Improving Tokamak Vertical Position Control in the Presence of Power Supply Voltage Saturation

J-Y. Favez\textsuperscript{1,2}, J.B. Lister\textsuperscript{1}, Ph. Müllhaupt\textsuperscript{2}, B. Srinivasan\textsuperscript{2}, F. Villone\textsuperscript{3}

\textsuperscript{1}Centre de Recherches en Physique des Plasmas, Association EURATOM-Confédération Suisse, EPFL, 1015 Lausanne, Switzerland
\textsuperscript{2}Laboratoire d'Automatique, EPFL, 1015 Lausanne, Switzerland
\textsuperscript{3}Associazione EURATOM/ENEA/CREATE, DAEIMI, Univ. di Cassino, Italy

Introduction
The control of the current, position and shape of a tokamak plasma is complicated by the instability of the vertical position when the plasma cross section is elongated. Linearised models of the system to be controlled all share the feature of a single unstable pole, attributable to this vertical instability, and a large number of stable or marginally stable poles (attributable to zero or positive resistances in all other circuit equations). The presence of only a single unstable pole is due to the system being stable before the vertical plasma movement is included. Due to the cost of ITER, there are relatively low voltage margins in the Poloidal Field coil power supplies, suggesting that the feedback control loop may experience actuator saturation during occasional large transients. We present a modified feedback controller which explicitly takes into consideration the saturation of the power supply voltages when generating the power supply demand signals. This novel approach has been tested on simulations using ITER and JET linearised plasma equilibrium response models.

Development of the control method
Our aim is to take an existing controller design, the reference controller, and to enlarge its region of attraction \( \mathbf{A} \) (the region in state space from which the closed loop system asymptotically reaches the origin \([1,3]) to the null controllable region \( \mathbf{C} \) (the region in state space where there exists an open loop input that can steer the system to the origin \([1,3,4])). In previous work described in detail in [2] we formally considered a system with a single unstable pole and a single stable pole. We derived the region of attraction of the closed loop system with saturation of the single input and we examined the performance of the controller.

Throughout this work, we use linearised tokamak models (CREATE-L for JET [5] and ITER [6]), which describe the tokamak by ODEs in the continuous time state space format given by

\[
\begin{align*}
\dot{x}_p &= A_p x_p + B_p u + E_p w \\
y &= C_p x_p + F_p w
\end{align*}
\]

(1). The state variables \( x_p \) explicitly represent the physical active coil currents, the passive structure currents and some plasma variables when the model is created. The coil voltages are the vector \( u \). The outputs of the system, the vertical and radial plasma positions, wall-separatrix gaps, plasma current and all the magnetic diagnostics measurements, are given by the variable \( y \). The input \( w = [\Delta \beta \ \Delta l]^T \) represents disturbances such as ELMs or sawteeth.

Traditionally, we talk of the vertical position as being unstable. However, when the position is unstable, the passive currents, coil currents and position also grow exponentially, although all these physical variables cannot be considered to be separately unstable. The eigenvalues of the matrix \( A_p \) in (1) determine the dynamical evolution of the physical variables \( x_p \). One of the eigenvalues of \( A_p \) is positive when the vertical field index is curved outwards. However, \( A_p \) is not diagonal and therefore the variables \( x_p \) are not the eigenvectors of the system. We transform these equations from the variables \( x_p \) to new combinations \( x \) for
which the transformed matrix $A$ is diagonal, generating the new equations:

$$
\dot{x} = Ax + Bu + E\hat{w}; \quad x_p = Tx; \quad A = T^{-1}A_pT; \quad B = T^{-1}B_p; \quad E = T^{-1}E_p
$$

$$
y = Cx + F\hat{w}; \quad C = C_pT; \quad F = F_p
$$

(2).

One of the new orthogonal states is unstable and this is now the unstable state. Unfortunately, there is no intuitive combination of the physically meaningful variables $x_p$ which describes the unstable state, even for the simplest 3-circuit model. If we can stabilise the unstable state, all other states, being stable, will decay and the closed loop system is stable. Provided there are enough diagnostic measurements, the unstable and stable states can be estimated using the pseudo-inverse of $C$, by $[\hat{x}, \hat{w}] = [C, F]^T \cdot y$, reconstructing the unstable state, the stable states, and the disturbance $w$. If we neglect $F$ (the direct influence on $y$ of $[\Delta \beta \Delta l]^T$ with respect to the nominal equilibrium) then $\hat{x}$ is simple to generate. However, coil saturation is most likely during a large perturbation and the influence of $F$ must be included, the object of future work.

