Effects of radio frequency waves on dissipative low frequency instabilities in mirror plasmas

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Recently, it has been observed that interchange modes in a magnetic mirror plasma can be suppressed by nonlinear interactions between applied rf waves and sideband waves (SBWs) [1]. An analysis based on the cold plasma model for the SBW stabilization in Ref. [2] explains well the experimental result. In the present work, we generalize the model in Ref. [2] so as to investigate the rf effects on the drift wave instability in mirror plasmas.

We consider a magnetized plasma that is subject to dissipative low frequency instabilities originated from the field line curvature of a confining magnetic field and a pressure gradient. Following a similar approach in Ref. [2], a coupled set of perturbed two-fluid equations provides a dispersion relation as follows:

\[ (\omega - \sigma_e)(\omega - \sigma_i) = k_z^2 u^2 + \frac{k_z^2}{k_i^2} H(\omega), \]  

where

\[ \sigma_e = \omega_e + \omega_gi - ib(1 + \theta) \nu_i - i \frac{m_e}{m_i} \nu_e - k_y \gamma_{RF}^2 - \frac{k_y k_a v_{RF}^2}{\Omega_i}, \]

\[ \sigma_i = -\frac{\theta}{2} \omega_e + \omega_gi - i \nu_i, \]

\[ u^2 = c_s^2 + v_{RF}^2, \]

\[ k_i^2 = \frac{(\omega - \sigma_i)(\omega - \sigma_e + \tau \omega_gi + i \nu_e)}{\Omega_i \Omega_e}. \]

with \( \omega_e = k_y k_n T_e / m_e \Omega_e \) being electron diamagnetic frequency (\( \Omega_{\alpha=\pm} \) is cyclotron frequency), \( \omega_gi = -k_y T_i / m_i \Omega_e R_c \) (\( R_c \) is curvature radius), \( k_y \) and \( k_z \) corresponding to the azimuthal and axial wave numbers of the mode, respectively, \( k_n = (\mathbf{e}_z \cdot \nabla n_0) / n_0 \) (\( n_0 \) is plasma density), \( b = -k_y \omega_e / k_y \Omega_i \), \( \theta = T_i / T_e \), \( \tau = T_e / T_i \), \( \nu_{\alpha=\pm} \) being plasma-neutral collision frequency, and \( c_s = \sqrt{(T_i + T_e) / m_i} \). In Eq. (1), \( H \) brings about the interchange mode and is defined as

\[ H(\omega) = \omega^2 + \tilde{\omega} \omega + \gamma_i^2, \]

where \( \tilde{\omega} = \theta \omega_e + (\tau - 1) \omega_gi + i \nu_i \) and \( \gamma_i^2 = \gamma_G^2 - \gamma_{RF}^2 + i (\theta \omega_e + \tau \omega_gi) \nu_i \) with

\[ \gamma_G^2 = (k_n / k_y)(1 + \tau) \omega_g \Omega_i \] the square root of which is the interchange growth rate.
In the dispersion, \( \gamma_{RF}^2 = \gamma_{RFi}^2 + \gamma_{RFe}^2 \), \( v_{RF}^2 = v_{RFi}^2 + v_{RFe}^2 \), and
\[
\frac{1}{k_z m_i n_{io}} \text{ and } v_{RFa}^2 = \frac{\nabla \cdot f_{par\perp}}{k_z^2 m_i n_{io}}
\]
represent the effects of high frequency waves on the low frequency mode in the form of ponderomotive force on species \( \alpha(i \text{ or } e) \) given by
\[
f_{p\alpha} = \sum_{\alpha'=0}^{n} \left( n_{\alpha'\alpha} E_{\alpha'} + \frac{\Gamma_{\alpha'\alpha}}{c} \times B_{\alpha'} \right) - \nabla \cdot \Pi_{\alpha\alpha'},
\]
where \( n \), \( \Gamma \), and \( \Pi \) denote density, flux, and rf-induced contribution to stress tensor, respectively, \( \omega' + \omega'' = \omega \) stands for the generation of a low frequency force through the coherent nonlinear interaction between high frequency waves.

Figure 1 (a) shows the low frequency wave branches at various mode numbers \( m \) \( (k_y = m / r_a \) \( r_a \) is the plasma radius) in the absence of rf-fields. We used \( \nu_i = 10^{-3} \omega_0 \) and \( \nu_e = 10^{-2} \omega_0 \) \( \omega_0 = 2\pi \times 3.5 \) MHz is the angular frequency of the rf wave. As \( k_z \) increases, the mode changes from an interchange type to a drift type and finally goes to an ion acoustic mode. In Fig. 2 (b), the growth rates of the modes are presented. One can see that the interchange mode is suppressed with increasing \( m \) by finite Larmor radius effect. The drift wave instabilities exist in the range of \( 10^{-4} \leq k_z / k_y \leq 10^{-3} \).

