Sphere in Flowing Plasma with non-zero Debye length: the unmagnetized Mach Probe part 2

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Abstract

The spatial distribution of ion flux to a sphere in a flowing collisionless plasma is calculated using a particle-in-cell code SCEPTIC. Comprehensive results are provided for ion temperatures 1 and 0.1 times the electron temperature, and for Debye length from 0.01 to 100 times the probe size. A remarkable qualitatively new result is obtained: over a range of Debye lengths, the downstream side of the probe receives substantially higher flux density than the upstream side. This unexpected reversal of the asymmetry renders the use of the flux ratio for Mach-probe purposes problematic, even for deriving the direction of the flow.

The Specialized Coordinate Electrostatic Particle and Thermals In Cell (SCEPTIC) code was described in [1], and used to solve the long-standing problem of the interaction of an ion-collecting sphere with an unmagnetized flowing collisionless quasineutral plasma. Here are reported the results of using the SCEPTIC code to solve the more general problem with non-zero Debye length.

The results are important for Mach probe measurements which try to measure the plasma flow velocity from the upstream/downstream asymmetry of ion collection flux. They are also important for determining grain charging in dusty plasmas and for spacecraft plasma interactions.

The SCEPTIC code uses standard PIC techniques except that the potential mesh is in spherical coordinates to match the geometry. Each ion, of charge $Ze$ and mass $m$, at Cartesian coordinate position $x$ is governed by the equation of motion in the electrostatic potential:

$$m \frac{d^2x}{dt^2} = -Ze \nabla \phi.$$  (1)

The electrons are taken to have density

$$n_e = Zn_{i\infty} \exp(e\phi/Te),$$  (2)

where $n_{i\infty}$ is the ion density where $\phi = 0$, far from the probe.

The self consistent potential satisfies Poisson’s equation:

$$\nabla^2 \phi = \frac{e}{\epsilon_0} (n_e - Zn_i),$$  (3)

where the ion density $n_i$ is obtained by integration over all velocities of the ion distribution function, $f(x, v)$, which is represented by typically 7 million individual ions in the calculation.
The potential (relative to zero at infinity) is fixed at the probe, which absorbs ions. At the outer computational boundary, rather subtle conditions are applied to $\phi$ and ions are reinjected with a distribution that arises from a drifting Maxwellian distribution at infinity and accounts approximately for finite potential drop in the external region. Details are given elsewhere [2].

In part as a benchmark, figure [1] shows a comparison of the best prior approximate results with those of SCEPTIC at a drift velocity $v_f = \sqrt{2}$ (in units of $\sqrt{Z T_e/m_i}$), which is the lowest non-zero velocity for which angular results were given.

At very large values of $\lambda_{De}$ the agreement is excellent to within draftsmanship uncertainties, as it should be since the potential is simply $1/r$ in this limit. For lower Debye lengths, where the plasma potential perturbation is substantial, the qualitative behaviour is correct but quantitative discrepancies of approximately 10 to 20% occur. These illustrate the errors introduced by the prior work’s approximations.

SCEPTIC results in the most interesting but most difficult parameter regime, where the Debye length is of order the sphere radius, are illustrated in Figure [2].

Figure 1: Variation of the flux density with position on the sphere, given as cosine of angle to the flow direction, for a flow $v_f = \sqrt{2}$, $T_i = T_e$, $\phi_p = -5$. The lines are SCEPTIC calculations and the points are from prior approximate calculations [3].

Figure 2: Potential and density contour plots ($\lambda_{De} = 1$, $T_i = 1$, $v_f = 1$, $\phi_p = -5$). The average ion velocity is indicated by the length of the vector arrows.

The potential is quite close to spherical symmetry, which is presumably why Godard’s symmetry approximation gives reasonable results. However the density is much less symmetric and shows enhancement on the downstream side.
The radial dependence of $\phi$ is strongly dependent on the value of the Debye length ($\lambda_{De}$ measured in units of the sphere radius) as illustrated in Figure 3.

In Figure 3 we show the flux density to the sphere as a function of the cosine of the angular position on the surface, $\cos \theta$. At large flow speed, the qualitative aspect meets the intuitive expectation that the downstream side ($\cos \theta > 0$) should receive less flux than the upstream. However, at low or moderate flow speed, this effect is almost absent in the low probe potential case ($\phi_p = -5$): there is hardly any asymmetry; and with high probe potential, ($\phi_p = -15$ for $v_f \lesssim 1.5$) the effect is reversed. That is, the downstream receives greater flux density than upstream. This result is quite contrary to the experience with infinitesimal Debye length reported previously[2]. When one investigates lower ion temperatures, the effects of the asymmetry reversal are much stronger.

![Figure 3: Angle averaged $\phi$-profile for three values of Debye length, all other parameters being equal ($T_i = 1$, $v_f = 1$, $\phi_p = -5$). The case $\lambda_{De} = 1$ corresponds to figure 2](image)

The ratio of upstream to downstream ion flux density is used as a measure of flow velocity in Mach probe applications. In quasi-neutral plasmas and magnetized plasmas, it is found that the ratio, $R = \Gamma_u/\Gamma_d$ is approximately proportional to $\exp(Kv_f)$. Therefore we use, as a measure of the flux asymmetry, the quantity $\ln |R|/v_f$, which is effectively the value of $K$.

![Figure 4: Flux density to sphere as a function of angle for various flow speeds at moderate (a) and high (b) potential bias; curves labelled with flow velocity ($v_f$) value.](image)
A systematic study of the flux ratio based on a large number of SCEP-TIC runs is documented in figure 5 as a contour plot of $ln|R|/v_f$ in the plane of probe potential and Debye length. It shows a trough deepening at larger probe potential with minimum as a function of Debye length at about $\lambda_{De} = 0.5$. The $K$-value reverses sign for $-\phi_p > 12$.

Because it decreases the angular averaging, lowering the ion temperature strongly enhances the asymmetry reversal, as illustrated in Figure 6 showing the variation of the $K$-factor with Debye length.

The enhancement of collection on the downstream side is due to focussing of the ions by the potential surrounding the probe. But purely Coulombic ($1/r$) potential variation does not produce the flux asymmetry reversal. It occurs only when there is substantial plasma shielding of the potential. As the flow velocity increases, eventually the reversal disappears, quite abruptly, as the high density wake detaches from the back of the sphere. This detachment is predominantly an orbit effect.

It is probable that different Mach probe geometries might show less susceptibility to the reversal of flux asymmetry. But in view of the results presented here, a reliable Mach probe configuration would certainly need detailed modelling to validate theoretically.

References

