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Study of nonlinear phenomena in a tokamak plasma using a novel Hilbert transform technique

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Experimental time series data from fusion devices like tokamaks and stellarators display a very rich structure indicative of a variety of intrinsic nonlinear activity in their plasmas. Some of these nonlinear phenomena are of a non-stationary nature and their characteristic features are best described in terms of instantaneous amplitudes and frequencies. Most conventional analysis tools employed in studying these phenomena are however based on the assumption that the experimental time series is the result of a sum of a finite number of linear modes with time independent amplitudes and frequencies. The widely employed Fourier- or wavelet-transform based techniques belong to this category. Of these two techniques, the wavelet-based method is slightly superior in the sense that the basis functions employed therein are localized in time. However the choice of the wavelet form is a highly subjective one and can significantly influence the analysis. In this work we consider an alternate technique that is based on a Hilbert transform [1] and apply it to the analysis of tokamak data.

The Hilbert transform (HT) of a time series, $X(t)$, can be defined as

$$Y(t) = \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{X(t')}{t-t'}dt'$$

where, P is the Cauchy principal value of the integral. All frequency components of $Y(t)$ are delayed by $\pi/2$ in phase and hence are orthogonal to $X(t)$. The analytic signal, $Z(t)=X(t)+jY(t)$ allows an appropriate definition of instantaneous amplitude, $A(t)=(X^2+Y^2)^{1/2}$, also known as the envelope, and instantaneous phase $\theta(t)=\tan^{-1}(Y/X)$. The instantaneous frequency (ω) is the local slope of $\theta(t)$. In principle one can directly work with the HT of raw data and examine its spectral properties. However since experimental data is likely to contain noise it is essential to work with a set of intrinsic mode functions (IMF) that are derived from the data and that preserve the essential nonlinear characteristics of the data. Such a method of extraction has been developed in Ref.[1] and is known as the empirical mode decomposition (EMD) technique. The Hilbert transforms of IMFs yield intrinsic amplitudes and intrinsic frequencies and the result can be presented in the form of a Hilbert-Huang spectrum, $H(t,\omega)$.

However such a spectrum violates the uncertainty principle constraint – essentially because the spectrum *H(t,ω)* has arbitrary resolution in time and frequency. Consequently the error in the spectral estimate can be 100%. It is more meaningful to consider time-averaged quantities. In this paper we consider one such quantity namely the degree of non-stationarity that is an important physical feature of tokamak data especially during start-up and termination phases of the discharge. The degree of non-stationarity is defined as,

$$DNS(\omega) = \frac{1}{T}\int_0^T \left[1 - \frac{H(t,\omega)}{h(\omega)}\right]^2 dt$$

Here, *h(ω)* is the mean marginal spectrum which is defined as the time-average of the Hilbert spectrum [1]. The DNS is a measure of the deviation of the Hilbert spectrum from the mean marginal spectrum. The latter is similar and comparable to the Fourier spectrum of the time series. The error (ε_{DNS}) is estimated as the counting error of the integrand.

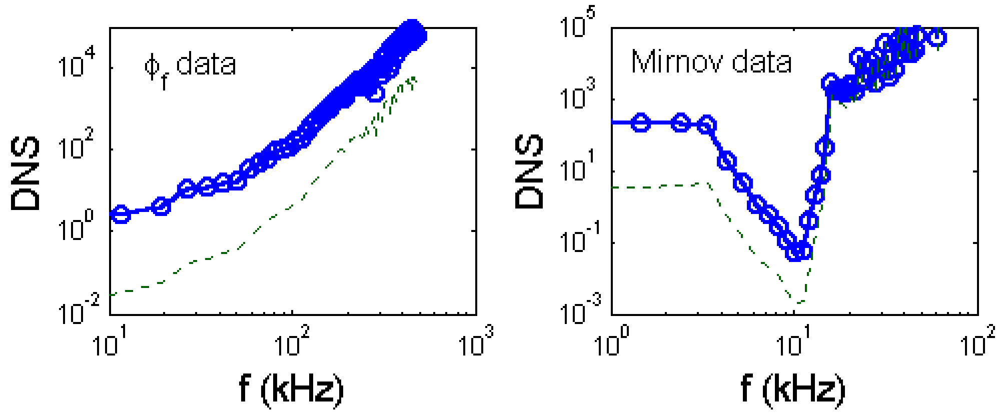

Figure 1: The degree of non-stationarity for the potential fluctuation data (left) and the Mirnov coil data (right). The dotted line indicated 1σ error in the estimate of DNS

We determine the degree of non-stationarity of two time-series, one representing the electrostatic fluctuations of the floating potential in the edge plasma of ADITYA tokamak and the other representing the fluctuations of the poloidal magnetic field as measured by a Mirnov coil. The potential fluctuations are recorded at a sampling frequency of 1 MHz whereas the Mirnov coil data are recorded at 125 kHz. Figure 1 shows the degree of non-stationarity (DNS) of the floating potential fluctuation (left) and the Mirnov coil (right) data. It is observed that the DNS increases with increasing frequency for the potential data. This is expected if the higher frequency components of the signal are more intermittent [2]. The error in the estimates of DNS also increases with frequency, making DNS estimates less reliable (i.e., not above 3σ level) at very high frequencies. Thus, frequency components in the range 20-200 kHz are non-stationary. The Mirnov oscillation data shows a non-stationary envelope (i.e., varying RMS amplitude). This is reflected in the high DNS values at low frequencies. The signal also has a coherent oscillation at about 10 kHz. The degree of non-stationarity has a dip of two orders of magnitude at this frequency. The DNS of higher frequency component, however, is not beyond the experimental error.