\[\text{Fig. 1. Dependency of (a) real and (b) imaginary parts of mode frequency on } k_z \text{ at various } m.\]

\[k_n = -1 / r_a, \ r_a = 15 \text{ cm, } T_e = 20 \text{ eV, } T_i = 100 \text{ eV, } n_0 = 10^{12} \text{ cm}^{-3} \text{ are used in calculation. Dotted lines in (a) represent } \omega = -\theta \omega_\ast \text{ (interchange), } \omega_\ast \text{ (drift), } k_z^2 c_s^2 \text{ (ion acoustic).}\]
EPM ($k_{\perp}^+ / k_n = -3$) dominant cases. $E_{\text{inc}}^2 = 30 \text{ V}^2 / \text{cm}^3$ are used except for RF-off and low power ($E_{\text{inc}}^2 = 5 \text{ V}^2 / \text{cm}^3$) cases. $E_{\text{inc}}^L = |E_{\text{inc}}^R| = 10^2 |E_{\text{inc}}|$, $k_{\perp}^+ = k_{\perp}^z = k_{\perp}^z / 5$ and $k_{\perp}^z = k_{\perp}^z = k_{\perp}^z = 1/10 l$ ($l = 4.2 \text{ m}$ is plasma length). Other conditions are same as Fig. 1.

In Fig. 2, growth rates are shown at various $\omega_0 / \Omega_i$ when rf power is applied. In order to take into account the equilibrium ponderomotive force (EPM) effect, we assumed here $\nabla |E_{\text{inc}}^j|^2 = (k_{\perp}^z \hat{e}_x + k_{\perp}^z \hat{e}_z) E_{\text{inc}}^j (j = L, R, z$ represent left-hand, right-hand, parallel components). In the small $k_z$ limit and under the conditions in Fig. 2, Eq. (1) is reduced to interchange mode dispersion $H(\omega) = 0$. In the intermediate $k_z$ range that drift modes prevail, an approximate solution of Eq. (1) is given by $\omega = \omega_i + i \gamma$, where

$$\gamma = \frac{(1 + \theta) \omega_i^2 + \gamma_G^2 - \text{Re} \gamma_{RF}^2}{\omega_i} \left[ b(1 + \theta) + \frac{k_{\perp}^z c_i^2}{|\sigma_d|^2} \right] v_i - \frac{m_e}{m_i} v_e + \text{Im} \left( k_{\perp}^z v_i^RFi f + \frac{k_i k_n v_i^RF e}{\Omega_i} - \frac{k_i^z v_i^RF e}{\sigma_d} \right)$$

with $\omega_i = -(k_{\perp}^z / k_n)^2 \Omega_i \Omega_e / v_e$ and $\sigma_d = \sigma_e - \sigma_i$. For both interchange and drift mode cases, terms involving $\gamma_{RF}^2$ are predominant over other terms at the large power. Near the resonance condition, $\gamma_{RF}^2$ in Eq. (3) can be approximated as

$$\gamma_{RF}^2 \approx \frac{\gamma_{RF}^2}{\omega_i(\omega_0 - \Omega_i)} |E_{\text{inc}}|^2,$$

where the number 2 is attributed to SBW-rf wave interaction and $k_{\perp}^z / k_n$ to EPM force. Therefore, the stability is determined by $k_{\perp}^z / k_n$ and $\omega_0 / \Omega_i$ at a constant rf power.
Fig. 3. Marginal stability boundaries as functions of $\omega_0 / \Omega_i$ and $|E_{\text{m}}|^2$ for (a) SBW ($k_{Ei}^+ / k_n = -1$) and (b) EPM ($k_{Ei}^+ / k_n = -3$) dominant cases. $k_z / k_r = 10^{-5}$ and $2.5 \times 10^{-4}$ are used for interchange and drift mode cases, respectively. Other conditions are same as Fig. 1.

Figure 2 (a) is presented to investigate when the SBW effect is dominant ($k_{Ei}^+ / k_n + 2 > 0$). As can be deduced from Eq. (4), both interchange and drift instabilities are suppressed by SBW-rf interactions when $\omega_0 / \Omega_i$ is lower than 1. On the other hand, for the EPM dominant case ($k_{Ei}^+ / k_n + 2 < 0$), the result shows opposite behavior as shown in Fig. 2 (b). Comparison with low power case shows that the stabilizing force is strong at high power.

Figure 3 represents stability boundaries as functions of $\omega_0 / \Omega_i$ and $|E_{\text{m}}|^2$. When the SBW effect is dominant over the EPM, the stable regions appear in favor of $\omega_0 / \Omega_i < 1$, and vice versa. We found that the stable window for drift modes is narrower and more localized near resonance compared to the interchange modes.

In summary, we studied the influence of an applied rf wave on the dissipative low frequency instabilities in mirror plasmas. It was shown that drift instabilities can be stabilized by the rf wave in a similar fashion to interchange modes.

References