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Analysis of Dissipation in MHD Turbulence Simulations in Two and Three Dimensions

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1 Introduction

Magnetohydrodynamic (MHD) flows display turbulent behaviour when the conditions $l_0 v_{rms}/\mu \gg 1$ and $l_0 v_{rms}/\nu \gg 1$ are satisfied. Here l_0 is the integral length scale of the flow, v_{rms} is the rms velocity, ν is the kinematic viscosity and μ is the magnetic diffusivity. Statistical investigations are frequently made by the construction of structure functions S_l^p, defined here in terms of the Elsässer field variables $\mathbf{z}^{\pm} \equiv \mathbf{v} \pm \mathbf{B}(\mu_0\rho)^{-1/2}$:

$$S_l^p = \langle |(\mathbf{z}^{\pm}(\mathbf{x}+\mathbf{l},t).\mathbf{l}/l - \mathbf{z}^{\pm}(\mathbf{x},t).\mathbf{l}/l|^p\rangle \tag{1}$$

This measure is sensitive to spatial correlation via dependence on the differencing vector $\mathbf{l}$, and is weighted to events of greater intensity as the order parameter p increases. Statistical self similarity is indicated by the scaling law $S_l^p \sim l^{\zeta_p}$. This can be expected in the inertial range $l_0 \gg l \gg l_d$ where l_d is the dissipation scale. Here energy can cascade between length scales without loss of energy, because these scales are sufficiently far from the integral scale not to be affected by the driver, and are sufficiently far from the dissipation scale not to be affected by viscosity. Numerical constraints inhibit the appearance of an extensive inertial range in direct simulations, so the phenomenon of extended self-similarity (ESS),

$$S_l^p \sim (S_l^q)^{\zeta_p/\zeta_q} \tag{2}$$

is instead considered. While this is found to extend scaling into the dissipation range down to $\approx 5l_d$, only ratios of scaling exponents ζ_p/ζ_q can be directly measured. Scaling arguments derived from generalised dimensional analyses predict the scaling exponents ζ_p. However, these dimensional arguments cannot account for the spatially intermittent distribution of eddy activity which is found to exist in both hydrodynamic and MHD turbulence, so an intermittency correction is required. A currently favoured model for intermittent turbulence is that of She and Leveque [1] (SL) which relates the scaling exponents ζ_p to the geometry of those structures that are most intensely dissipating. The argument relies on the refined similarity hypothesis, which provides a relationship between the assumed statistical self-similarity in the local rate of disspation and that found in S_l^p. This hypothesis can be recast in terms of ESS as

$$S_l^p \sim \langle \epsilon_l^{p/3} \rangle \left(S_l^3 \right)^{p/3} \quad \text{or} \quad S_l^p \sim \langle \epsilon_l^{p/4} \rangle \left(S_l^4 \right)^{p/4} \tag{3}$$

after Eq.(12) in [2]. The first example follows Kolmogorov (K41) [3] in that the non-linear interaction dominating the cascade process is eddy scrambling, while the second follows Iroshnikov and Kraichnan (IK) [4, 5] where the dominant process is Alfven wave collisions. Both relations imply ESS in the local rate of dissipation: this was recently recovered for the 3D incompressible MHD decaying turbulence simulation of Biskamp and Müller, and was found to agree with the SL model [6]. However it was not possible to verify Eq.(3), which links dissipative and fluid scaling, to high order. The present work investigates this relationship. We study driven two dimensional (2D) turbulence, because higher Reynolds numbers are attainable in 2D simulations, compared to 3D, for given computational resources. These simulations are physically relevant to plasma turbulence where the perturbed magnetic field is small compared to the mean magnetic field, and is essentially perpendicular to it. We drive the simulations so as to maximise our ability to harvest statistics over many eddy turnover times. However, we note that driven simulations, as opposed to decaying ones, usually require longer run times to harvest reliable statistics because of their more bursty nature. The 2D isothermal MHD equations are solved on a periodic square grid using a finite difference method which is sixth order in space and fourth order in time. Fourier modes of the magnetic and velocity fields are initialised with random phase and a power spectrum peaking at the driving wave number k_0 taking the same form as the initial conditions in [7]. The simulation is driven in the wave number range $k_0 \pm 0.5$ using a forcing function which forces shear magnetic and velocity Fourier modes at random phase. A mode is selected randomly within the shell $k_0 \pm 0.5$ for driving at each time step. Simulations presented here have a resolution of 1024^2 grid points with $k_0 = 4$. Once the magnetic and kinetic energies have reached quasi equilibrium, statistics are gathered over a period of one hundred eddy turnover times from snapshots separated by one eddy turnover time. Here we define an eddy turnover time as $1/(k_0 \overline{E_T}^{1/2})$, where $\overline{E_T}$ is the space averaged steady state magnetic plus kinetic energies. The average Mach number of the flow, once in quasi equilibrium, is ≈ 0.25 and the values of kinematic viscosity and magnetic diffusivity are set equal to each other.

