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Kinetics of particle ensembles with variable charges

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In complex (dusty) plasmas the grain charges are not constant, but fluctuate randomly with time around some equilibrium value which, in turn, is some function of spatial coordinates. The charge variations affect the interaction between the grains [1, 2, 3] and therefore the total energy of the grains is not conserved. We propose generalized kinetic theory which allows us to investigate the evolution of the particle ensembles with variable charges. We employ the fact that for most of the realistic situations the charge variations are relatively small and the spatial and temporal variations are not correlated. Therefore, we can study separately the cases of (i) *inhomogeneous* charges – when Q depends only on the particle coordinates, and (ii) *fluctuating* charges – when Q changes randomly with time around the equilibrium constant value.

In the absence of external fields, kinetics of charged particles (grains) is governed by the mutual collisions and by the collisions with neutrals, so that the kinetic equation is

$$\frac{df}{dt} = \mathrm{St}_d f + \mathrm{St}_n f. \tag{1}$$

The grain-neutral collision integral does not depend on particle charges and can be written in the Fokker-Planck form [4], $\mathrm{St}_n f(\mathbf{p}) = \gamma(\partial/\partial\mathbf{p})[\mathbf{p}f + mT_n(\partial f/\partial\mathbf{p})]$, where m is the grain mass, T_n is the neutral gas temperature and γ is the neutral friction damping rate in the Langevin equation. As regards the grain-grain collisions, we investigate *gaseous* ensembles and hence focus on the binary interactions only. The corresponding collision integral is [4]

$$\mathrm{St}_d f(\mathbf{p}) = \int \Big[w(\mathbf{p}',\mathbf{p}'_1;\,\mathbf{p},\mathbf{p}_1) f(\mathbf{p}')f(\mathbf{p}'_1) - \\ w(\mathbf{p},\mathbf{p}_1;\,\mathbf{p}',\mathbf{p}'_1) f(\mathbf{p})f(\mathbf{p}_1)\Big] d\mathbf{p}_1 d\mathbf{p}' d\mathbf{p}'_1. \tag{2}$$

Here, $w(\mathbf{p},\mathbf{p}_1;\,\mathbf{p}',\mathbf{p}'_1)$ is a probability function for a pair of colliding particles with momenta $\mathbf{p}$ and $\mathbf{p}_1$ to acquire momenta $\mathbf{p}'$ and $\mathbf{p}'_1$, respectively, after the scattering. Equation (2) counts for all possible transitions $(\mathbf{p}',\mathbf{p}'_1) \to (\mathbf{p},\mathbf{p}_1)$ (sources) and $(\mathbf{p},\mathbf{p}_1) \to (\mathbf{p}',\mathbf{p}'_1)$ (sinks), and then is averaged over $\mathbf{p}_1$. Function w can be determined by solving a mechanical problem of the binary scattering with given interaction between the particles. Mechanics of binary grain collisions can be conveniently considered in terms of the center-of-mass and relative coordinates. For a pair of particles with momenta $\mathbf{p}$ and $\mathbf{p}_1$, the center-of-mass and relative momenta are $\mathbf{p}_\mathrm{c} = \frac{1}{2}(\mathbf{p}+\mathbf{p}_1)$ and $\mathbf{p}_\mathrm{r} = \mathbf{p}_1 - \mathbf{p}$, respectively. In the absence of external forces, the center-of-mass momentum is conserved, and the relative momentum is changed during the collision: $\mathbf{p}'_\mathrm{c} = \mathbf{p}_\mathrm{c}$ and $\mathbf{p}'_\mathrm{r} = \mathbf{p}_\mathrm{r} + \mathbf{q}$.

The charge variations cause the variation of the kinetic energy of colliding particles. Therefore, the exchange of the relative momentum can be divided into the elastic and inelastic parts, $\mathbf{q} = \mathbf{q}_0 + \delta\mathbf{q}$. The corresponding energy variations are $\delta\varepsilon_c = 0$ and $\delta\varepsilon_r = p_r\delta q/2m + (\delta q)^2/4m$. The magnitude of inelastic momentum exchange δq is generally a function of $\mathbf{p}_c$ and $\mathbf{p}_r$ and is determined by the characteristics of the charge variations.

The relative smallness of the inelastic processes associated with the charge variations introduces very important *hierarchy of the time scales* in the particle kinetics: Each interparticle collision is accompanied by (i) elastic momentum exchange $\mathbf{q}_0$, which provides the relaxation of the distribution function to the Maxwellian equilibrium [4] – yet keeping the mean energy E constant, and (ii) inelastic momentum exchange $\delta\mathbf{q}$, which causes variation of the mean energy. For $\delta q \ll q_0 \sim p_r$, the latter process is much slower than the former one. Therefore, the velocity distribution is close to the *Maxwellian form* with the temperature $T = \frac{2}{3}E$, and the inelastic processes simply cause an *adiabatic variation of the temperature* [5].

