Shape Optimization for DIII-D Advanced Tokamak Plasmas

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The advanced tokamak program on DIII-D is targeting the full integration of high $\beta$ and high bootstrap/non-inductive current fraction for long pulse lengths, and the high confinement consistent with these features. Central to achieving these simultaneously is access to the highest ideal $\beta$ limits possible to maximize the headroom for experimental operation with RWM control. A study of the ideal MHD stability is done for plasmas modeled after DIII-D advanced tokamak plasmas, varying the plasma elongation, triangularity, and outboard squareness. The highest $\beta_N$ limits reach 6-7 for the $n=1$ kink mode for all $\kappa$ and $\zeta_o$ values and $\delta = 0.8$.

Equilibrium and Stability Analysis

In order to examine the benefits of divertor modifications and establish the maximum $\beta_N$ headroom for advanced tokamak plasmas in DIII-D, equilibrium and ideal MHD stability calculations were done varying the elongation, triangularity and outer squareness. Free-boundary equilibria for diverted plasmas, within the DIII-D PF coil and vacuum vessel/limiter boundary, were done to determine the fixed boundary shape parameters that best approximates the actual shapes. Only up-down symmetric double null plasmas were examined. The fixed boundary plasma shapes, to be used in the stability studies, were described by the following,

\[ R = R_o + a \cos(\theta + \sin^{-1} \delta \sin \theta) \]
\[ Z = \kappa a \sin(\theta + \zeta_o, \sin 2\theta) \]

where $\delta$, $\kappa$, $\zeta_o$, and $\zeta_i$ are the plasma triangularity, elongation, outer squareness, and inner squareness, respectively. The free-boundary studies indicated that the inner squareness should be fixed at approximately $-0.25$ to best approximate the plasma boundary on the inboard side and near the x-point. The inner squareness is applied for poloidal angles between 90 and 180 degrees, measured from the outboard midplane, and the outer squareness is applied from 0 to 90 degrees. The plasma major radius is fixed at 1.688 m, the minor radius at 0.587 m, and the toroidal field is 1.85 T.

The current profiles in the advanced tokamak plasmas produced in DIII-D experiments have contributions from the NBI, EC, bootstrap, and inductive current drive. For this study the current profile is taken as fully non-inductive, with the NBBCD at 300 kA and ECCD at 120 kA, and their profiles fixed to approximate those in AT experiments[1]. The bootstrap current is then self-consistently determined in the equilibrium calculation using the Sauter single ion formulation[2]. An example of one of these equilibria is shown in Fig.1.

The density profile is fixed and the temperature profile is determined as the ratio of pressure over density. The model pressure profile is fixed for these studies, with core and pedestal components, the pedestal comprising 10% of the peak pressure value.
The pedestal was modeled with a hyperbolic tangent term and was placed at a normalized poloidal flux of 0.875 with a width in poloidal flux of 0.05:

\[ p(\psi) = p_o [0.6(1 - \tilde{\psi}^{1.5})^{1.5} + 0.3(1 - \tilde{\psi})^{3.5} + 0.1\tanh(\tilde{\psi}_o, \Delta \psi)] \]

The pressure profile is then scaled to generate various \( \beta_N \) values. The plasma current floats since the bootstrap current is being determined consistently with the equilibrium. The plasma current ranges in values from 0.9 to 2.0 MA.

The fixed boundary flux coordinate equilibrium code JSOLVER[3] is used to produce the equilibrium. The input functions are the plasma pressure and the parallel current density from external sources (NB and EC). The grid used is 257 flux surfaces by 257 equal arc theta points, and up-down symmetry is used since only DN plasmas are examined. The high-n ballooning stability is evaluated with BALMSC[4]. The low-n kink stability is examined with PEST2[5], and a conforming ideal wall is assumed at 1.5a measured from the plasma center. The toroidal mode numbers \( n=1-3 \) are evaluated.