2 Results

Figure 1 shows the power spectrum of the z^+ and z^- Elsässer field variables (labelled E^+ and E^- respectively). Both spectra overlie perfectly, suggesting that the overall alignment of v with B is small. This is desirable since a high degree of field alignment

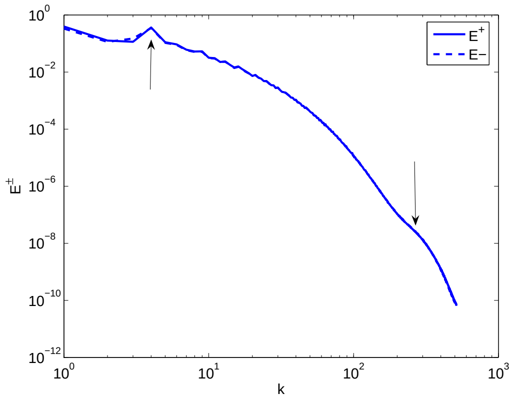

Fig. 1: Power spectra of the z^+ and z^- Elsässer field variables (labelled E^+ and E^- respectively). Marked on the spectra are the driving shell at $k_0 = 4$ showing enhanced power, and the Kolmogorov dissipation scale where the power falls off rapidly.

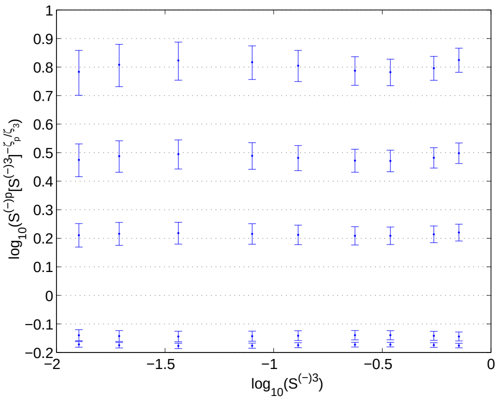

Fig. 2: ESS obtained from z^- Elsässer field variables orders $p = 6$ to $p = 1$ top to bottom (excluding $p = 3$ since this necessarily gives perfect scaling). The y axis is normalised by ζ_p/ζ_3 obtained from linear regression on a log-log plot. Error bars show the standard error present in the time averaging process.

kills the turbulent dynamics. Marked on the spectra are the driving shell at $k_0 = 4$ showing enhanced power, and the Kolmogorov dissipation scale where the power falls off rapidly. Structure functions S_l^p are constructed from the Elsässer field variables, and ESS is used to extract the ratio of scaling exponents ζ_p/ζ_3 in line with Eq.(2). These scaling relations are shown in Fig. 2, where perfect self-similar scaling would appear as a horizontal line. Errors are estimated as the standard error in the time averaging process. Standard procedure is to compare these ratios of scaling exponents to a model of turbulence. Currently favoured is the SL model mentioned above, which relies on scaling relations of the form Eq.(3) for theoretical consistency. We are able to test these relations directly, and present preliminary results here.

We calculate $\langle \epsilon_l^p \rangle$ directly from our simulation using the gradient squared one dimensional proxy as in [6, 8], and combine this with calculations of S_l^p in line with Eq.(3). Table 1 then shows the scaling relations found by evaluating Eqs.(3).The values in the table correspond to the gradient recovered when plotting the scaling relations Eq.(3) on a log-log plot as p is varied: an entry of 1 represents perfect agreement. Neither the K41 nor the IK variants of the refined similarity hypothesis fit our data perfectly, since a trend can be observed in the gradient as p is increased. However it is reasonable to place greater emphasis on our low order results, since we have greater statistical confidence in these. This suggests that the K41 variant of the refined similarity hypothesis is more appropriate for 2D MHD turbulence than the IK variant.

- -	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$
K41 +	1.02	1.01	—	0.97	0.93
K41 $-$	1.02	1.02	—	0.96	0.91
IK +	1.12	1.10	1.06	—	0.94
IK $-$	1.14	1.11	1.07	—	0.93

Table 1: Test of the refined similarity hypothesis as modified for consistency with extended self-similarity. IK or K41 refers to the versions of this hypothesis Eq.(3). The symbols + or $-$ represent scaling derived from the z^+ and z^- Elsässer field variables respectively. Perfect agreement would be indicated by a value of 1.0 across all columns

3 Conclusions

We have extended our investigation [6] of the self-consistency of an SL interpretation of the scaling of the Elsässer field variables for MHD turbulence. Development of a 2D driven numerical simulation has now enabled us to address the combined scaling properties of dissipation and field variables. We find that the ESS-adjusted refined similarity hypothesis Eq.(3) can be tested directly using this simulation. Neither the Kolmogorov (K41) or Iroshnikov-Kraichnan (IK) phenomenologies of this hypothesis agree with our numerical results perfectly. However the low order investigations, for which statistical confidence levels are higher, are better approximated by the K41 variant of the hypothesis than by the IK version. Some previous high resolution simulations of 2D MHD turbulence report [7] that the scaling exponent of the third order structure function ζ_3 has a value considerably less than 1, ($\zeta_3 = 1$ is a necessary condition of K41). Biskamp and Schwartz report that the power law index of the energy spectrum is very close to $-3/2$ which suggests that the IK phenomenology is more appropriate for 2D MHD turbulence [7]. The Reynolds numbers we have simulated so far are not high enough for direct tests of these scaling laws, which do not rely on ESS. A satisfactory SL model for the scaling exponents ζ_p has not yet been agreed upon for 2D MHD turbulence, and perhaps the theoretical divergence uncovered here may contribute to this. Definite conclusions will require simulation runs at resolutions sufficiently high that non-ESS scaling relations can be evaluated.

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