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Lévy distributions in plasma diffusion through a magnetic field

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Plasma transport across a confining magnetic field occurs by random motion resulting from turbulence. The true properties of this random motion are most usually ignored, however it is assumed to imply a brownian statistics and a diffusive process. The mean square motion would increase linearly with time, and the out flowing plasma flux is assumed to be proportional to the density gradient times a diffusion coefficient.

It is useful to directly observe the random plasma motion, but this cannot be simply obtained. The use of single or multiple probes gives only local information on the time fluctuating density or electric field. We like to show however that the signal obtained from collective light scattering, under proper observation conditions, provides the appropriate information. This is done by a twofold approach: an analysis of the relation between signal and the plasma motion statistics; an experiment that is conducted in a plasma discharge confined by a toroidal magnetic field, where a collective scattering device observes the plasma transport properties. This transport is found relevant to the class of "Lévy processes" [1, 2].

A collective scattering experiment consists in lighting the plasma by a laser beam. We use a far infrared, 10.6 microns, five watts DC single mode, CO_2 laser, with a 11 mm waist. The scattered light at a small angle from the incident beam direction ($1 < \theta < 3$ mrad) is received by a photo current diode and beats with a reference LO beam. In this way, the photo current j(t) is proportional to a complex space Fourier transform of the plasma density [3, 4]

$$j(t) = C^{st} \int_V dr^3\, n(r,t) \exp[- i\, k.r] \quad (1)$$

Two conditions are frequently met: when the motion at the scale of the inverse wave number k, is divergence free; and when the space distribution of the displacement Δ of a plasma element in a time τ, is independent from the density space distribution at the same scale. The current signal time auto correlation can then be written as [5]

$$C(\tau \mid k) = \langle j(t).j^*(t+\tau)\rangle = C_2\, S(k) \langle \exp[-ik.\Delta(\tau \mid r,t)]\rangle \quad (2)$$

the scattered light intensity is proportional to the "Form factor" at k-scale

$$S(k) = \langle n(k)\, n^*(k)\rangle / n_0 V \quad (3)$$

The time variation is that of the statistical "characteristic function" $\varphi(k \mid \tau)$ of the displacement Δ random distribution $P(\Delta \mid \tau)$

$$\varphi(k \mid \tau) = \langle \exp[-ik.\Delta(\tau \mid r,t)] \rangle = \int d\Delta\, P(\Delta \mid \tau) \exp(ik.\Delta) = (1/T)\int_T dt \exp[-ik.\Delta(\tau \mid r,t)] \quad (4)$$

In the familiar (but peculiar) case of a gaussian probability distribution $P_G(\Delta \mid \tau)$ with a variance $\delta^2(\tau)$, the "characteristic" is a gaussian function of k

$$\varphi_G(k \mid \tau) = \exp[-k^2\delta^2(\tau)/2] \quad (5)$$

The experiment is conducted in a plasma confined in a toroidal magnetic field (0.36 Tesla) of 0.6 and 0.1 meter mean and poloidal radius, respectively, the "ToriX" device. An Argon plasma is produced by a primary electron filament source. A Langmuir probe provides plasma density (10^{17} m^{-3}) and electron temperature (2eV) measurements. Very large fluctuations of the density (50% rms., in relative values) can be observed on the probe, in the kHz range of frequencies. This observation is common to similar devices [6]. When a weak vertical B field (1-2 mT) is added by use of two large horizontal coils in the Helmholtz positions, the plasma density fluctuation level is reduced [7] by a factor of three, and the spatial density distribution is almost uniform along a vertical axis in the poloidal plane, until the plasma edges. This is the plasma regime that has been investigated. The incident laser lights the plasma vertically across a poloidal plane, the probing collective scattering k vector is horizontal and directed inwards the torus.

The time correlation functions $C(t \mid k)$ of the detector current obtained for six different k values at 0.36T toroidal B-field intensity, are shown in Fig.1. These functions have been normalised by their value at time t=0. From that origin, the slope is horizontal (unlike an exponential) followed by a slow decay and a long tail, longer than a gaussian distribution; the smaller is the wave number, the longer is the characteristic time.

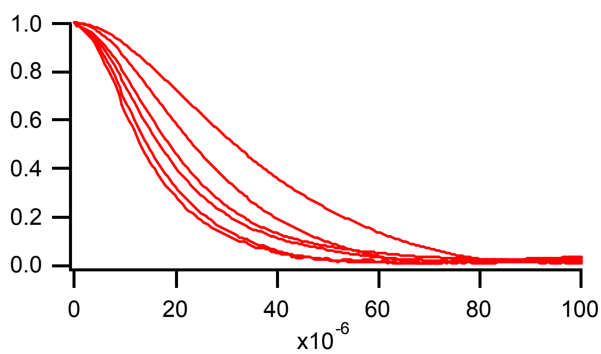

Fig.1: Auto correlation of the collective scattering signal as a function of time, for different wave number k. The correlation have been normalised to their amplitude at time t=0. k increases from top to bottom