**Results and Discussion**

The plasma shaping ranges examined were \( 1.7 \leq \kappa \leq 2.1, 0.5 \leq \delta \leq 0.8, \) and \( 0.0 \leq \zeta_o \leq 0.2 \). Shown in Fig. 2 are the results for \( n=\infty \) ballooning and \( n=1 \) kink modes. The \( n=1 \) kink mode limits increase significantly with triangularity, reaching \( \beta_N \)’s of 6-7. The highest values are obtained with the lowest outboard squareness, as this tends to strengthen the magnetic shear near the plasma edge. Higher plasma elongation in combination with low outer squareness at lower triangularity, and higher outer squareness at higher triangularity maximize the ballooning stability limits. The \( n=2 \) and \( n=3 \) modes set lower \( \beta_N \) limits than for \( n=1 \) over the entire range of shapes. This reflects the theoretical result that with wall stabilization, \( n=1 \) obtains the highest \( \beta_N \), while \( n=2, 3 \) and possibly higher values set progressively lower limits[6]. This effect normally turns around as \( n \) increases, with the wall no longer benefiting higher mode numbers. Their behavior is similar to \( n=1 \), in that these modes prefer higher elongation with high triangularity and low squareness which maximizes the magnetic shear near the plasma edge.

The interpretation of ideal MHD instabilities is not always clearly understood. Since the model is linear it does not describe how the unstable modes will manifest themselves, for example, as a disruption or as enhanced transport. Experimentally high-n ballooning modes do not result in disruptions, although high quality equilibrium reconstructions suggest that in the core the plasma pressure gradient does not exceed the ballooning critical gradient by any significant amount. In addition, the ballooning instability near the edge associated with the pedestal pressure gradient, is considered a primary candidate for ELMs, which do not disrupt the plasma. On the other hand, the \( n=1 \) external kink mode, or the resistive wall mode in the presence of a conducting wall, is disruptive.

The \( n=2 \) and \( n=3 \) modes fall in between, and are not found to be the cause of disruptions. Modes with toroidal numbers of 4 up to high values have been classified as peeling modes due to their compressed mode structure at the plasma boundary.
is possible that the n=2 and 3 modes are peeling modes, although their mode structure appears to extend deeper into the plasma. An interesting observation in ref.[7] is that these modes may ultimately cause neoclassical tearing modes, rather than manifest themselves as large ideal disruptive instabilities. This is due to a rapid positive increase in $\Delta$ as the ideal mode begins to grow, initiating a classical tearing mode. This provides the seed island for a neoclassical tearing mode to grow in a high $\beta$ plasma. This would indicate that comparing plasma shapes that have high n=2 and 3 $\beta_N$ limits to those that have low values, to look for a systematic sensitivity to NTM's.

The experiment routinely obtains $\beta_N$ in the range of 3.1-3.5 in AT plasmas, depending on the pressure peaking and plasma shape. The peak to volume average for this stability analysis is 2.3. Very recent shaping experiments[8] on DIII-D indicate 10-15% improvements in the $\beta_N$ when the elongation and triangularity increase together from 1.8 and 0.5 to 2.0 and 0.8, respectively at fixed $I/a_B$, $q_{\text{min}}$, and pressure peaking.

In summary, a shape study has been performed to examine the impact of plasma elongation, triangularity and outer squareness on the ideal MHD stability of fully non-inductive plasmas modeled after DIII-D advanced tokamak plasmas. The results indicate that higher elongation is uniformly beneficial to all modes, while the combination of triangularity and outer squareness vary between low-n kink modes that prefer high edge magnetic shear and high-n ballooning modes that prefer the opposite. Although the n=1 mode can access high $\beta_N$ limits in the presence of a wall, the n=2 and 3 modes appear to be setting lower limits. However, how they manifest themselves in limiting access to high $\beta$ in the plasma is not understood.

Figure 1. An example of the model DIII-D AT plasmas in this study showing the equilibrium profiles and plasma shape, this particular plasma reached a stable $\beta_N$ of 3.65.
Figure 2. Maximum $\beta_N$ for high-n ballooning and n=1 external kink modes as a function of plasma elongation, triangularity, and outboard squareness.

Figure 3. Maximum $\beta_N$ for n=2 and 3 external kink modes as a function of plasma elongation, triangularity, and outboard squareness.

Work supported by U.S. DOE contracts DE-AC02-76CH03073 and DE-AC03-99ER54463

References