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Zero Dimensional Model for Transport Barrier Oscillations in Tokamak Edge Plasmas

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Abstract. Transport barriers at the plasma edge are key elements of high confinement regimes in fusion devices. In typical configurations, such barriers are not stable but exhibit quasi-periodic relaxation oscillations. In this work, a zero-dimensional model for such oscillations is presented describing the non linear dynamics of mode amplitudes. The relevant modes are determined by applying a proper orthogonal decomposition to the results from three dimensional turbulence simulations with a transport barrier generated by an imposed shear flow[4]. It is found that the relevant modes depart from linear modes. This leads to a zero dimensional model which reproduces barrier oscillations.

High confinement regimes in thermonuclear fusion devices are characterized by the appearance of a transport barrier at the plasma edge . Such barriers are regions where the turbulent flux of particles and energy is reduced by a sheared rotation of the plasma. As a consequence of the local flux balance, the density, temperature and pressure profiles steepen in these regions. In the most promising operational regime of future fusion reactors, the transport barrier is not stable but exhibits quasi-periodic relaxation oscillations. A relaxation is characterized by an increase of turbulent transport through the barrier and a decrease of the pressure inside the barrier. Currently, transport barrier relaxations are modeled by phenomenologically constructed dynamical equations for the amplitudes of relevant modes [2].

Here we propose 1D and 0D models based on a fluid description of the plasma. We start from 3D turbulence simulations with a transport barrier generated by an imposed $E\times B$ shear flow. In this simulations barrier relaxes quasi-periodically and, during such a relaxation a mode grows at the center of the barrier. A 1D model for the amplitude of this mode is derived from a 3D system of normalized reduced resistive MHD equations for the electrostatic potential ϕ and pressure p [4]. For this purpose, the pressure is decomposed into a mean profile $\bar{p}(r,t)$ and a perturbation $\delta p = \tilde{p}(r,t)e^{i(m\theta - n\varphi)}$ localized at the barrier center ($r = r_0$). The model is further simplified by using a linear relation between potential and pressure fluctuations, $\tilde{\phi} = ik_\theta/(\gamma_0 k_\perp^2)\tilde{p}$ with $k_\theta = m/r_0$ and $k_\perp$ representing the poloidal and perpendicular wave numbers. Here γ_0 is the linear growth rate in the presence of a mean pressure gradient κ and in absence of dissipation and $E\times B$ shear flow. The poloidal shear flow is assumed to have the form $\bar{u}_\theta = \partial_r\bar{\phi} = \omega_E(r-r_0)$.

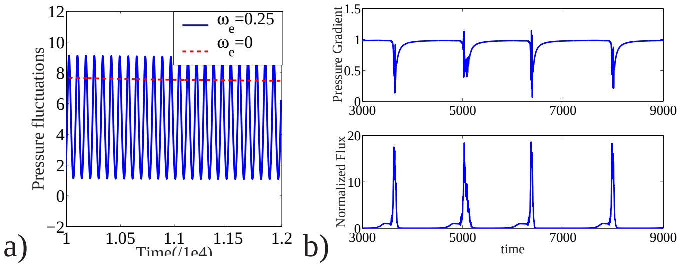

Figure 1: a)Time evolution of the pressure fluctuations $\sqrt{\int \tilde{p}^2\,\mathrm{d}r}$ with and without frozen shear flow b)Time evolution of pressure gradient $[|\partial_r \bar{p}|/(\Gamma_{tot}/\chi_\perp)]$ and turbulent flux $\left[2\gamma_0\,|\tilde{p}|^2/\Gamma_{tot}\right]$ at the barrier center observed from the 1D model (1-2)