Let us study how the charge variations affect the temperature. Assuming the distribution function to be normalized to unity, $\int f\, d\mathbf{p} = 1$, the mean kinetic energy per particle is $E = \int (p^2/2m) f\, d\mathbf{p}$. In accordance with Eq. (1), the temporal dependence $T(t)$ is determined by

$$\dot{T} = \frac{2}{3}\int \frac{p^2}{2m}(\mathrm{St}_d f + \mathrm{St}_n f)\, d\mathbf{p}. \tag{3}$$

For the grain-neutral collisions we have simply $\int (p^2/2m)\mathrm{St}_n f\, d\mathbf{p} = -3\gamma(T - T_n)$. For the nonlinear grain-grain collision integral, it is convenient to introduce the "binary" distribution function and the probability function expressed in terms of the center-of-mass and relative momenta, $F(\mathbf{p}_c, \mathbf{p}_r) \equiv f(\mathbf{p})f(\mathbf{p}_1)$ and $W(\mathbf{p}_c, \mathbf{p}_r; \mathbf{q})\delta(\mathbf{p}'_c - \mathbf{p}_c) \equiv w(\mathbf{p}, \mathbf{p}_1; \mathbf{p}', \mathbf{p}'_1)$. The first integrand in Eq. (3) can be expanded into a series over $\delta q/p_r$. Retaining the linear and quadratic terms and integrating in parts, we obtain,

$$\int \frac{p^2}{2m}\mathrm{St}_d f\, d\mathbf{p} \simeq \frac{1}{4m}\int (p_r\mathscr{A} + \mathscr{B})F\, d\mathbf{p}_c d\mathbf{p}_r, \tag{4}$$

where $\mathscr{A}(\mathbf{p}_c, \mathbf{p}_r) = \int \delta q W d\delta q$ and $\mathscr{B}(\mathbf{p}_c, \mathbf{p}_r) = \frac{1}{2}\int (\delta q)^2 W d\delta q$ are analogous to the Fokker-Planck coefficients [4]. The magnitude of the charge variations and hence of coefficients $\mathscr{A}$ and $\mathscr{B}$ is measured in terms of the smallness parameters – dimensionless charge gradient $\tilde{\varepsilon} = |\nabla Q/Q|\lambda$ for inhomogeneous charges, and dispersion $\tilde{\sigma} = \sqrt{\langle \delta Q^2 \rangle}/|Q|$ for fluctuating charges.

For the particles interacting via the Yukawa potential with the screening length λ, the mutual collisions are characterized the ratio of the Coulomb coupling energy at the distance λ to the particle temperature, $Q^2/\lambda T$. When the ratio is large the interaction is of the *hard-spheres* type [6]. In the opposite case, when the ratio is small, the interaction is of the *Coulomb* type [4], similar to that between electrons and ions in usual plasmas. Below, these two limits are refereed to as the *low-temperature* and *high-temperature* regimes, respectively, with "transition" temperature $T_{\rm tr} = Q^2/\lambda$.

Coefficients $\mathscr{A}$ and $\mathscr{B}$ in Eq. (4) are calculated in Ref. [7] for different types of the charge variations and the temperature regimes. For *inhomogeneous charges* we obtain the following equations for the particle temperature:

$$\begin{aligned} T \ll T_{\rm tr}: &\quad \dot{T} \sim \frac{\tilde{\varepsilon}^2}{m^{1/2}\lambda}\, T^{3/2} - 2\gamma(T - T_n), \\ T \gg T_{\rm tr}: &\quad \dot{T} \sim \frac{\tilde{\varepsilon}^2 T_{\rm tr}}{m^{1/2}\lambda}\, T^{1/2} - 2\gamma(T - T_n). \end{aligned} \tag{5}$$

Introducing the thermal velocity of the grains, $v_T = \sqrt{T/m}$, we can see that in the hard-sphere regime the temperature exhibits an explosionlike growth, $T \propto (t_{\rm cr} - t)^{-2}$ with the critical time $t_{\rm cr} \sim \tilde{\varepsilon}^{-2}\lambda/v_{T_0}$, provided that $\gamma t_{\rm cr} < 1$. In the Coulomb regime, however, the temperature growth is always saturated due to finite friction. For the *fluctuating charges* we have

$$\begin{aligned} T \ll T_{\rm tr}: &\quad \dot{T} \sim \frac{\tilde{\sigma}^2}{m\nu_{\rm ch}\lambda\ell}\, T^2 - 2\gamma(T - T_n), \\ T \gg T_{\rm tr}: &\quad \dot{T} \sim \frac{\tilde{\sigma}^2 \omega_{\rm pd}^2}{\nu_{\rm ch}}\, T - 2\gamma(T - T_n). \end{aligned} \tag{6}$$

Here $\nu_{\rm ch}$ is the grain charging frequency [8], $\omega_{\rm pd}$ is the dust plasma frequency, and ℓ is the mean free path due to grain-grain collisions. The explosionlike growth in the hard-sphere regime, $T \propto (t_{\rm cr} - t)^{-1}$ with $t_{\rm cr} \sim \tilde{\sigma}^{-2}\nu_{\rm ch}\lambda\ell/v_{T_0}^2$, occurs when $\gamma t_{\rm cr} < 1$. For the Coulomb interaction, the temperature does not saturate but grows exponentially if the damping rate is below the threshold value, $\gamma\nu_{\rm ch}/\omega_{\rm pd}^2 < \tilde{\sigma}^2$.

