Nonlinear and self-consistent treatment of ECRH

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The interaction of electrons with EC waves plays a key role in fusion experiments for plasma heating and current drive [1,2]. The flux power density in all current experiments is relatively small and the linear theory is used to simulate the wave absorption. The quasilinear theory is also used to simulate the evolution of the electron distribution function. However, in cases where nonlinear effects are important, the validity of these theories becomes questionable [3,4]. Furthermore, it is well known that for relatively strong wave amplitudes, the quasilinear theory breaks down due to the presence of resonant islands in the phase-space (see [5] and refs. therein). In the present work, we perform a self-consistent analysis of the interaction of magnetized relativistic electrons with EC waves. A closed set of nonlinear equations is used, which consists of the equations of motion in the presence of an electromagnetic wave with wave vector **A**. The electrons collectively drive the temporal evolution of the wave amplitude and the frequency through the current density source term. As an application, we study the absorption of an EC beam in a finite tokamak slab geometry, for plasma parameters relevant to ECRH experiments in ASDEX Upgrade.

Self-consistent model for wave-particle interaction

Our model for wave-particle interaction relies on the coupling of the equations of particle motions under the wave field with the Maxwell equations. The electromagnetic wave (ω,**k**) propagates in the x-z plane at an angle θ with respect to the uniform magnetic field $\mathbf{B}=B_0\hat{\mathbf{z}}$. The wave field is described by the normalized vector potential

$$\mathbf{A}=A_0(t)\left[\pi_x\cos\theta\sin\varphi(t)\hat{\mathbf{x}}+\pi_y\cos\varphi(t)\hat{\mathbf{y}}-\pi_z\sin\theta\sin\varphi(t)\hat{\mathbf{z}}\right] \quad (1)$$

where $\varphi(t)=\mathbf{k}\cdot\mathbf{r}-\int_0^t\omega(t')dt'$ is the wave phase and π_x, π_y, π_z are integers that determine the polarization of the wave. The amplitude $A_0(t)$ is normalized with m_ec^2/e, where c is the speed of light and e, m_e are the electron charge and rest mass. The spatial coordinates are normalized with c/ω_c, the time with ω_c^{-1}, the frequencies with ω_c and the wave-vectors with ω_c/c, where $\omega_c=eB_0/m_ec$ is the cyclotron frequency. The normalized equations of motion are

$$\dot{\mathbf{r}}=\mathbf{p}/\gamma,\quad \dot{\mathbf{p}}=-\dot{\mathbf{A}}+\hat{\mathbf{z}}\times\mathbf{p}/\gamma+\mathbf{p}\times(\nabla\times\mathbf{A})/\gamma,\quad \dot{\gamma}=-\mathbf{p}\dot{\mathbf{A}}/\gamma \quad (2)$$

where **p** is the relativistic mechanical momentum, normalized with $m_e c$, and γ the Lorentz factor. The normalized wave equation for the evolution of the vector potential reads

$$\nabla^2 \mathbf{A} - \ddot{\mathbf{A}} = -\omega_p^2 \mathbf{j} \tag{3}$$

with **j** the current density, normalized with $en_e c$, and $\omega_p=(4\pi e^2 n_e/m_e\omega_c^2)^{1/2}$ the normalized plasma frequency. Replacing (1) for **A** in the general relation (3), one may obtain the equations describing the temporal evolution of A_0 and ω

$$\ddot{A}_0+(k^2-\omega^2)A_0=-\omega_p^2\left(j_x\cos\theta\sin\varphi+j_y\cos\varphi-j_z\sin\theta\sin\varphi\right) \tag{4.a}$$

$$A_0\dot{\omega}+2\omega\dot{A}_0=-\omega_p^2\left(-j_x\cos\theta\cos\varphi+j_y\sin\varphi+j_z\sin\theta\cos\varphi\right) \tag{4.b}$$

Assuming an initial electron distribution $f_0(\mathbf{r_0},\mathbf{p_0})$, the normalized current density is given by $\mathbf{j}(\mathbf{r},t)=-\iint d^3\mathbf{r}_0 d^3\mathbf{p}_0 f_0 \delta[\mathbf{r}-\tilde{\mathbf{r}}(\mathbf{r}_0,\mathbf{p}_0,t)]\mathbf{p}/\gamma$, where $\tilde{\mathbf{r}}(\mathbf{r}_0,\mathbf{p}_0,t)$ is the position of the particle with initial conditions $(\mathbf{r_0},\mathbf{p_0})$. The right-hand sides in (4) depend on the particle positions through the wave phase and the current density. Since the temporal evolution of A_0, ω is of interest, we may average (4) over space in order to obtain the time evolution of the wave amplitude and the frequency. Taking into account the form of the current density, we obtain

