Statistical analysis of plasma edge fluctuations in MAST and comparison with BOUT simulation results

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Introduction

In recent years many statistical techniques have been applied to edge turbulence measurements from Langmuir probes and reflectometers in order to examine properties such as long-time correlations which differentiate between models of turbulence. Fluid codes have developed to the point where these statistical methods can be applied to their output and compared with experiment. The aim of the current work is to compare characteristics of edge turbulence from the BOUT code, written by X.Xu at LLNL [1], with measurements taken from the Mega-Amp Spherical Tokamak (MAST) [2] using some of these methods.

BOUT is a two-fluid Braginskii code which has a realistic geometry and includes closed and open field lines as well as the magnetic x-point [1]. In the radial-poloidal plane, the grid is obtained from UEDGE, based on an EFIT reconstruction. The code has sources of many ideal and non-ideal instabilities which can lead to turbulence including high-n ballooning, dissipative drift, shear Alfvén and peeling modes. Due to several specific features of spherical tokamaks, the time-step is smaller than for a conventional tokamak. This makes observing saturated turbulence over a long time in an ST quite challenging. See [3] for details of the BOUT simulation of MAST.

Results

BOUT simulations were run for a MAST single-null L-mode plasma using temperature and density profiles from Thomson scattering. A simulation was also run with double the density and half the temperature. The results are compared with data from MAST ohmic L-mode single-null plasmas with plasma currents of 700kA. Table 1 shows temperature and density values at the separatrix for these plasmas and simulations.

Fig. 1a shows an ion saturation current (\(I_{\text{SAT}}\)) signal from MAST taken at the outboard mid-plane with a reciprocating Langmuir probe. This shows intermittent bursts of varying sizes...
Table 1: Plasma parameters

<table>
<thead>
<tr>
<th></th>
<th>$T_{SEP}$ (eV)</th>
<th>$N_{SEP} \times 10^{18} m^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOUT Low density case</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>BOUT High density case</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>12679</td>
<td>25 ± 10</td>
<td>5 ± 1</td>
</tr>
<tr>
<td>12850</td>
<td>25 ± 10</td>
<td>7 ± 1</td>
</tr>
<tr>
<td>12851</td>
<td>25 ± 10</td>
<td>9 ± 1</td>
</tr>
</tbody>
</table>

Figure 1: Isat signals

The most widely used method of analysing turbulent signals is the Fourier power spectrum. The spectra from three MAST plasmas are shown in Fig. 2a, whilst spectra from the two BOUT simulations are shown in Fig. 2b. For both MAST plasmas and BOUT simulations, the power spectrum follows a power law for large frequencies, but the gradient of this fall-off is much larger for the BOUT data than for the MAST data. This indicates that small-scale structures present in experimental turbulence are suppressed in the simulation. This could reflect the absence of kinetic effects in BOUT which may give rise to the small-scale structure seen in experimental data. The effect of numerical dissipation will also be to damp high-frequency components. It is also possible that after only $\sim 200 \mu s$ the turbulence is not yet fully developed and so there is less energy in small scale structures than
would be the case in a longer run.

In order to analyse the behaviour of turbulent signals over different temporal and spatial scales, methods based on Hurst exponents have been applied to turbulence measurements from tokamaks and other complex systems [4, 5, 6]. Differencing and rescaling is one such method and involves calculating a set of probability distribution functions (PDFs) \( P(\delta x, \tau) \) of a quantity \( \delta x(t, \tau) = \sum_{i=t}^{t+\tau} (I_{SAT}(i) - \overline{I_{SAT}}) \) where \( \overline{I_{SAT}} \) is the average \( I_{SAT} \) signal. This is a measure of the total density flux passing the probe during a time period \( \tau \). The PDF of this then gives the probability that during a time period of length \( \tau \) a given flux passes the probe. If the signal is statistically the same on all time-scales (self-similar), then these PDFs can be rescaled by a simple power-law transformation onto a single underlying PDF - see [4] for further details.

The peak \( P(\delta x = 0) \) of the PDFs as a function of \( \tau \) is shown in Fig. 3. Time-scales over which the signal is self-similar are indicated by power-law scaling of the form \( P(\delta x = 0) \propto \tau^{-H} \), where \( H \) is the Hurst exponent. A typical result from MAST has a Hurst exponent \( H \approx 1 \) for \( \tau \) less than \( 10 - 50 \mu s \) and a value of \( H \approx 0.7 \) for times longer than this [4]. The high density case does show a change of gradient at around \( 20 \mu s \), but the value of the Hurst exponent obtained is approximately 0.5, usually indicating no correlations. A longer BOUT simulation would be needed to confirm this scaling at long times.

A simpler way of looking for self-similarity is by looking at how the \( I_{SAT} \) signal changes in time by examining the skewness and kurtosis of \( \delta I = I_{SAT}(t+\tau) - I_{SAT}(t) \). Fig. 4 shows the kurtosis of \( \delta I \) as a function of \( \tau \): \( \kappa = \frac{\langle (\delta I)^4 \rangle}{\langle (\delta I)^2 \rangle^2} - 3 \). This gives a measure of intermittency at different scales - low values imply that the signal is constantly fluctuating by small amounts, whereas large values imply that the signal consists of quiet periods and large fluctuations. For self-similar signals, \( \kappa \) should...
be independent of $\tau$. This does not appear to be the case for either the BOUT data or the measured $I_{SAT}$ data for small values of $\tau \leq 20\mu s$. The value of the kurtosis at small values of $\tau$ does not appear to depend in a simple way on the density since intermittent signals may be caused by many different effects. For long times, both signals do appear to have approximately constant kurtosis. Note that the values of the kurtosis from BOUT are smaller than for MAST plasmas and in fact in BOUT are approximately zero for long times, corresponding to the kurtosis of a Gaussian. This could be due to missing physics in BOUT which give rise to a more intermittent signal seen in MAST plasmas, or because as with the power spectrum, the simulation has not been run long enough for the rare large events which contribute to the kurtosis to be observed.

**Conclusion**

Quantitative methods have been used to characterise and compare turbulent signals from MAST edge plasmas and from BOUT simulations. Significant similarities have been observed, but also several differences. The kurtosis results are encouraging, since intermittency is one of the main characteristics of edge turbulence, and its dependence on timescale appears to be reproduced by BOUT simulations. However the differencing and rescaling results suggest that the process which creates the long-term correlations seen in experimental data may not be present in BOUT. There could be several reasons for the suppression of high-frequency components in the Fourier power spectrum, which could be resolved by further simulations using a finer mesh. Longer simulation runs will assist in determining whether the observed differences are due to an incomplete description of the physics in the BOUT model, or arise from the short duration of the simulations so far.

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**References**