The molecular dynamics simulations show that the velocity distribution indeed keeps an isotropic Maxwellian form, provided the magnitudes of the charge variations, $\tilde{\varepsilon}$ and $\tilde{\sigma}$, are small enough. If the neutral damping rate in the simulations is chosen above the instability thresholds, the particle temperature remains constant, $T \sim T_n$. Otherwise, the temperature increases with time. Curves 1 and 2 in Fig. 1 are obtained for "zero" damping rate and different magnitudes of (a) charge gradient and (b) charge dispersion: At small t, the simulations reveal with good accuracy a quadratic dependence on $\tilde{\varepsilon}$ and $\tilde{\sigma}$, viz., $T/T_0 - 1 \propto \tilde{\varepsilon}^2 t$ or $\propto \tilde{\sigma}^2 t$, respectively, in agreement with Eqs. (5) and (6). Curves 3 illustrate the scaling with the initial temperature and reveal very good agreement with the theoretical results as well.

The obtained results are of significant importance for laboratory complex plasmas as well as for space plasma environments. For instance, the charge variations can cause heating of microparticles in low-pressure complex plasma experiments and be responsible for the melting

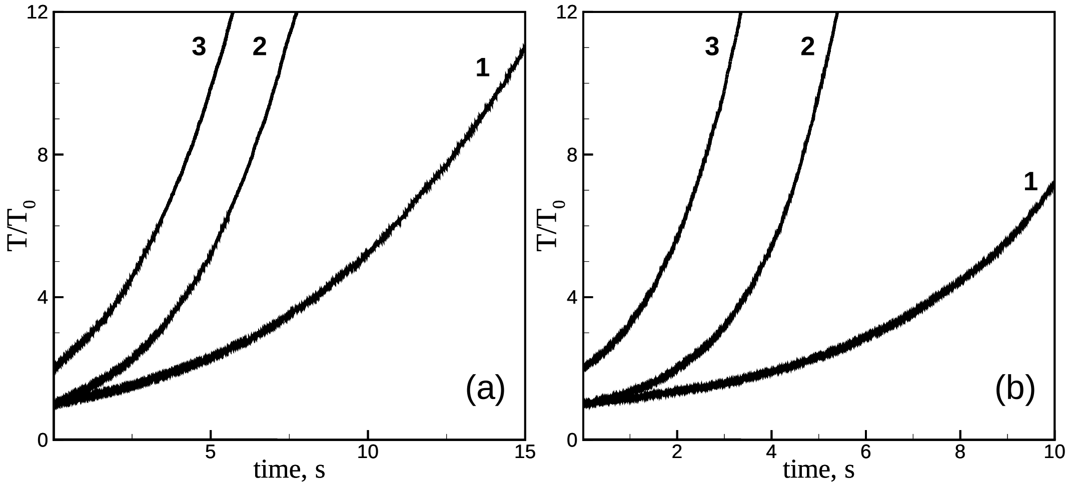

Figure 1: Temperature of particles versus time for (a) inhomogeneous and (b) fluctuating charges, as obtained from the molecular dynamics simulations. In (a), curves (1) and (2) are plotted for initial temperature $T_0 \simeq 0.3T_{\rm tr}$ and different values of the normalized charge gradient: (1) $\tilde{\varepsilon} \simeq 0.1$ and (2) $\tilde{\varepsilon} \simeq 0.15$, curve (3) is for $2T_0$ and $\tilde{\varepsilon} \simeq 0.15$. In (b), curves (1) and (2) are plotted for T_0 and different values of the charge dispersion: (1) $\tilde{\sigma} = 0.2$ and (2) $\tilde{\sigma} = 0.3$, curve (3) is for $2T_0$ and $\tilde{\sigma} = 0.3$.

of plasma crystals, operate in the protoplanetary disks and affect the kinetics of the planet formation, cause anomalous heating of dust in the interstellar clouds and contribute to the cosmic ray generation, etc. Ensembles of particles with variable charges represent a new class of open systems with spatially-temporal dependence of the parameters characterizing the “internal interactions”. Regardless of nature and the absolute magnitude of the interactions, the kinetics of such systems may exhibit similar behavior. Therefore complex plasmas, which allow investigations at the individual particle level, can bring us important insights into generic properties of the open systems.

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