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Expansion of a finite-size plasma in vacuum

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INTRODUCTION

In the last ten years, the advent of laser systems operating in the multi-terawatt regime has renewed the interest in laser-plasma ion acceleration. This kind of acceleration processes has gained high interest also because of the wide variety of physical applications in which it is expected to play a significant role, e.g., hadrontheraphy [1], proton imaging [2] and inertial confinement fusion (ICF) [3]. In this framework the expansion process of the plasma produced by a laser impinging on a solid target has been the subject of several theoretical papers where semi-infinite [4] as well as finite [5], self-similar plasma models have been taken into account. Although both these two approaches may be useful when some peculiar feature is addressed, they include severe limitations. In a semi-infinite approach, because of the infinite amount of energy available, the electron temperature remains constant during the whole expansion process. In fact, each electron experiences a single collision with the ion front and never comes back. While this kind of approximation is certainly acceptable during the early phase of the expansion, it becomes quite rough already on a time scale of the order of few tens the time required by the thermal electrons to cross the target from one side to the other. In a finite-size self-similar approach instead a certain number of constraints have to be introduced, e.g., the charge distribution must depend on the velocity distribution. As the electric field intensity has a strong dependence on the charge profile, such approximation can easily affect the results. Moreover an assumption of quasi-neutrality must be introduced, in spite of the fact that a huge charge separation takes place at the ion front soon after the plasma formation. For these reasons we believe that taking into account both effects, i.e. the electron cooling and the non quasi-neutrality of the front region is of crucial role in order to provide a more detailed estimation of the energy the ions can acquire during the expansion of a plasma bunch. In the following an analytical model where these two features are included is presented . The analytical results are compared with numerical results obtained with a particle in cell (PIC) code and an overall good agreement is found.

ANALYTICAL MODEL

We now summarize the key points of the model for the expansion of a two component plasma with an hot electron population in vacuum that we discussed in Ref. [6]. The starting configuration is represented by an onedimensional, finite-size, globally charge neutral plasma of hot electrons and cold ions whose initial density profiles are equal and are given (for the sake of simplicity we assume that the ion charge is equal and opposite to that of the electrons) by $n_{e0} = n_{i0} = n_0\ \theta(1-|x|/a)$, where we have normalized lengths with respect to the initial electron Debye length $\lambda_{d,0} = \sqrt{T_{e0}/4\pi n_0 e^2}$, with T_{e0} the initial electron temperature, e the modulus of the electron charge, x the linear coordinate and a the plasma initial half-thickness. Because of the symmetry of the configuration under reflection with respect to the point $x = 0$, we will only be concerned with the expansion of the right half of plasma which initially occupies the region $x > 0$. As the electrons are hot, they start to expand and on timescales of the order of the electron plasma period $\omega_{pe}^{-1} = \left(4\pi n_0 e^2/m_e\right)^{-1/2}$, with m_e the electron mass, they reach an equilibrium configuration with the ions in which the pressure exerted by the former at the plasma-vacuum interface is balanced by the self-consistent electric field which sets up, and in such an equilibrium state the electrons are distributed according to a Boltzmann profile which, by measuring energies in units of T_{e0}, is given by $n_e = n_a e^{\phi}$, where $\phi = e\Phi/T_0$, Φ is the electrostatic potential satisfying Poisson equation and n_a the electron density at point $x = a$ where we set the zero of Φ. While the electric field in the region $x > a$ can be calculated analytically one has to solve the Poisson equation in the region $0 < x < a$ numerically, and by imposing the conditions that the electric field at the plasma-vacuum interface is continuous and that it is zero at infinity (which expresses charge neutrality of the plasma configuration) one obtains the value of n_a [6] which, for $a \gg 1$, i.e. for slabs much thicker than the Debye length, is equal to the value obtained in the case of semi-infinite plasma [4], given by $n_a = n_0/\mathrm{e}$, with e the Neper number[1]. An approximated analytical expression for the electrostatic potential in the region $0 < x < a$ can be obtained by defining a layer of thickness δ such that the plasma is taken to be locally charge neutral in the whole region $0 < x < a-\delta$, while in the layer $a-\delta < x < a$, where charge separation occurs, the electrostatic potential ϕ is approximated with a parabolic fit, given by $\phi = C_1(x-a) + C_2(x-a)^2/2$, while the parameters C_1, C_2 and δ are determined requiring that both the electric field and the electrostatic potential are continuous at points $x = a-\delta$ and $x = a$. One easily obtains $C_1 = -\sqrt{2/\mathrm{e}}$, $C_2 = C_1/\delta$ and $\delta = \sqrt{2\mathrm{e}}$.

In order to describe the time evolution of such an equilibrium configuration and the energy

[1] We observe that the electron density at the plasma-vacuum interface is nearly equal to n_0/e already for slabs of length $a \gtrsim 4$.