$$\ddot{A}_0+(k^2-\omega^2)A_0=\omega_p^2\iiint dp_{0x}dp_{0y}dp_{0z}f_0\left\langle\left(p_x\cos\theta\sin\varphi+p_y\cos\varphi-p_z\sin\theta\sin\varphi\right)/\gamma\right\rangle_{\mathbf{r}} \tag{5.a}$$

$$A_0\dot{\omega}+2\omega\dot{A}_0=\omega_p^2\iiint dp_{0x}dp_{0y}dp_{0z}f_0\left\langle\left(-p_x\cos\theta\cos\varphi+p_y\sin\varphi+p_z\sin\theta\cos\varphi\right)/\gamma\right\rangle_{\mathbf{r}} \tag{5.b}$$

The equations (2), (5) form a closed set of equations describing the wave-plasma system.

Application: EC beam absorption in a tokamak plasma slab

We apply the model described above to the case of EC absorption of the 2nd harmonic X-mode in ASDEX Upgrade, considering a finite slab geometry for the region of wave-particle interaction. In the directions perpendicular to the beam propagation, the slab is defined by the projection of the beam width onto the magnetic axis, while along the propagation the slab width corresponds to a 2% relative variation of the toroidal field, assuming a typical profile $B_0(1+x/R_{maj})^{-1}$ with R_{maj} the major tokamak radius. The particles are initialized within the slab region and interact with the wave while inside the slab. In order to have a proper comparison with the linear theory, we follow the particles for the time needed by the beam, propagating with group velocity c, to cross the slab region; this time is denoted by t_{int}. The refraction index is calculated as the solution of the cold plasma linear dispersion relation for the X-mode polarization, while in the calculation of the linear absorption coefficient hot plasma effects are taken into account [6]. In our simulations, the magnetic field is B_0=2.5T, the plasma is initially Maxwellian with $n_e=10^{13}cm^{-3}$, T_e=1KeV, and the ECRH power is injected at angles $\theta=70^0$ and 90^0, which are typical values in ASDEX Upgrade experiments. For these wave injections, the values of the polarization indices in (1) are $\pi_x=\pi_y=\pi_z=1$. In fig. 1 we show the wave power, normalized over its initial value, as a function of the fraction of interaction time, together with the linear result for ECRH power P_0=1MW and (a) $\theta=70^0$, (b) $\theta=90^0$. The wave power is absorbed by the plasma electrons, which gain energy, with a rate moderately smaller from what predicted by the linear theory. Especially in the case of perpendicular injection, the nonlinear calculations show a significant reduction in the absorption. This is in accordance with recent results suggesting that the nonlinear effects during ECRH play a significant role (see [3]). The wave frequency remains close to the initial 2^{nd}-harmonic value, whereas the velocity distribution is not strongly affected by the ECRH.

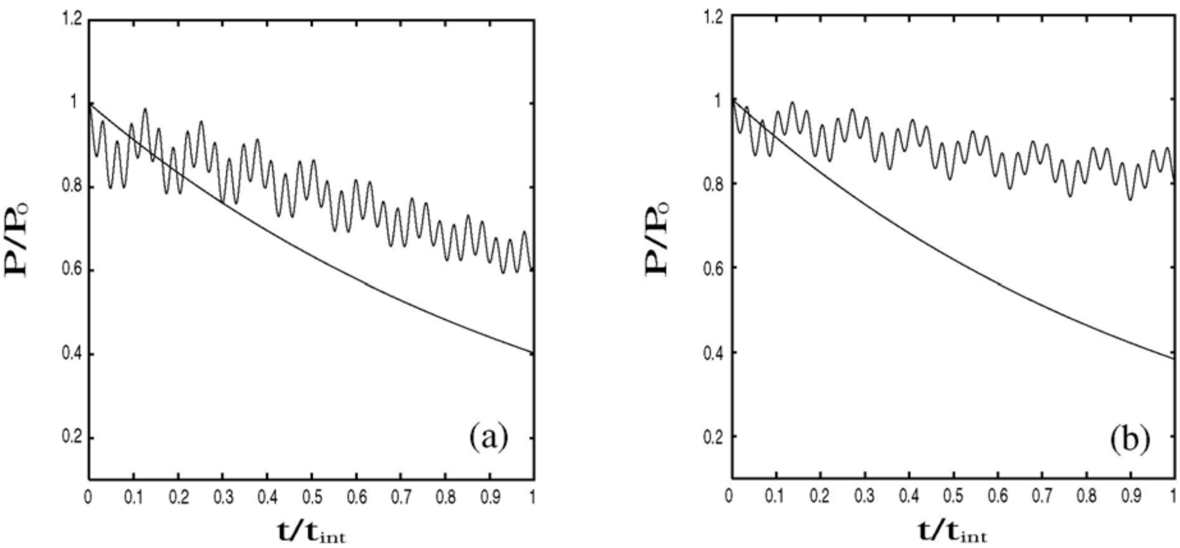

Fig. 1: P/P_0 and the linear result vs t/t_{int} for P_0=1MW and (a) $\theta=70^0$, (b) $\theta=90^0$.

