Measurement of local electrical conductivity and thermodynamical coefficients in JT-60U

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1. Introduction

Transport properties of high temperature plasma parallel to the magnetic field such as electrical conductivity and bootstrap current were found to be consistent with neoclassical theory as reviewed by M. Kikuchi and M. Azumi [1]. But, those observations rely on the agreement of global measurement of surface voltage as compared with time dependent simulation or quasi-stationary resistive loop voltage [2],[3]. Progress of motional Stark effect (MSE) measurements in tokamaks enables local measurement of transport coefficients parallel to the magnetic field [4] and local beam and EC driven current profiles were measured and compared with theory in JT-60U [5, 6]. Local bootstrap current profile was measured and compared with theory in bootstrap current dominated discharge with sustained duration less than current relaxation time $\tau_R$ in JT-60U [7]. More recently bootstrap current dominant discharges are produced in JT-60U [8]. In this paper, methods of numerical evaluation of local transport coefficients and its comparison with MSE measurement are summarized and typical comparisons are shown.

2. Method of Equilibrium Analysis

In the tokamak geometry, plasma equilibrium follows so-called Grad-Shafranov equation, $L\psi=-\mu_0 RJ_\phi(R,\psi)$ where $L=R\partial/\partial R(1/R\partial/\partial R)+\partial^2/\partial Z^2$. Equilibrium code MEUDAS for JT-60U solves this equation fitted to the measurements of poloidal fluxes at 15 poloidal locations, poloidal magnetic field $B_\theta$ at 12 poloidal locations and radial magnetic field $B_r$ at 15 poloidal locations, and up to 21 MSE angles (12 for this study). Here, the current profile is parametrized as $J_\psi(R,\psi)=RdP/d\psi+\mu_0FdF/d\psi=J_0[R\rho^2+(1-\rho^2)R\rho^2/R]Y(\psi)^1$, where $Y(\psi)$ is the 3rd order spline function of $\rho$ ($\rho$ is normalized radius $(V(\psi)/V_s)^{0.5}$ where $V$ is volume inside $\psi$ surface and $V_s$ is a volume inside the separatrix). Functional form of $P$ and $F$ can be treated separately. But it did
not affect the result, so far. Magnetic fitting MEUDAS code provides flux surface averaged $J_\psi (J_\psi = \langle J_\psi / R \rangle / \langle 1 / R \rangle)$ as a function of $\rho$. The toroidal plasma current $J_\psi (R, \psi)$ is related with parallel current $\langle B \cdot J \rangle$ by,

$$J_\psi (R, \psi) = B_\phi \langle B \cdot J \rangle / \langle B^2 \rangle - [1 - B_\psi^2 / <B^2>] \cdot R \cdot d\psi$$  \hspace{1cm} (1)

A generalized Ohm’s law $\langle B \cdot J \rangle = \langle B \cdot J \rangle_{bs} + \langle B \cdot J \rangle_{NBRF} + \sigma_{eff} <B \cdot E>$ is satisfied for a parallel current $\langle B \cdot J \rangle$. Here, $\langle B \cdot J \rangle_{bs} = -F(\psi) \cdot n_e \cdot \Sigma(T_e / |Z|) \cdot [L_{31} \cdot \partial \ln \rho \cdot \partial \psi + L_{32} \cdot \partial \ln \rho \cdot \partial \psi] \cdot \langle B \cdot J \rangle_{NBRF} = \Sigma(\tau_{ab} / m \cdot n_e)$ is bootstrap current driven by thermodynamic forces, $\langle B \cdot J \rangle_{bs} = \langle B \cdot J \rangle_{NBRF} = \langle B \cdot J \rangle_{NC} = \Sigma \cdot e \cdot n_e (M - L)^{-1} \cdot \tau_{ab} \cdot \tau_{bb} / m_b$ is neoclassical electrical conductivity, respectively. And $M$ and $L$ are parallel viscous and friction matrices normalized by $\tau_{ab} / m_b \cdot n_e$. Using Eq. (1), the flux surface averaged toroidal current density $J_\psi (= \langle J_\psi / R \rangle / \langle 1 / R \rangle)$ from this generalized Ohm’s law consists of 4 terms as, $J_\psi = J_{OH} + J_{bs} + J_{CD} + J_{VP}$, where $J_{OH} = \langle B \cdot J \rangle_{bs} / \langle 1 / R \rangle / \langle B^2 \rangle$, $J_{bs} = \langle B \cdot J \rangle_{bs} / \langle 1 / R \rangle / \langle B^2 \rangle$, $J_{CD} = \langle B \cdot J \rangle_{CD} / \langle 1 / R \rangle / \langle B^2 \rangle$, $J_{VP} = -\langle B_p^2 \rangle / \langle B^2 \rangle / \langle 1 / R \rangle$. Here, flux surface averaged toroidal electric field $\langle B \cdot E \rangle = \langle B \cdot E \rangle / \langle \partial \psi / \partial t \rangle_b$ is calculated using the identified $\psi$ evolution ($\Phi$ is toroidal flux inside the magnetic surface). Change in toroidal flux is small in flat top phase and $\partial \psi / \partial t$ is almost equal to $\partial \psi / \partial \tau$ at fixed location for typical JT-60 experimental conditions. These quantities are evaluated in ACCOME code based on MI method described in [1] using measured temperatures ($T_e(\rho)$: ECE, Thomson), density ($n_e(\rho)$: Thomson), effective charge ($Z_{eff}(\rho)$) profiles.

