

Nonlocality and memory effects in grain dynamics on a 2D dust plasma quasi-crystal

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We report a dust plasma mono-layer experiment exhibiting a state with high mobility and large scale fluid flows of the dust grains. Yet the global hexagonal lattice structure is maintained.

The experiment is performed in a capacitively coupled RF discharge operated in Argon at the pressure 4 Pa and RF power of 19 W. Injected 600 plastic spheres with a diameter of $2a = 8.9 \mu\text{m}$ levitate as a monolayer in the sheath above the lower electrode. The dust grains are illuminated by a horizontal laser sheet, and 30000 images are taken by a video camera at a sampling rate of 30 Hz and spatial resolution of $24 \mu\text{m}/\text{pixel}$. The particle cloud diameter is $\sim 16 \text{ mm}$ with interparticle distance $\Delta \sim 0.6 \text{ mm}$. All particles have been tracked through all images and the statistical analysis is performed on the ensemble of 600 time series (particle coordinates), each consisting of 30000 points sampled at time intervals $\delta t = 1/30 \text{ s}$.

A strongly ordered (basically hexagonal) structure of the system is presented in Fig. 1(a). The pair correlation function is shown in Fig. 1(b) and reveals presence of order on distance of a few (~ 5) Δ . On longer time scales, however, pronounced fluid like motion can be observed as seen from Fig. 1 (c-d). Statistical analysis is performed on the cumulative sum $\xi_j = \sum_{i=1}^j \delta\xi_i$ of the azimuthal position displacement during the sampling interval δt , defined as $\delta\xi_i = r_i \delta\varphi_i$. Here r_i is the distance from the center of the cluster at time $i \delta t$ and $\delta\varphi_j$ is the increment in the azimuthal angle from time $(i-1) \delta t$ to $i \delta t$. The *variogram* of the process ξ_j is defined as the variance of the PDF of azimuthal position increments $\Delta\xi_j(\tau) = \xi_{j+\tau/\delta t} - \xi_j$ over the time lag τ . An example of such a PDF (with $\tau = 2 \text{ s}$) is shown in Fig.2(a). A log-log plot of the standard deviation $\sigma(\tau)$ of the PDF