Consider system (2) with zero disturbance input. We can split this system into an anti-stable and a stable subsystem [4]

$$
[\hat{x}_1, \hat{x}_2] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & A_j \end{bmatrix} \begin{bmatrix} x_1 \\ x_j \end{bmatrix} + \begin{bmatrix} \lambda_1 \\ 0 \end{bmatrix} u
$$

(3). Here $x_j$ describes the anti-stable subsystem and $x = [x_2 \ x_3 \ ... \ x_j]^T$, $A_j$ and $b_j$ describe the stable subsystem. The null controllable region is only restricted by the unstable state (anti-stable system) while the stable states (stable subsystem) can be controlled for any arbitrary values. We assume that there exists an adequate algebraic state reconstruction available such that $\hat{x} \sim x$ (in what follows we use these interchangeably). We can therefore replace our reference input-output controller $v(s) = K(s)y(s)$ by a linear state feedback controller

$$
v(x) = f\hat{x} = f_1x_1 + f_2x_2 + f_3x_3 + ... + f_nx_n
$$

(4)

With this feedback controller the closed loop system becomes $\dot{x} = Ax + Bs\text{at}(f\hat{x})$. By considering the linear controller (4) we can see that $A = C$, if and only if $f_2 = f_3 = ... = f_n = 0$ and if the linear stability condition $1 + f_1 < 0$ is satisfied. For all other linear controllers in which at least one of the parameters $f_2, f_3, ... f_n$ is nonzero, $A \subset C$.

We have been able to enlarge the region of attraction to include the full null controllable region $A = C$, without loss of local performance, by introducing a continuous nonlinear function in the controller [2]. Consider the modified controller $v(x) = f_1x_1 + k(x)(f_2x_2 + f_3x_3 + ... + f_nx_n)$; $u = \text{sat}(v)$ (5). Assume that $f$ has been chosen to obtain the desired performance of the closed-loop system near the origin, for small disturbances. Compared to (4), the new controller differs by the introduction of a smooth nonlinearity by choosing $k(x) = (1 - x_i^2)$ where $0 < k(x) \leq 1$ since $|x_i| < 1$ within the null controllable region. $k(x) = 0$ for $|x_i| > 1$. The idea behind this nonlinear controller is as follows. If $x_i \approx 0$, then $k(x_i) \approx 1$ which implies that the controller is approximately the linear state feedback controller $v = f_1x_1$. In this case, the controller concentrates on local performance. On the other hand, if the unstable state approaches the boundary of the null controllable region, $C$, $x_i \approx \pm 1$ and $k(x) \approx 0$. This implies that the controller is approximately the linear state feedback $v = f_1x_1$, and it focuses on the stabilisation of the unstable state and global stability ($A = C$). Moreover, since this controller is a continuous one, chattering is avoided.

**Testing the method on ITER** We first implemented this approach on a closed loop model of ITER. The state was not estimated, but taken directly from the equations. We compare via simulation the reference controller [7], given by $v = f_1x_1$, against the new continuous nonlinear controller (5) using phase diagrams. Since we are dealing with a high order system (50 .. 100 states) we cannot show the evolution of all states. Thus, the phase diagrams show the evolution of only two states: the unstable state $x_1$, and one of the most disturbed stable states,
conditions. The phase diagram (Fig.1) shows the initial conditions point \( x_{\text{init}} \), located inside the null controllable region. Since for the nonlinear controller the initial conditions are located in the region of attraction, the trajectory converges to the origin. For the reference controller the trajectory diverges, thus confirming by simulation that \( A_n \subseteq C \).

The second case shows the evolution of the trajectories for both controllers during and after a large perturbation (Fig.2). At \( t_2 \) the states of the systems with both controllers are inside \( C \). Since for the nonlinear controller \( A_c = C \), the trajectory converges to the origin. For the reference controller the trajectory diverges and thus, the state is not in \( A_n \).

The third case shows the trajectory for a much larger perturbation amplitude (Fig.3). Both trajectories leave the null controllable region \( C \) and only the trajectory for the system with the nonlinear controller reenters \( C \). For all these cases, the unstable state \( x_1 \) is brought back to the origin faster when the continuous nonlinear controller is used. This is the benefit of the nonlinear function \( k(x) \) which helps the controller concentrate on the unstable state in the proximity of the boundaries of \( C \) and beyond it.
**Testing the method on JET** We implemented this technique on the CREATE-L model of JET, including the closed loop controller. We generated an observer of the unstable state \( \dot{x} \) directly from the documented diagnostics. We increased the amplitude of the disturbance (resembling ELMs) until the closed loop model lost control due to saturation of the FRFA supply. The simulation was repeated with the modified controller and control was no longer lost.

Fig.4 shows an example of the evolution of the vertical position \( z \) and the FRFA control voltage for a very large ELM perturbation in JET. The ELM perturbation starts at \( t_0 \), reaches its maximum at \( t_1 \) and vanishes at \( t_2 \) (vertical dashed lines). The reference controller loses stability just after \( t_1 \).

**Discussion** A simple continuous nonlinear controller for the stabilisation of the ITER tokamak unstable vertical position in the presence of voltage saturation is proposed. The principle is to modify an existing linear controller by introducing a simple nonlinear term into the control law. This new controller enlarges the region of attraction to the maximal reachable region of attraction under input saturation, which is the null controllable region. Additionally, its local performance around the origin is similar to that of the existing linear controller. An additional advantage of the nonlinear controller is that the unstable state is brought back faster to the origin and thus, the rejection of the perturbation is more efficient. This is a benefit of the nonlinear function where the controller concentrates on the control of the unstable state in the proximity of the boundaries of null controllable region and beyond it.

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**References**