The evolution equations for the pressure become,

$$\partial_t \bar{p} = -2\gamma_0\partial_x\,|\tilde{p}|^2 + \chi_\perp\partial_x^2\tilde{p} + S \quad (1)$$

$$\partial_t \tilde{p} = \gamma_0\,(-\partial_x\bar{p} - \kappa_0)\,\tilde{p} - i\omega_E' x\tilde{p} - \chi_{\|}x^2\tilde{p} + \chi_\perp'\partial_x^2\tilde{p} \quad (2)$$

where $x = r - r_0$, $\omega_E' = k_\theta\omega_E$, $\chi_\perp' = k_\theta\omega_E$, and $\kappa_0 = k_\theta^2\chi_\perp/\gamma_0$. The system is driven by a source $S(r)$ modelling the total flux $\Gamma_{tot} = \int S\,\mathrm{d}r$ In absence of shear flow ($\omega_E = 0$), the system evolves to a stationary state. With increasing $\omega_E > 0$, the system first shows regular oscillations (Fig. 1a) and then reproduces relaxation oscillations (Fig. 1b). The mechanism for relaxations oscillations is as follows. During a quiescent phase, the pressure gradient increases on a slow timescale. When crossing the linear instability threshold fluctuations start growing rapidly and are stabilized by the velocity shear only after a time delay of the order $\tau = \left(\frac{1}{4}\chi_\perp\omega_E'\right)^{-1/3}$ [4]. This essentially non linear mechanism reveals that the role of the velocity shear is different from a modification of the linear instability threshold. Indeed, if the coupling term with the shear flow is replaced by a shift of the instability threshold, no relaxations oscillations are observed except if the instability term is further modified (e.g. by a Heavyside function [2]).

We now derive a system of amplitude equations (0D model), describing the evolution of the amplitudes of the relevant radial structures and reproducing main features of the differents states observed in the 1D model. For this purpose, the relevant radial structures are determined by applying a proper orthogonal decomposition (POD) method [1] to a spatio-temporal signal obtained from the 1D model. The data are converted to an $M \times N$ matrix P_i^j in which columns correspond to time series. With the POD, the matrix is decomposed into a set of modes A_n, P_n which are orthogonal in space and time,

$$P_i^j = P(x_i, t_j) = \sum_n W_n A_n(t) P_n(r) \quad (3)$$

These modes are sorted in a series of decreasing weight W_n.

In a first time, the POD will be applied to the system when the frozen shear flow is absent. In that case, the dynamics can be described using only one mode for the perturbed part of the field:

$$\bar{p}(r,t) = -\kappa(r-r_0) + a_0(t)p_0(r), \tag{4}$$

$$\tilde{p}(r,t) = a_R(t)p_R(r) \tag{5}$$

The modes p_0 and p_R obtained from the POD can be approximated by the following analytic expression :

$$p_0 = \beta x \exp(-2ax^2) \quad p_R = \alpha \exp(-ax^2) \quad \text{with } a = \sqrt{\frac{\chi_\parallel}{\chi_\perp}} \frac{k_\theta}{2} \tag{6}$$

and the normalization constants α, β are such that $\int_{-\infty}^{\infty} p_0^2 \, \mathrm{d}x = \int_{-\infty}^{\infty} p_R^2 \, \mathrm{d}x = 1$.

The projection of the 1D model equations onto these modes leeds to the following evolution equation for the amplitude :

$$\partial_t a_0 = -3\gamma_s a_0 + \delta_0 (a_R)^2 \tag{7}$$

$$\partial_t a_R = (\Gamma_0 - \frac{\delta_0}{2} a_0) a_R \tag{8}$$

with : $\gamma_s = k_\theta \sqrt{\chi_\parallel \chi_\perp}, \quad \kappa_0 = k_\theta^2 \chi_\perp / \gamma_0, \quad \Gamma_0 = \gamma_0(\kappa - \kappa 0) - \gamma_s, \quad \delta_0^2 = \frac{\gamma_0^2}{\sqrt{\pi}} \sqrt{\gamma_s/(2\chi_\perp)}^3$

The system evolves to a stable fixed point defined by : $a_0 = \Gamma_0/\delta$ and $a^R = 1/\delta\sqrt{3\Gamma_0\delta/2}$. The coupling between a_R and a_0 is not sufficient to produce oscillations. But, when the velocity shear is included in the system, the POD reveals that more radial modes play a role in the dynamics. However the steepness of the weight distribution suggests that the system can be described using the first modes only (Galerkin approximation)

$$\bar{p}(r,t) = -\kappa(r-r_0) + a_0(t)p_0(r)$$

$$\tilde{p}(r,t) = a_R(t)p_R(r) + i a_I(t) p_I(r)$$

The modes p_R and p_I can be approximated by :

$$p_R(x) = \alpha \cos(bx) \exp\left(-ax^2\right), \quad p_I(x) = -\zeta\alpha \sin(bx) \exp\left(-ax^2\right), \quad b = \frac{\omega_E}{2\sqrt{\chi_\parallel \chi_\perp}} \tag{9}$$

The normalization constants α and ζ are such that $\int_{-\infty}^{\infty} p_R^2 \, \mathrm{d}x = \int_{-\infty}^{\infty} p_I^2 \, \mathrm{d}x = 1$.

