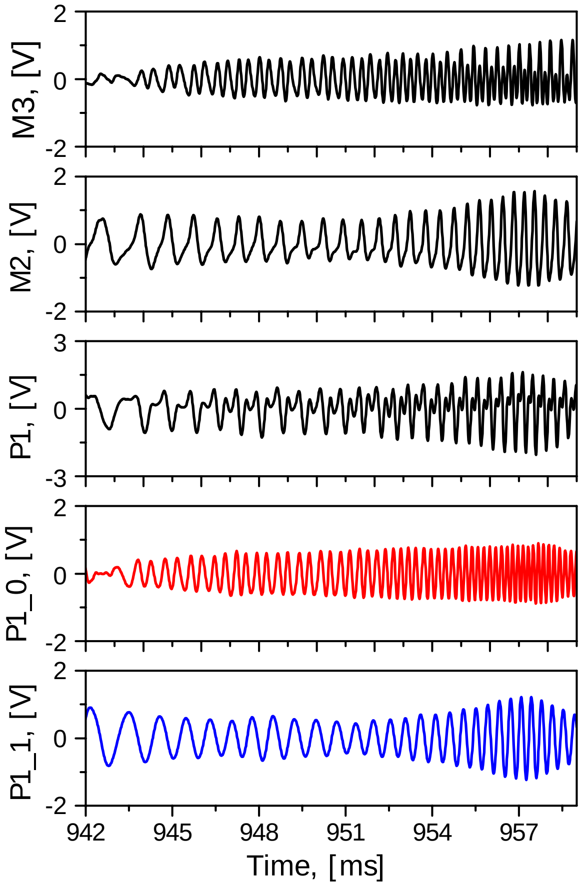

Fig. 2. Cylindrical Fourier harmonics with m=3 (M3) and m=2 (M2); signal of a magnetic probe (P1); P1_0 and P1_1 are the components of P1 decomposed by EMD.

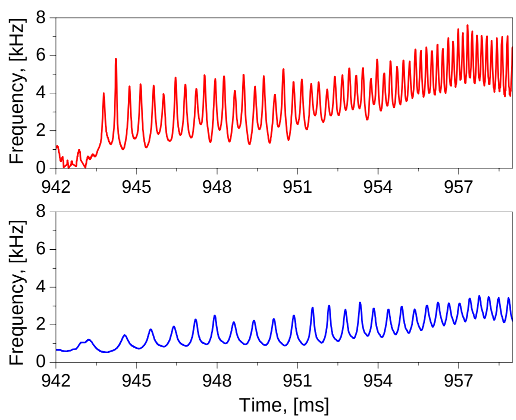

Fig. 3. Instantaneous frequency of P1_0 and P1_1, calculated by HSA.

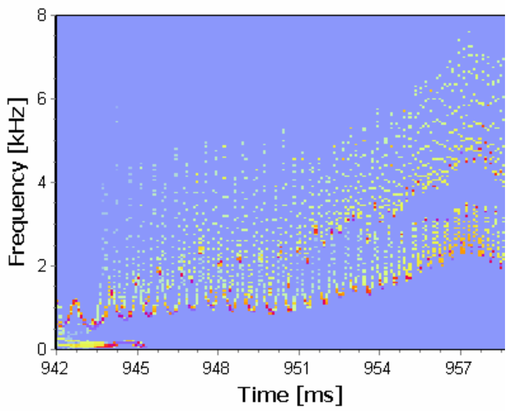

Fig. 4. Hilbert spectrum of P1_0 and P1_1.

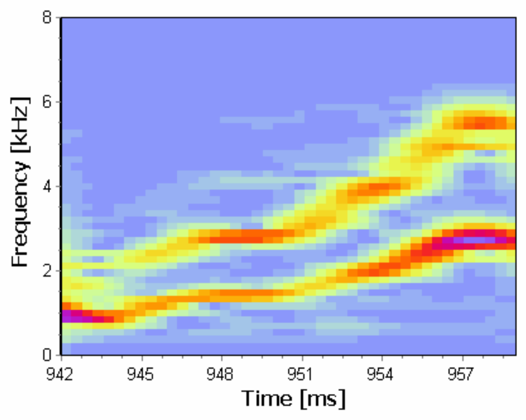

Fig. 5. Spectrogram of P1.

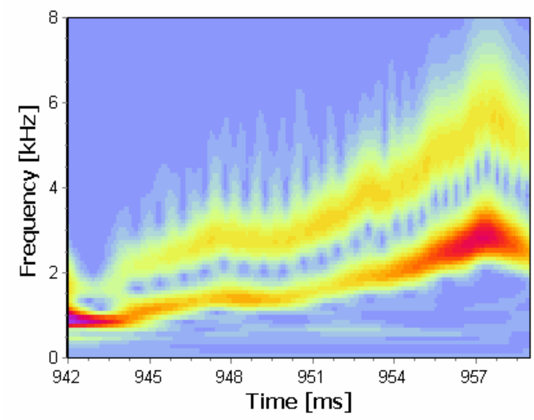

Fig. 6. Wavelet transform of P1.

On the contrary, limited time-frequency resolution of spectrogram does not allow determining intrawave modulation of the signal (fig. 5); wavelet transform just qualitatively indicates the presence of the modulation (fig. 6) while quantitative analysis is still complicated.

Conclusions.

Combination of HSA and EMD provides qualitatively new and higher level of investigation of large-scaled MHD perturbations with maximal possible resolution that allows quantitative analyzing irregularities of mode rotations. Such detailed information about frequency variation can be used in identification of error field [6]. The method can be also applied to the signals of any diagnostics whose amplitude and frequency vary in time.

References.

1. *Dose V., Venus G., Zohm H.* Phys. Plasmas 4 (2), 1997, pp. 323-327.
2. *Taylor E.D., Cates C., Mauel M.E.* Rev. Sci. Instrum, Vol. 70, 1999, p. 4545.
3. *Huang N.E., Shen Z., Long S.* Proc. R. Soc. London A, 454, 1998, pp. 903-995.
4. *Flandrin, P., G. Rilling, P. Goncalves*. IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing, NSIP-03.
5. *Kakurin A.M., Orlovsky I.I.* Plasma Phys. Rep., Vol. 30, no. 4, 2004, pp. 370-375.
6. *Ivanov N.V, Kakurin A.M., Orlovskiy I.I.* This conference, P5.068.