The nonlinear effects become more intense as the wave power increases. In fig 2(a) the evolution of the normalized wave power is shown for oblique injection ($\theta=70^0$) of high ECRH power P_0=1GW, relevant to a FEL source. In this case, the deviation from linear theory is larger than in the case of P_0=1MW treated above. From fig. 2(b), where the energy distribution function is shown, we can see that the velocity distribution differs significantly from its initial Maxwellian form (the black curve). More specifically, a high-energy tail appears due to the nonlinear wave-particle interaction. All the above results are consistent with the energy conservation law. This consistency can be verified by estimating the variation of the total wave-particle energy $\omega_p^2\Delta\langle\gamma\rangle+\Delta(A_0^2\omega^2)=0$ inside the plasma volume occupied by the test particles. The resulting accuracy was of the order 10^{-4}-10^{-7}.

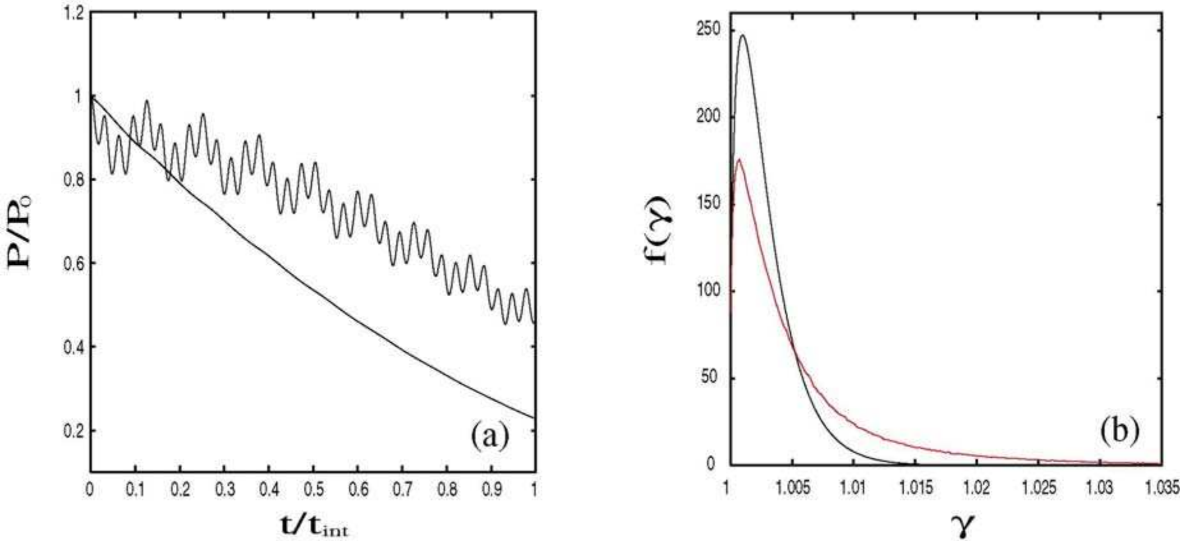

Fig. 2 (a) P/P_0 and the linear result vs t/t_{int} and (b) $f(\gamma)$ for $\theta=70^0$, P_0=1GW.

Conclusions

A self-consistent analysis of the nonlinear wave-particle interaction during ECRH was performed. Our initial results suggest that (within the limits of our model) the absorption of the EC wave is in disagreement with the linear theory. This disagreement may in some cases (eg. using high-power EC beams) be significant. We believe that there is a need to reconsider the importance of nonlinear effects on ECRH, especially when the power of the injected EC wave increases dramatically, as it will be the case for the ITER experiments. We are currently working on more realistic plasma and wave beam geometries.

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References

[1] V. Erckmann and U. Gasparino, Plasma Phys. Control. Fusion 36, 1869 (1994).

[2] B. Lloyd, Plasma Phys. Control. Fusion 40, A119 (1998).

[3] R. Kamendje et. al., Phys. Plasmas 10, 75 (2003).

[4] H. P. Laqua et. al., 31st EPS Conference, ECA Vol. 28G, P-1.209 (2004).

[5] C. Tsironis and L. Vlahos, Plasma Phys. Control. Fusion 47, 131 (2005).

[6] M. Bornatici, Plasma Phys. 24, 629 (1982).