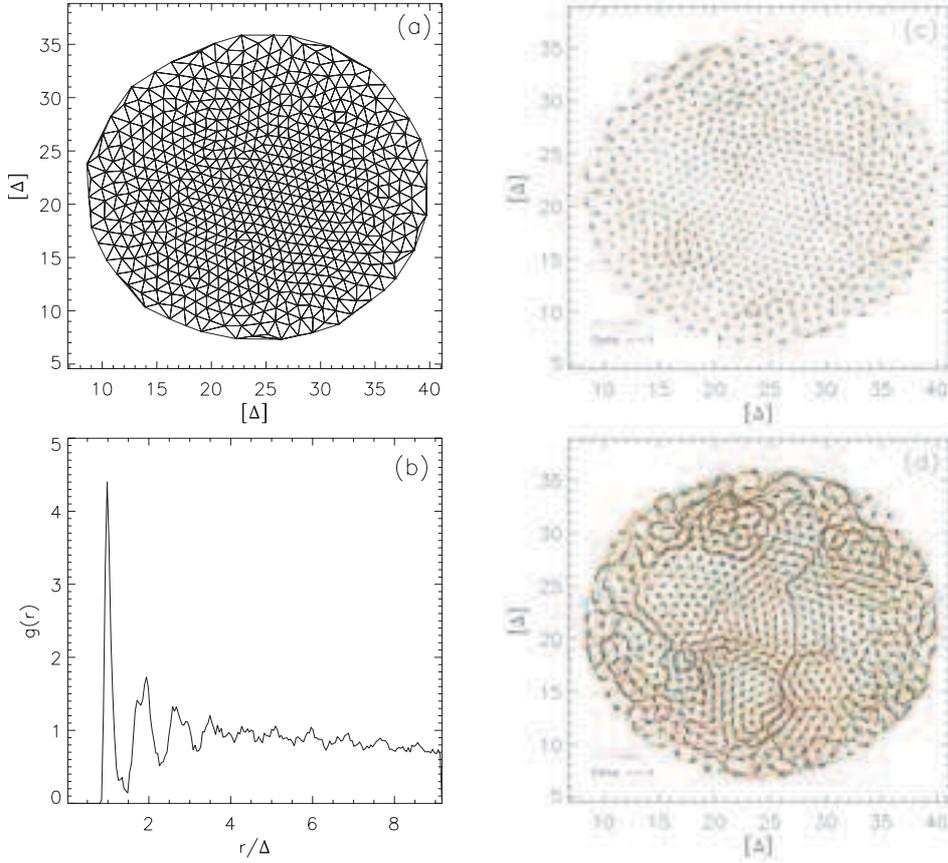


Figure 1: (a) A strongly ordered structure of the system. (b) Pair correlation function. (c),(d) Motion of particles during ~ 1 s and ~ 10 s.

versus τ is given in Fig. 2(b). This plot shows that $\sigma(\tau) \sim \tau^{H_a}$ with Hausdorff exponent [1] $H_a = 0.84$ for $t < 10$ s and $H_a = 0.68$ for $10 < t < 500$ s, indicating superdiffusive transport of dust grains on all time scales accessible for study.

On time scales up to $\tau \approx 30$ s the PDFs of azimuthal position increments can be fitted with high accuracy [Fig. 2(a)] by a stretched Gaussian PDF;

$$P(\Delta\xi, \tau) = A(\tau) \exp \left[-B(\tau)(\Delta\xi)^{2\mu} \right]. \quad (1)$$

From analysis we find that $B(\tau) \sim \tau^{-2\eta}$ with different values of η for $\tau < 1$ s and $1 < \tau < 30$ s. That means $A(\tau) \sim \tau^{-\eta/\mu}$ in order satisfy the condition $\int_{-\infty}^{\infty} P(\Delta\xi, \tau) d\Delta\xi = 1$. For displacements distributed according to Eq.(1) we have $\sigma(\tau) \sim \tau^{\eta/\mu}$, and hence $H_a = \eta/\mu$. The Hausdorff exponent obtained from the variogram can also be found from plotting

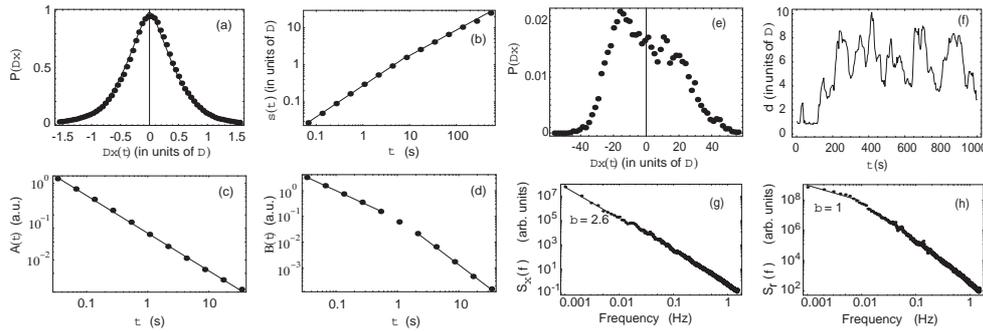


Figure 2: (a) PDF of azimuthal position increment $\Delta\xi(\tau)$ for time lag $\tau = 2$ s (dots). Nonlinear fit by function given by Eq.(1) (full curve). (b) Log-log plot of square root of variogram $\sigma(\tau)$ for azimuthal displacements. (c),(d) Log-log plot of $A(\tau)$ and $B(\tau)$ (see Eq.(1) for definition). (e) PDF of azimuthal position increment on long ($\tau = 500$ s) time scale. (f) An example of relative diffusion (distance d between two particles as a function of time). (g),(h) Power spectra of azimuthal ($S_\xi(f) \sim f^{-\beta}$) and radial ($S_r(f) \sim f^{-\beta}$) displacements, with corresponding $\beta \sim 2.6$ and $\beta \sim 1$ for $f < 0.01$ Hz.