transfer between the electrons and the ions simplified profiles of the particle distributions are thus introduced. The ion profile depends on the time-dependent position of both the rarefaction front and of the plasma-vacuum interface, denoted with x_s and x_f, respectively. The former is initially located at the boundary between the neutral and the charged plasma region, placed at point $x = a - \delta$ and propagates towards the charge neutral region with the time dependent ion-acoustic velocity c_s which, by measuring masses in units of the ion mass m_i and time in units of $\omega_{pi}^{-1} = \left(4\pi n_0 e^2\right)^{-1/2}$, is given by $c_s = \sqrt{T_e}$, where T_e is twice the time-dependent average kinetic energy of the electron population, thus we have $x_s = a - \delta - \int_0^\tau c_s d\tau$, in adimensional units. The time evolution of the coordinate x_f of the ions placed on the plasma-vacuum interface is ruled by the the equation of motion given, in the adimensional units adopted, by $\ddot{x}_f = \dot{v}_f = E_f$, where E_f is the self-consistent electric field at the plasma-vacuum interface. The simplified ion density profile n_i adopted in paper [6] is obtained taking the ion density to be equal to the unperturbed density n_0 in the whole region which has not been reached by the rarefaction front yet, specified by $0 < x < x_s$, and to uniformly redistribute the ions laying between points x_s and a on the interval $x_s < x < x_f$. Therefore one has $n_i = n_0\theta(x_s - x) + n_0 D\theta(x - x_s) - n_0 D\theta(x - x_f)$, where $D = (a - x_s)/(x_f - x_s)$. The ion profile is qualitatively sketched in Fig. 1. The spatial distribution assumed for the electrons is such that the plasma is quasi-neutral in the whole region $0 < x < x_f - \delta$ apart from a small layer $[x_s - \Delta, x_s]$ where charge separation occurs, with Δ a parameter to be determined by imposing $-\int_{x_s-\Delta}^{x_s} E_{cs} dx = T_e[\phi(x_s) - \phi(x_s - \Delta)]$, Thus, by approximating the electric field E_{cs} in such layer with the field of a plane capacitor with surface charge density given by $\sigma = (1 - D)/4\pi$ one has $\Delta = -T_e(\ln D)/(1 - D)$. Assuming the electrons to be in hydrostatic equilibrium with the ions at every stage of the expansion process one obtains that the electron density at the plasma-vacuum interface n_f is related to the ion density in the region $x_s < x < x_f$ by relation $n_f = Dn_0/\mathrm{e}$.

The cooling of the electron population is determined by the reflections they undergo with the two potential barriers located in the region $[x_s - \Delta]$ (*internal barrier*) and $[x_f, +\infty]$ (*external barrier*), and taking such reflections to be elastic one obtains that the energy loss of an electron with velocity v is given by $\Delta U(v) = 2m_e(v - \bar{v})\bar{v}$, where $\bar{v}$ is the barrier velocity, given by $\bar{v} = -c_s, v_f$. Assuming the electrons to be distributed according to a Maxwellian with average energy $T_e/2$ the total electron energy is given by $U = n_0 a T_e/2$. Thus by taking T_e to be uniform and $c_s/v_{th} \ll 1$, where $v_{th} = \sqrt{T_e/m_e}$ is the electron thermal velocity one obtains the electron cooling equation, given by

$$\frac{1}{T_e}\frac{dT_e}{d\tau} = -(2/a)\left[Dv_f - (1 - D)c_s\right], \qquad (1)$$

which must be coupled to the equations describing the time evolution of both x_s and of x_f and finally integrated numerically.

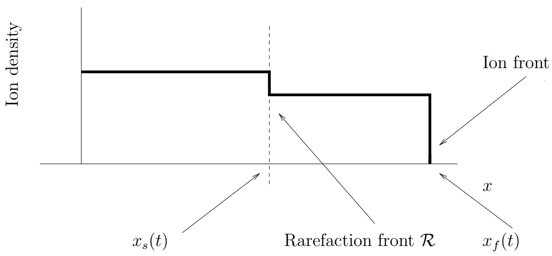

Fig. 1. Ion profile as expansion takes place.

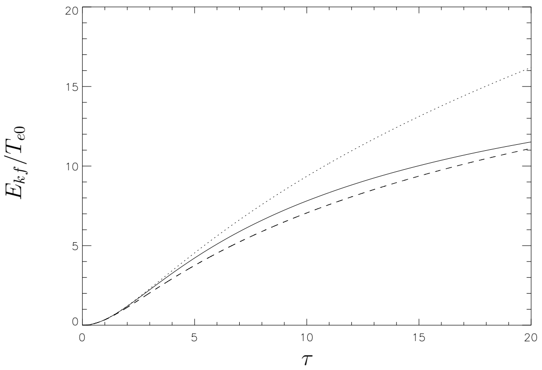

Fig. 2. Time evolution of the kinetic energy of the fastest ion obtained with the numerical simulations (solid line), with the analytical theory (dashed line) and with the semi-infinite model (dotted line). The plasma thickness is $2a = 50\lambda_d$.

Because of the ion front significant spread, the numerical value of E_f is smaller than the one obtained with the analytical model, and this can be corrected by introducing the reduction factor $1-(1-D)^{\alpha}$, with α a numerically fixed coefficient which is found to be equal to 1.4.

The analytical predictions regarding the time evolution of the kinetic energy of the fastest ion, which is identified with the ion placed at the plasma-vacuum interface, is compared in Fig. 2 with both the numerical and the semi-infinite results. It is worth to note that for plasma of sizes of the order of those available in present experiments the results obtained with our finite analytical scheme and the numerical simulations are significantly different from those predicted by the semi-infinite model already at early times .

References

[1] T. Zh. Esirkepov *et al.*, Phys. Rev. Lett. **89**, 175003 (2002)

[2] M. Borghesi *et al.*, Phys. Plasmas **9**, 2214 (2002).

[3] S. Atzeni and J. Meyer-ter-vehn, *The physics of inertial confinement fusion, beam plasma interaction, hydrodynamics, hot dense matter*, Clarendon press, Oxford (2004).

[4] A. V. Gurevitch *et al.*, JETP **21**, 449 (1966), J. E. Allen *et al.* J. Plasma Physics, **4**, 187 (1970), P. Mora, Phys. Rev. Lett. **90**, 185002 (2003).

[5] D. S. Dorozhkina and V. E. Semenov, Phys. Rev. Lett. **81**, 2691 (1998), V. F. Kovalev and V. Yu. Bychenkov, Phys. Rev. Lett. **90**, 185004 (2003).

[6] S. Betti *et al.* Plasma Phys. Contr. Fusion **47**, 521-529 (2005).