Design of Optimal Plasma Position and Shape Controller for KSTAR

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Introduction

Non-rigid plasma equilibrium response model, which can predict perturbed responses of plasma equilibrium with conducting structures in the presence of external magnetic perturbations, is developed and applied to the design of optimal plasma position and shape controller for KSTAR. Plasma equilibrium response model for KSTAR (K-PERM) is formulated by coordinating a perturbed Grad-Shafranov equation and perturbed plasma evolution equations based on reference equilibrium [1, 2]. K-PERM is validated by comparing vertical growth rates and vertical displacement control characteristics with nonlinear MHD evolution models (TSC) and linear rigid current displacement model (RZIP) [3]. Diagnostic matrix for plasma position and shape in K-PERM is organized with real-time plasma boundary identifications by equivalent filament approach method, of which accuracy and efficiency are assessed for various MHD equilibria with measurement errors and noises. In addition, a methodology to trace eddy currents (on passive plate, limiter, divertor, and vacuum vessel) in real-time is implemented to the design of plasma shape controller, which is a crucial kernel to delicately relate perturbed plasma responses from magnetic measurements with a severe noise source of magnetic diagnostics. Eddy currents on various conductors can be estimated by defining the equivalent filamentary elements on conductor surfaces and integrating the time variations of magnetic signals.

Perturbed Equilibrium Response Model

With the time-scale of much longer than the Alfven time scale, the magnetic flux evolutions for mass-less plasmas can be described by Grad-Shafranov equation and generalized Ohm’s law:

\[ \Delta \psi = -\mu_0 R J_p, \quad E = \nabla \times B = \eta_p J_p \]

\[ J_p = \lambda \left[ \beta_0 R \psi_s^{\alpha_0} + \frac{\psi_s^{\alpha_0}}{\mu_0 R} \left( 1 + \alpha_0 (1 - \psi_s) \right) \right] \]

At the flat-top phase in a slow dynamics, the time behaviour of equilibrium quantities can be approximated with linear dependency on the reference values by ignoring any transient. With
linear perturbation theory and finite difference method, the perturbed equilibrium response model for KSTAR (K-PERM) can be formulated from perturbed equilibrium relations and magnetic diffusion relations. Perturbed magnetic flux distributions are obtained for various perturbation sources as shown in Fig. 1.

**Fig. 1** Perturbed magnetic flux distributions from (a) magnetic flux variation at magnetic axis, (b) plasma profile variation, and (c) in-vessel control current variation.

To confirm the reliability of the K-PERM, the vertical growth rates are estimated and compared with those from different plasma dynamic models such as TSC (nonlinear MHD evolution and transport) and RZIP (rigid linear displacement model). Linear plasma response models, K-PERM and RZIP, have shown very similar vertical growth times of usually 7~8 msec while those by the nonlinear model, TSC, are higher by factor of 2 as shown in Fig. 2. Also, it is found that the vertical stabilizing effects of passive plates are more efficient for much higher beta plasmas since their equivalent centers are closer to the position of passive plates.

**Fig. 2** Vertical growth rates by different plasma dynamics models

**Real-time plasma boundary identifications**

Diagnostic matrix for plasma position and shape in K-PERM is constructed by embodying the plasma boundary identifications from magnetic measurements. As a simple
and efficient method, continuously distributed plasma currents are assumed to be collections of current filaments. Figure 3(c) shows that reconstructed equilibria are very sensitive to measurement noises, especially near x-points.

**Fig. 3** Real-time plasma boundary identification by equivalent filament approach. (a) Reconstructed magnetic flux distribution (b) Effects of MHD equilibrium variations (c) Effects of measurement noises

**Tracing the equivalent eddy currents from magnetic variations**

In order to reduce effects of measurement noises for the improvement of accurate boundary identification, real-time equivalent eddy currents are evaluated. To estimate the eddy currents effects, additional time-dependant relations are formulated as follows:

\[
\delta \mathbf{m} = \int_{\Omega_p} \tilde{G}_{mp} \delta \mathbf{I}_p \, d\Omega + \tilde{G}_{ms} \delta \mathbf{I}_s + \tilde{G}_{ma} \delta \mathbf{I}_a
\]

\[
\tilde{M}_{ss} \frac{d\delta \mathbf{I}_s}{dt} + \tilde{M}_{sp} \frac{d\delta \mathbf{I}_p}{dt} \left( \int_{\Omega_p} \delta \mathbf{I}_p \, d\Omega \right) + \tilde{M}_{sa} \frac{d\delta \mathbf{I}_a}{dt} + \tilde{R} \delta \mathbf{I}_a = 0
\]

By combining with equivalent plasma currents, time-dependant equivalent eddy currents relation can be formulated.

\[
\tilde{M}_{ss} \delta \mathbf{I}_s + \tilde{R} \delta \mathbf{I}_a = T_s \delta \mathbf{I}_a + T_m \delta \mathbf{m}
\]

With initial value, \( \delta \mathbf{I}_a(0) \), the above time-dependant equivalent eddy current equation can be solved by integrating the time derivatives of active control currents, \( \delta \mathbf{I}_a(t) \), and the magnetic measurements, \( \delta \mathbf{m}(t) \).

**Plasma position and shape control simulations**
Figure 4 shows plasma control simulations with real-time boundary identifications. Vertical offset responses are shown with control current, voltage, and eddy currents at passive plates in figure 4(a). And figure 4(b) shows two plasma shape control simulations; one for shifting the lower x-point, and the other for oscillating the up-down symmetric x-points.

![Fig. 4 Plasma control simulations with real-time boundary identifications. (a) vertical offset response (b) lower x-point shift and up-down symmetric x-points oscillating simulation](image)

**Conclusions**

For the design of optimal plasma position and shape controller for KSTAR, control system modelling within a controller design framework has been performed. To build the state-space matrixes for plasma dynamics, perturbed equilibrium response model has been developed and evaluated. A real-time plasma boundary identification method has been examined for MHD equilibrium variations with various noise effects, and it has been embodied into diagnostic matrixes of the control system model. In addition, real-time plasma boundary identification algorithm has been improved by implementing an equivalent eddy current tracing method.

**References**