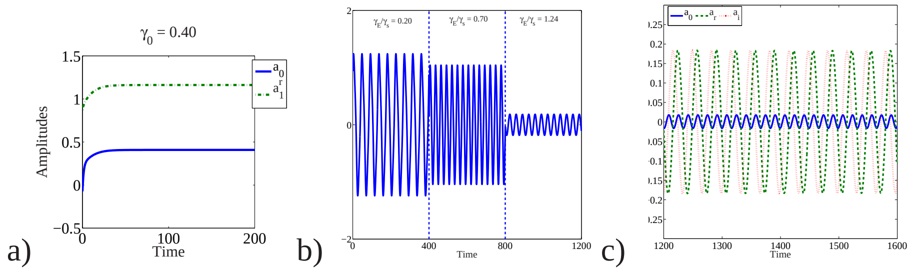

Figure 2: Dynamics of the mode in the absence of flow shear (a) Time evolution of the amplitudes a_0, a_R, and a_I for $\gamma_E/\gamma_s = 1.24$ (b). Dynamics of the perturbation amplitude a_R for three different values of the flow shear (c). The other parameters are $\gamma_s = 0.25$, $\gamma_0\,(\kappa - \kappa_0) = 1$, and $\delta_0 = 0.22$.

Now, the projection leeds to the system :

$$\mathrm{d}_t a_0 = -3\gamma_s a_0 + 2\delta a_R^2 + 2\delta' a_I^2, \tag{10}$$

$$\mathrm{d}_t a_R = (\Gamma - \delta a_0)\, a_R + \Omega \left(\frac{a_R}{\zeta} - a_I \right) \tag{11}$$

$$\mathrm{d}_t a_I = (\Gamma - \delta' a_0)\, a_I - \Omega\,(\zeta a_I - a_R) \tag{12}$$

with the coefficients : $\Gamma = \Gamma_0 - \gamma_E$, $\quad \Omega = 2\gamma_E \left[\exp\left(\frac{\gamma_E}{\gamma_s}\right) + 1 \right]^{-1}$, $\quad \zeta = \tanh^{-1/2}\left(\frac{\gamma_E}{2\gamma_s}\right)$,

$$\delta = \delta_0 \left[\exp\left(\frac{\gamma_E}{2\gamma_s}\right) + 1 + \frac{\gamma_E}{\gamma_s} \right] \cosh^{-1}\left(\frac{\gamma_E}{2\gamma_s}\right) \quad \delta' = \delta_0 \left[\exp\left(\frac{\gamma_E}{2\gamma_s}\right) - 1 - \frac{\gamma_E}{\gamma_s} \right] \sinh^{-1}\left(\frac{\gamma_E}{2\gamma_s}\right)$$

The system (10-12) reproduces oscillations due to the coupling between a^R and a^I. For low values of γ_E/γ_s, the oscillation frequency is found to increase with ω_E but for higher values of γ_E/γ_s, the frequency decrease. Note that $\tilde{p}$ is different from a linear mode. The linear mode corresponds to $a_R = \zeta a_I$, and in that case, the coupling between a_R and a_I disappears. The system becomes similar to the one obtained in the case without shear. Note also that in the system (10-12) the effect of ω_E is different from a shift of the linear instability threshold.

In conclusion we have derived a 0D model based on a 1D system which reproduces relaxation oscillations of transport barriers. The dominant radial modes are different from the linear modes. The dynamical system for the amplitudes of this modes (0D model) reproduces the frequency dependence with velocity shear. In particular, for large shear, the frequency decreases with shear. This can leed to a frequency decrease with heating power in the case where velocity shear increases strongly with heating power.

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