We next discuss an extension of the HT technique to explore the occurrence of `non-linear mode-coupling' events in a turbulent plasma. Since the IMF components are represented as, $Z_i(t)=A_i(t)e^{j\theta i(t)}$,the interaction among the IMF components can be studied by evaluating the IMF coherency factor, namely,

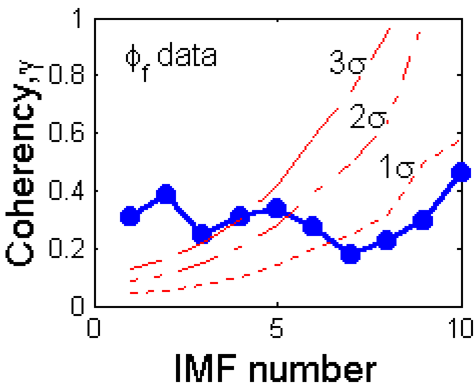

Figure 2: Coherency of potential fluctuation data. Frequencies of the first six mode numbers are 200 kHz, 120 kHz, 65 kHz, 39 kHz, 22 kHz and 11 kHz respectively.

$$\gamma_i = \frac{\left\langle Z_i^* Z_{i+1} Z_{i+2} \right\rangle}{\left\langle A_i A_{i+1} A_{i+2} \right\rangle}$$

where, the angular bracket represents time averaging. It should be noted that if the instantaneous phases, θ_i, θ_{i+1} and θ_{i+2} are random, the time averages would yield $\gamma_i=0$. On the other hand, if there is an interaction such as $\theta_i=\theta_{i+1}+\theta_{i+2}$, the coherency $\gamma_i=1$. We have estimated the coherency factor for three types of test signals: (a) white Gaussian noise, Wgaus, (b) coloured Gaussian noise with 20 μs correlation time, Zcor20, and (c) a coherent mode signal having $\theta_1=\theta_2+\theta_3$ type of phase coupling, Cohmode. We find that the IMF coherency factor is able to effectively differentiate between the random phase mixed data and Cohmode data and to efficiently detect the triplet interaction in Cohmode data. The error ε_γ is estimated by counting the number (M) of the largest wave period in the triplet, $\varepsilon_\gamma=1/\sqrt{M}$. We, then, apply this technique to the experimental signals for floating potential fluctuations and Mirnov oscillations. Figure 2 shows the result for potential fluctuation data. It is observed that γ_i in the three lowest IMFs are larger than the 3σ error and hence statistically significant. Significant coherencies are also observed in the frequency range 20-200 kHz, and hence we conclude that higher frequencies are generated by triplet interaction of lower frequencies. At lower frequencies, however, the statistical error ε_γ

becomes large and hence estimates of IMF coherency are not reliable. Figure 3 shows similar results for Mirnov coil data, in two separate time segments. The first time-segment (0-24 ms) represents the rise time of discharges and the second (24-51.2 ms) represents the flat top of the discharge. It is observed that coherencies are not significant at 3σ level of confidence except for the third mode (4.4 kHz).

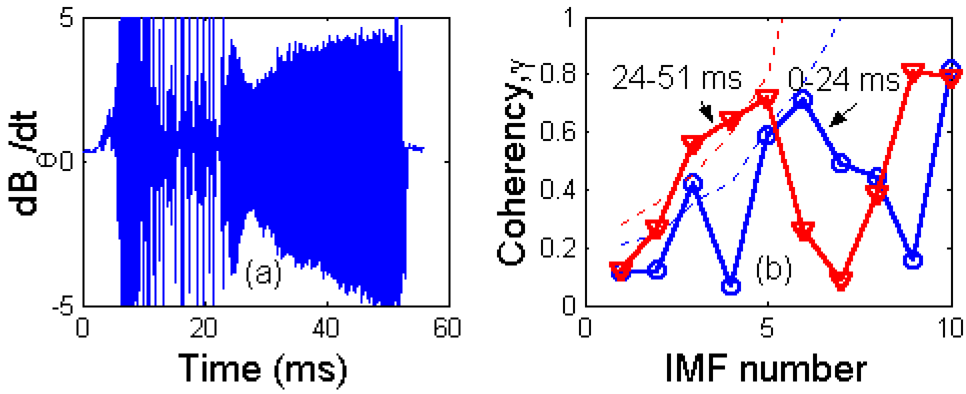

Figure 3: (a) The time series of Mirnov oscillation, and (b) phase coupling during the current rise (bullets, 0-24 ms) flat top (inverted triangle, 24-51 ms) time. The frequencies of the first six modes are 10 kHz, 7 kHz, 4.4 kHz, 2.7 kHz, 1.7 kHz and 1 kHz respectively. The dotted lines are 3σ errors in the estimates for the two cases.

Interestingly, there is no phase coupling for the most dominant (m=2, 10 kHz) mode. No physical meaning to phase coupling at mode 3 is possible at present. In conclusion, it is observed that triplet interaction among high frequencies and that among low frequencies characterise potential fluctuation in the edge plasma. Magnetic fluctuation does not exhibit triplet interaction.

Reference:

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[2] R. Jha *et al.*, "Perspectives of intermittency in the edge turbulence of fusion devices" In *Nonequilibrium Phenomena in Plasmas*, (Ed. A. S. Sharma and P. K. Kaw), Published by Springer , Dordrecht, The Netherlands, 199-218 (2005).