It is interesting to look for an appropriate similitude between these auto correlations at different k's. At each time τ, we know these correlations should be the characteristic $\varphi(k|\tau)$. For the general class of "Lévy" conservative random process, these "characteristics" must be exponential functions of a power of k, k^{α}, where $0< \alpha < 2$:

$$\varphi(k) \cong \exp[-(d.k)^{\alpha}]. \qquad (6)$$

The experimental correlation of Fig.1 were thus re-normalised by a factor $\{\exp[-(d.k)^{\alpha}]\}$, for two different values of α, α=2 (Fig.2, left), and α=1 (Fig.2, right). α=2 is the gaussian process (cf. Eq. 5), and α=1 is an exponential characteristics that correspond to a Lorentz probability distribution. One sees the Lorentz distribution (α=1 Lévy exponent) shows a much better bunching of the different correlation.

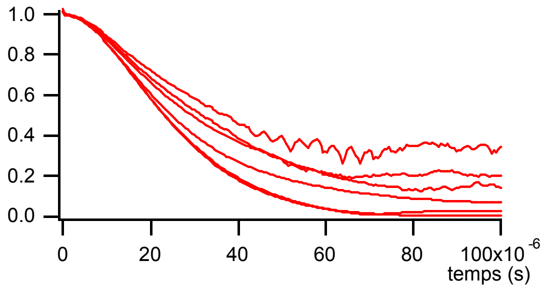

Fig. 2a k-Gaussian distribution normalised scattered E-field time correlation, for k ranging from 10^3 to 2.10^3 rad/m (bottom to top).

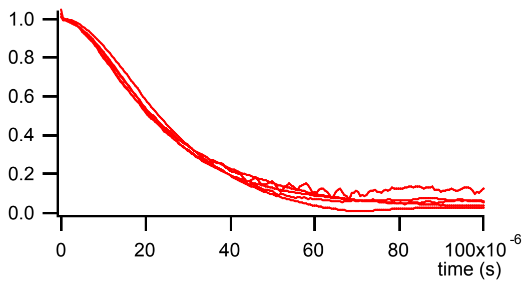

Fig. 2b k-Lorentz distribution normalised signal time correlation (same k's as on the left).

A more direct way to check the Lévy behaviour of the characteristics, is to plot, at any given time τ, the variation as a function of k. Namely we looked for a slope, if it exists, of Log{Log[C(k)]} as a function of Log(k) (see Eq. 2 and 6). A linear variation is indeed found, and the slope (i.e. the α Lévy exponent) is found to depend on the elapsed time τ, as shown in Table 1:

time (μs)	2.5	5	10	15	20	25	30	35	40
Lévy exponent	2	1.8	1.7	1.6	1.4	1.3	1.2	1.1	1.1

Table 1: Lévy exponent of the displacement distribution characteristics, at different times.

The α exponent decreases form 2 to 1.1 as τ increases. For times that are shorter than a correlation time, the plasma motion is almost rectilinear, and the displacement probability reflects that of the plasma (turbulent) velocities: The fact that α=2 indicates that not only the displacement but also the turbulent velocities are gaussian processes at short times. It means that a typical velocity exists, as proposed in Lévy walk model [8]. For longer times instead, the displacement probability is no more related to the velocity distribution; it steadily transforms into a Lorentz distribution.

The plasma transport in a simple toroidal magnetic field configuration without shear shows a random motion with very broad and long tailed probability distribution. Such distributions imply a very large transport rate across the confining B-field that cannot be accounted for by use of the familiar random diffusive model and the diffusion equation [9]. Collective scattering could be used on plasma fusion devices to check the occurrence of such plasma motion probability distribution.

References

1. R. Balescu, Phys. Rev. E 51, 4807 (1995).
2. J.Klafter, M.F.Shlesinger, G.Zumofen, "Beyond Brownian Motion", Physics Today, Febr. 1996, pp. 33-39.
3. E. Holzhauer and J. Massing, Plasma Phys. 20, 867 (1978).
4. R. Slusher and C. Surko, Phys. Fluids 23, 472 (1980).
5. N. Lemoine & D. Grésillon, "Plasma Diffusion across a Magnetic Field Observed by Collective Light Scattering : Experimental Evidence for Lévy Stable Distributions", (submitted for publication, March 2005).
6. K. Rypdal and S. Ratynskaia, Phys. Plasmas 11, 4623 (2004).
7. S. Müller, A. Fasoli, B. Labit, M. McGrath, M. Podestà, and F. Poli, Phys. Rev. Lett. 93 (2004).
8 M. F. Schlessinger, J. West, B. And J. Klafter, Phys. Rev. Lett. 58, 1100 (1987)
9. B. Van Milligen, R. Sánchez, and B. Carreras, Phys. Plasmas 11, 2272 (2004).