Ion Distribution Function and Momentum Transfer in the Presence of an Electrostatic Pump Wave in a Magnetized Plasma

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Abstract
The time evolution of the ion distribution function (IDF) when an electrostatic pump wave is applied, with a frequency in the range of the low-order ion cyclotron harmonics, is investigated by numerically integrating the Vlasov-Poisson system of equations. The case of the wave propagation perpendicular to the external magnetic field is considered. Previous extended analysis \cite{1,2}, with a reduced ion mass, have shown the appearance of a regular periodic structure of the IDF, in the velocity plane normal to the magnetic field, which seems to be caused by the periodic “accumulation” of detrapped ions, leaving the trapping region.

In this paper, the numerical analysis is extended to the case of ions with their actual mass, and the resulting moments of the IDF are computed and discussed.

Introduction
Suppression of turbulence in tokamak plasmas by means of a sheared poloidal flow \cite{3} can be driven by externally applying radiofrequency power \cite{4}, and specifically by exciting radially propagating ion Bernstein waves (IBW) \cite{5}. It is therefore of primary importance to have a clear understanding of the coupling processes which govern the energy and momentum transfer from the excited IBW to the magnetized plasma. The linear theory of IBW collisionless dissipation is well established \cite{6} and foresees that small amplitude IBW, propagating normal to the magnetic field, are not absorbed. However, due to their relative low frequency (tens or hundreds MHz), and since, close to the cold (lower hybrid) resonance, IBW are almost electrostatic plasma oscillations, the system can depart from the linearity conditions, and new effects can come into play. The process of generation of a transverse (to both the magnetic field, \( B_0 = B_0 e_z \), and to the wave propagation direction, \( k_0 = k_0 e_x \)) ion flow can be thought as the result of several effects, which have seeds of nonlinearities. For example, it is known that in the presence of fluctuations the Reynolds stress tensor, originating from
the quadratic term $<\mathbf{V}_{osc} \cdot \nabla \mathbf{V}_{osc}>$, contributes to the generation of a sheared transverse flow, this term deriving from a suitable average of the $v \nabla f_i$ in the ion Vlasov equation. In addition, the transverse flow direction is consistent with that of the cross product between an average wave electric field, $\mathbf{E}_{osc} = -i \mathbf{k} \phi_{osc}$, and the magnetic field, $\mathbf{B}_0$, and this effect is contained in the non linear part of the ion Vlasov equation [1,2].

With the aim of investigating the nonlinear processes taking place during the propagation of a finite amplitude IBW wave in a magnetized plasma, we have performed a series of kinetic numerical simulations of a purely electrostatic wave, propagating perpendicularly to the magnetic field, with frequency $\omega_0$ and wavenumber $k_0$ typical of IBW experiments [7]. The physical model and the relevant equations have been described in detail elsewhere [1,2,8] and will not be reported here. In short, the Vlasov equations for electrons and protons have been numerically integrated, together with the Maxwell equations, in a slab geometry where $x$ and $y$ represent the “radial” and the “poloidal” coordinates, respectively [1,2]. The strictly perpendicular propagation of the pump wave allows one to reduce the system to one spatial dimension, $x$, and two velocity dimensions, $v_x$ and $v_y$. In an early set of simulations [1,2] we have solved both the ion and the electron Vlasov equations, introducing a reduced ion mass $m_i = 50 m_e$ (case A), in order to limit the run time to a reasonable level. More recently, by exploiting the fact that electrons follow the ExB drift, and that the electron density remains almost unperturbed [2], it became possible to integrate the Vlasov equation for ions only, with a considerable shortening of computational time [8]. This has resulted in fast runs of a new version of the code for the physical parameters corresponding to $m_i = 1836 m_e$ (case B) [9].

Here we present new results for $m_i = 1836 m_e$. The structure of the IDF under the action of the pump wave, and the characteristics of its first order moment, which represents the driven ion flow in the $y$ (i.e., the “poloidal”) direction are discussed. The physics of the wave-plasma interaction as obtained in case A is confirmed, and quantitative estimates in actual experimental conditions are now possible.

The ion distribution function and the induced transverse ion flow
It is well established [1,2,8] that in perpendicular propagation ($k_\parallel = 0$), even at frequencies in the range of the low order ($n < 5$) ion cyclotron harmonics, the principal mechanism of wave-plasma coupling is the perpendicular ion Landau
damping at $v_x = \omega_e/k_0$ and at lower extent at $v_x = 2\omega_e/k_0$: ions gain energy and momentum along $v_x$ within the trapping region (a kind of plateau is formed), and the magnetic field redistributes them along $v_y$. **Fig.1** shows two sections of the IDF, for $m_i=1836\ m_e$. The trapping region, around $v_{x,y} = \omega_e/k_0$ ($\approx 1.22\times10^{-3}c$, for $\omega_0 = 0.35\ \omega_{pi}$ and $k_0 = 285.6\omega_{pi}/c$), extends between $0.62\times10^{-3}c$ and $1.82\times10^{-3}c$. Independently of $x$ and $t$, the IDF is quite regular in the half-plane $v_x > 0$, while it presents a complicated structure for $v_x < 0$, as it is seen in **Fig.2** where the IDF contours are shown for $m_i=1836\ m_e$ at $t = 0, 20, 40, 60\ \omega_{pi}, a=10^{-4}, \omega_0 = 0.35\ (= 4\omega_{ci})$.

![Fig.1](image1.png)

![Fig.2](image2.png)

The “finger-like” structure already observed in case A [1,2], is formed in case B, as well. From the plateau created in the trapping region, ions are detrapped periodically by the Lorentz force, and accumulate in bunches in the velocity plane, thus producing the “lowest finger”. Then, the Larmor rotation makes the whole structure to rotate clock-wise until each finger reaches the upper part of the trapping region, where it is again acted upon by the wave. It has been verified that changing $k_0 \rightarrow -k_0$, the finger-like structure occupies the half-plane $v_y > 0$, and the rotation is counter-clock-wise.
In order to calculate the average induced ion velocity \( <U_{i,y}> \) in the \( y \) direction, the IDF has been multiplied by \( v_y \), integrated in the velocities \( v_x, v_y \), and averaged over the spatial coordinate \( x \), resulting in the plots in Fig. 3. Here, \( -<U_{i,y}>/c vs \omega_0/\omega_p (\omega_0 = 0.32 = 4\omega_n) \) is shown for \( a = 5 \times 10^{-5} \) (a) and \( 10^{-4} \) (b).

![Fig.3](image)

(a)                                                                 (b)

A negative peak in \( <U_{i,y}> \) appears in both cases, upshifted in frequency by \( \approx 5\% \) with respect to \( \omega_0 = 4\omega_n \), corresponding to an average ion velocity varying with the ratio \( \omega_0/\omega_p \), and then with the spatial (radial) coordinate.

For typical parabolic density profiles and for the \( 1/R \)-dependence of the magnetic field, conditions can be found such that the resulting \( x \)-variations of the average ion velocity satisfy the criterion for turbulence suppression [10]. Indeed, for \( T_e = 1keV \), \( n_e = 10^{14} \text{cm}^{-3} \), \( B_0 = 7.5T \), \( a = 33\text{cm} \), \( \delta x_k = \rho_L/e \), \( a = 10^{-4} \), at \( x = 0.75a \), \( |<U_{i,y}>_x'| \approx cT/eB_0L_n\delta x_k \approx 2 \times 10^5 \text{sec}^{-1} \) (for drift turbulence).

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