Evaluated flux surface averaged toroidal current density $J_\psi (= \langle J_\psi / R \rangle / \langle 1 / R \rangle)$ is compared with that from magnetic fitting MEUDAS code for various experimental conditions to check the validity of neoclassical theory for Generalized Ohm’s law.

### 3. Local Evaluation of Neoclassical Resistivity Enhancement Factor

The electrical conductivity of tokamak $\sigma_{eff} = \Sigma \cdot e \cdot n_e (M - L)^{-1} \cdot \tau_{ab} \cdot \tau_{bb} / m_b$ is strongly influenced by the existence of trapped particle. The circulating particle fraction $f_c$ is expressed as, $f_c = (3/4) \cdot B^2 / \lambda \cdot \partial \psi / \partial \lambda / \langle (1 - \lambda \cdot B)^{0.5} \rangle$ where $\lambda$ integral is 0 to $1 / B_{max}$. Figure 1 shows typical $T_e$, $n_e$ profiles, radial profiles of effective toroidal electric field and “measured” current profile and “calculated” current profiles with and without NC correction to electrical conductivity. The $Z_{eff}$ profile is also measured by multi-channel bremsstrahlung emission measurement showing flat profile ($Z_{eff} = 1.42$) towards the edge.
So, we can check the effect of trapped particle correction to the electrical conductivity locally. The comparison supports the existence of trapped particle correction to electrical conductivity.

4. Bootstrap Current

The bootstrap current is calculated in ACCOME/TOPICS codes by solving parallel momentum and heat momentum equations as follows,

\[
\sum_{a} \begin{bmatrix}
    \mu_1^a \\
    \mu_2^a 
\end{bmatrix} \left\langle Bq_{//a} \right\rangle = \left\langle Bq_{//b} \right\rangle \frac{2}{5P_b} - \left\langle Bq_{//a} \right\rangle \frac{2}{5P_a} \left\langle Bq_{//b} \right\rangle - \left\langle Bq_{//a} \right\rangle F_{e_a} T_a \partial \ln P_a / \partial \psi
\]

where \( \mu_1, \mu_2, \) etc are normalized friction and viscous coefficients. Solving above equation, bootstrap current is calculated as,

\[
\left\langle BJ_{//b} \right\rangle_{bs} = \sum_{a=e,l,f} e_a n_a \left\langle Bq_{//a} \right\rangle = F(\psi) \sum_{a=e,l,f} e_a n_a \sum_{b=e,l,f} \frac{T_{e_b}}{e_b} \left\{ [(M-L)^{-1} M_{ab}] \frac{\partial \ln P_{e_b}}{\partial \psi} - [(M-L)^{-1} M_{ab}] \frac{\partial \ln T_{e_b}}{\partial \psi} \right\}
\]

Thus bootstrap coefficients \( L_{31}, L_{32} \) are determined from \((M-L)^{-1} M\) matrix components. High bootstrap current fraction up to \( f_{bs}=75\% \) (calculated) was sustained for 7.4s in RS ELMy H-mode (3.4T, 0.8MA, \( q_{95}=8.3, \delta=0.42 \)) whose discharge waveform is shown in Figure 2. Toroidal current profile identified by magnetic fitting code MEUDAS is compared with the calculated current profile using neoclassical theory and measured kinetic profiles in Figure 3. There is a relative shift of the current profile between measurement and calculation in \( \rho=0.5-0.8 \) region. But the peak value is in good agreement within 10\%. This agreement between theory and measurement tells us \( L_{31} \) of 0.3-0.4 is consistent with experiment. As we calculate from NC theory, \( dT/d\rho \) term is small compared with \( dP/d\rho \) term since \( L_{32} \) coefficients for electron and ion are \( \sim 0 \).

References

[8] Y. Sakamoto et al., This conference.
Figure 1 $T_e$, $n_e$ profiles (top) current density and toroidal electric field profiles. (middle) and comparison of measured and calculated current profiles with and without NC correction to electrical conductivity.

Figure 2 Discharge waveform of high bootstrap current discharge in JT-60U..

Figure 3 $T_e$, $T_i$, $n_e$ profiles and comparison of calculated and measured current profiles and effective toroidal electric field $E^\text{eff}_\phi$.

Figure 4 Radial profile of theoretical bootstrap current coefficients $L_{31}$ and $L_{32}$ for high bootstrap current fraction discharge.