$A(\tau)$ in a log-log plot as in Fig. 2(c) and yields the same value $H_a = 0.84$ for $\tau < 10$ s. However, from Fig. 2(d) we observe that plotting the function $B(\tau)$ in a log-log plot yields two distinct regimes; $\tau < 1$ s for which $\eta \approx 0.55$, and $1 < \tau < 30$ s for which $\eta \approx 0.85$. From the relation $\mu = \eta/H_a$ we thus have $\mu \approx 0.65$ for $\tau < 1$ s, and $\mu \approx 1.0$ for $1 < \tau < 30$ s. This means that on the time-scale $1 < \tau < 30$ s the transport is a persistent Gaussian process [1], i.e. there is a long-range memory due to some cooperative motion up to 30 s scale. On shorter scales ($\tau < 1$ s) the transport is still superdiffusive with $H_a \approx 0.84$, but the stretched Gaussian PDF ($\mu < 1$) indicates that the cause of this enhanced diffusion may not be memory effects. Superdiffusion in absence of long-range memory is normally ascribed to Lévy processes, for which the PDFs exhibit algebraic tails $P(\Delta\xi) \sim (\Delta\xi)^{-\mu}$. Such tails are not observed here, but it is still conceivable that the underlying increment process on the short scales exhibits such tails if the dust grains experience random kicks from grain-grain interactions, and that these kicks are heavy-tailed distributed. The particle displacements on longer scales are the cumulative sum of these kicks, and would be Lévy distributed with heavy tail, unless some process effectively truncates this tail at a fraction of the interparticle distance. The observed displacements would then only display the features of the Lévy distribution corresponding to the core

of this distribution, i.e. the stretched Gaussian Eq.(1).

On longer ($\tau > 30$ s) time scales the system exhibits fluid-like motion as visualized in Fig. 1 (c-d). There are vortical structures in the core of the cluster, and in Fig. 1(d) it can be observed that in the boundary region vortices of size down to the interparticle distance let particle trajectories mix rapidly, leaving the impression of a strongly turbulent motion. Figures like these give a vivid impression of how even the ordered motion in the core region evolves into into a braided “spaghetti” of trajectories on sufficiently long time-scales. These direct observation can be supported by the results of statistical analysis. After approximately 30 s the nearly Gaussian PDF $P(\Delta\xi, \tau)$ develops an asymmetry and at $\tau \approx 500$ s it has split up into two large and some smaller humps [Fig. 2(e)]. These humps show that the superdiffusive transport observed on time scales up to 30 s is replaced by advection of subpopulations of particles trapped in vortices of varying size. Additional support for this interpretation can be found in results for relative diffusion of the particles. Fig. 2(f) shows the evolution of the distance d between two particles that start out as nearest neighbors ($d \approx \Delta$) at $t = 0$. On longer time scales we observe oscillatory motions on a broad spectrum of frequencies corresponding to trapping of the two particles in vortices of varying size up to the size of the system. The same can be deduced from Fig. 2(g-h) where the spectra of azimuthal and radial displacements demonstrate power law behavior, $S_\xi(f) \sim f^{-\beta}$ for frequencies below 0.01 Hz.

The experiment demonstrates that fluid dynamics can take place on a strongly ordered structure. With kinematic viscosity $\eta = 1$ mm²/s, hydrodynamic scale $L = 16$ mm and large scale fluid velocity of $u = 0.02$ mm/s, we find Reynolds number $R_L = Lu/\eta \sim 1$. Such a low value of R_L indicates that there is no “inertial range” for the turbulence in this system, it is all “dissipation range”. This means that our system is very suitable for studying 2D turbulence on the “molecular level”, and thus complements the recent study of 3D turbulence given in Ref.[2].

References

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